Sant'Anna Pisa, July 2009

Alternative approaches to Macroeconomic and Micro-Macro Links

M. Lippi, Macroeconometrics: Beyond Demand and Technology Shocks

Plan of the talk

1. Microfoundations of macroeconomics, I

1.1 Rational Expectations—Representative Agent

1.2 Aggregation(s)

2. Microfoundations of macroeconomics, II

2.1 VAR models. One-step-ahead prediction error

2.2 Estimated representation and SVAR

2.3 Relative importance of technology and demand

2.4 The RBC, Blanchard and Quah, etc., Gali

Plan of the talk

3. Two problems with SVAR analyis

3.1 Fundamentalness

3.2 Aggregation.

3.3 On Dahlem report

4. A possible way out: Dynamic factor models

4.1 A restricted model

4.2 The assumption of fundamentalness. A strong motivation.

4.3 Still problems remain

1. Rational Expectations and Representative agent

Closing the model. Agents are not predicting nature but a system which includes themselves.

–Heterogeneity impossible (extremely difficult): otherwise agents would have to predict other agents' decisions (learning, convergence).

–As a consequence aggregation is only seldom mentioned.

Moreover, we should recall all these systems never mention crucial problems that arise form heterogeneity:

–capital, consumers and stability.

When I say "Aggregations" I also refer to temporal aggregation and measurement errors. Wrong idea that aggregate data are micro data aggregated.

2. Microfoundations II.

VAR analysis.

You start with

$$(I - A_1 L - \cdots - A_p L^p)\mathbf{x}_t = \mathbf{u}_t$$

The next step is

$$\mathbf{x}_t = (I - A_1 L - \cdots - A_p L^p)^{-1}\mathbf{u}_t = (I + B_1 L + B_2 L^2 + \cdots)\mathbf{u}_t = B(L)\mathbf{u}_t$$

Then

$$\mathbf{x}_t = [B(L)G]\left[G^{-1}\mathbf{u}_t\right] = C(L)\mathbf{v}_t$$

with the matrix G determined by economic theory, institutional settings, common sense, etc.

2. Microfoundations II.

As an example, consider the debate on the relative importance of technology shocks and demand shocks, beginning in the eighties with I(1) variables, Nelson and Plosser, Beveridge and Nelson, the RBC. Everything begun with the fall of deterministic trends,, so that what is permanent and what is transitory is no longer clear.

After a decade of discussions Blanchard and Quah proposed a SVAR that became the standard. The matrix G was obtained by imposing that

1. Technology and demand are orthogonal
2. Only technology has a permanent effect on output.

3. Microfoundations. Fundamentalness

However, we know that there exist moving average representations of $\mathbf{x}_t$ that are not fundamental. In other words, why do we start with $\mathbf{x}_t = B(L)\mathbf{u}_t$ to introduce our economic theory considerations ? Why don't we start with any other moving average representation ?

Consider

$$y_t = x_t - x_{t-1} = p_0 w_t + p_1 w_{t-1}, \quad p_0 + p_1 = 1$$

where w_t is a white noise representing shocks to technical progress, while x_t is the log of productivity. Thus p_0 is the fraction of the shock w_t which becomes an increase in productivity during period t, while p_1 is the fraction of w_t which becomes an increase in productivity during period $t+1$.

Rewrite the process in standard form

$$y_t = (p_0 w_t) + \frac{p_1}{p_0}(p_0 w_{t-1}) = z_t + \frac{p_1}{p_0} z_{t-1} = (1 + \alpha L) z_t$$

Now, $\alpha < 1$ iff $p_0 > p_1$, which is not necessarily the case. If $\alpha > 1$ then z_t is not fundamental for y_t.

$$y_t = (1 + \alpha L) z_t$$

Now, $\alpha < 1$ iff $p_0 > p_1$, which is not necessarily the case. If $\alpha > 1$ then z_t is not fundamental for y_t. If that is the case

$$z_t = \frac{1}{1 + \alpha L} y_t = \frac{F}{F + \alpha} y_t = \frac{\alpha^{-1} F}{1 + \alpha^{-1} F} y_t = \alpha^{-1} y_{t+1} + \alpha^{-2} y_{t+2} + \cdots$$

Thus z_t belongs to the space spanned by future values of y, not present and past.

If you want the fundamental representation of y_t:

$$y_t = (1 + \alpha L) z_t = (1 + \alpha^{-1} L) \left[\frac{1 + \alpha L}{1 + \alpha^{-1} L} z_t \right] = (1 + \alpha^{-1} L) \zeta_t$$

But of course the fundamental and the structural representation do not coincide.

3. Microfoundations. Fundamentalness

In general there is no reason to believe that the white noise emerging from the solution of the prediction problem has a structural interpretation. In our case further information is necessary.

3. Microfoundations. Fundamentalness

Consider the following model (D. Quah)

$$
\begin{aligned}
y_t - y_{t-1} &= a(L)u_t + (1-L)b(L)v_t \\
c_t - c_{t-1} &= a(\beta)u_t + (1-\beta)b(\beta)v_t
\end{aligned}
$$

where y_t and c_t are income and consumption respectively, $\beta = 1/(1+r)$. Consumption is determined according to life-cycle-permanent-income, the agents observing the two components of income. Then the matrix

$$
\begin{pmatrix} a(L) & (1-L)b(L) \\ a(\beta) & (1-\beta)b(\beta) \end{pmatrix}
$$

has a root for $L = \beta$, wich is a value smaller than unity in modulus, thus the wrong side of the unit circle. In this case the structural shocks are non fundamental. On the other hand, the econometrician, who uses a VAR for he observables y_t and c_t, does not recover u_t and v_t.

3. Microfoundations. Fundamentalness

Start with the fundamental representation

$$A(L)\mathbf{x}_t = \mathbf{u}_t, \quad \mathbf{x}_t = B(L)\mathbf{u}_t$$

You get all possible MA representations by inserting functions

$$\mathbf{x}_t = [B(L)G(L)]\,[G'(F)\mathbf{u}_t]$$

under the condition $G(e^{-i\theta})G'(e^{i\theta}) = I$ for a.a. $\theta \in [-\pi\ \pi]$.

For example, in the univariate case

$$\mathcal{B}_a(z) = \frac{\bar{a}}{|a|}\frac{a-z}{1-\bar{a}z} \quad a < 1$$

which is called a Blaschke factor and

$$\prod_{i=1}^{n} \mathcal{B}_{a_i}(z)$$

which is called a Blaschke product.

In the multivariate case you can take diagonal Blaschke matrices, scramble them using invertible matrices

$$\begin{pmatrix} \mathcal{B}_{a_1}(z) & 0 & \cdots & 0 \\ 0 & \mathcal{B}_{a_2}(z) & \cdots & 0 \\ & & \ddots & \\ 0 & 0 & \cdots & \mathcal{B}_{a_2}(z) \end{pmatrix} M$$

make products etc.

3. Microfoundations. Fundamentalness

Fundamentalness has great potential importance. The fundamentalness problem implies that there is room for further information when we try to achieve identification. What you know on the diffusion of technical progress, for example, might be used.

3. Microfoundations. Aggregation

Suppose the variables x_{it} are I(1) and that

$$y_{it} = \alpha_i x_{it} + u_{it}$$

with u_{it} stationary. This means that each of the y's is cointegrated the corresponding x. Now, what about the aggregates

$$Y_t = \sum y_{it}, \quad X_t = \sum x_{it}$$

Are they cointegrated ? The anwer is NO, unless the coefficients α are equal (this is not completely accurate but almost).

My conclusion is that cointegration between macrovariables is not the consequence of microcointegration.

3. Microfoundations. Aggregation

By the way, I did not find the cointegration section of the Dahlem report very convincing. I do not know that cointegration has interpretations that go beyond the representative agent. As you know, when y_t and x_t are cointegrated, Granger representation theorem has that an ECM links them

$$\Delta y_t = a(L)\Delta x_t + \beta(y_{t-1} - \gamma x_{t-1}) + r_t.$$

This is a mathematical result. But if cointegration of the aggregate leads you to suppose that agents are using an ECM, then you are assuming a representative agent. In conclusion, there is nothing in cointegration as such that leads us out of the mainstream paradigm.

However, cointegration people are bravely fighting against bad econometric practice.

Aggregation and VARs. Consider

$$\begin{pmatrix} \Delta y_t \\ U_t \end{pmatrix} = \begin{pmatrix} a_{11}(L) & a_{12}(L) & b_{11}(L) & b_{12}(L) \\ a_{21}(L) & a_{22}(L) & b_{21}(L) & b_{22}(L) \end{pmatrix} \begin{pmatrix} u_{1t} \\ u_{2t} \\ v_{1t} \\ v_{2t} \end{pmatrix}$$

This is Blanchard-Quah, but we have 2 technology and two demand shocks. On the other hand

$$\begin{pmatrix} \Delta y_t \\ U_t \end{pmatrix} = \begin{pmatrix} A_{11}(L) & A_{12}(L) \\ A_{21}(L) & A_{22}(L) \end{pmatrix} \begin{pmatrix} W_t \\ Z_t \end{pmatrix}$$

is what we would get by following the standard procedure.

Problem. Is W_t influenced by the micro demand shocks? YES, unless very special conditions on the polynomials $a_{ij}(L)$ are fulfilled.

4. Dynamic Factor Models

$$\begin{aligned} x_{it} &= \chi_{it} + \xi_{it} = \mathbf{b}_i(L)\mathbf{u}_t + \xi_{it} \\ &= b_{i1}(L)u_{1t} + b_{i2}(L)u_{2t} + \cdots + b_{iq}(L)u_{qt} + \xi_{it}. \end{aligned}$$

– for $i = 1, 2, \ldots, n$; typically n is huge; consistency results are obtained for T, the number of observations for each series, and n, tending to infinity

– q is very small as compared to n in empirical applications

– χ_{it} are the common components; u_{jt} the common shocks; the vector $\mathbf{u}_t$ is an orthonormal white noise

– ξ_{it} are the idiosyncratic components; $\xi_{it} \perp u_{j\tau}$ for all $i,\ j,\ t,\ \tau$

– the filters $b_{ij}(L)$ are square summable

– the vectors $\boldsymbol{\chi}_{nt} = (\chi_{1t} \ \cdots \ \chi_{nt})$ and $\boldsymbol{\xi}_{nt} = (\xi_{1t} \ \cdots \ \xi_{nt})$ are stationary

$$\chi_{it} = b_{i1}(L)u_{1t} + b_{i2}(L)u_{2t} + \cdots + b_{iq}(L)u_{qt}$$

We assume that the common components are pervasive. Suppose for example that

$$\chi_{it} = b_i u_t.$$

Then pervasiveness means that

$$\sum_{i=1}^{\infty} b_i^2 = \infty.$$

Idiosyncratic components

Strictly idiosyncratic

$$\xi_{it} \perp \xi_{j\tau} \quad \text{for all } i,\, j,\, i \neq j,\, t,\, \tau.$$

We do not need so much. Let $\Sigma_n^{\xi}(\theta)$ be the spectral density of $\boldsymbol{\xi}_{nt}$. We assume that $\lambda_{1n}^{\xi}(\theta) < \Lambda$ for all n.

Local correlations among idiosyncratic variables are allowed.

Estimation. Example (all you need to know to understand everything)

Assume the elementary example

$$x_{it} = b_i u_t + \xi_{it}.$$

Take the average

$$\frac{1}{n}\sum_{i=1}^{n} x_{it} = \left(\frac{1}{n}\sum_{i=1}^{n} b_i\right) u_t + \frac{1}{n}\sum_{i=1}^{n} \xi_{it}$$

The variances are

$$\mathrm{var}\frac{1}{n}\sum_{i=1}^{n} x_{it} \leq \left(\frac{1}{n}\sum_{i=1}^{n} b_i\right)^2 \sigma_u^2 + \frac{1}{n^2} n \max_i \mathrm{var}\xi_{it} = \bar{b}_n^2 \sigma_u^2 + \frac{1}{n} M$$

Thus the average of the x's converges in mean square to u_t.

Example (continued)

Back to

$$\mathrm{var}\frac{1}{n}\sum_{i=1}^{n} x_{it} \leq \left(\frac{1}{n}\sum_{i=1}^{n} b_i\right)^2 \sigma_u^2 + \frac{1}{n^2} n \max_i \mathrm{var}\xi_{it} = \overline{b}_n^2 \sigma_u^2 + \frac{1}{n} M$$

What if

$$\overline{b}_n \to 0$$

This problem is solved using principal components of the x's

4. Dynamic Factor Models: Fundamentalness

Let us stick to rational spectral density and rational representations.

Suppose $\mathbf{x}_t$ is n-dimensional, has rational spectral density $f(\theta)$ and that rank$f(\theta) = n$ for a.a. $\theta \in [-\pi\ \pi]$. We say that $\mathbf{x}_t$ is full rank. Let

$$\mathbf{x}_t = B(L)\mathbf{u}_t \qquad (*)$$

be a rational MA representation. Then $(*)$ is fundamental iff $\det B(z) = 0$ has no roots of modulus smaller than unity. All other rational representations are obtained as

$$\mathbf{x}_t = [B(L)\mathcal{M}(L)]\,[\mathcal{M}'(F)\mathbf{u}_t]$$

where $\mathcal{M}$ is a Blaschke matrix.

If the representation

$$\mathbf{x}_t = C(L)\mathbf{v}_t \qquad (**)$$

is non-fundamental, there exist a neighborhood of $(**)$ in which representations are non-fundamental. Indeed, $\det C(z) = 0$ has roots of modulus smaller than unity.

3. Rational spectral density. Rational representations. Non-full rank, tall systems

Now suppose that rank $f(\theta) = q < n$ for a.a. $\theta \in [-\pi \ \pi]$. In this case the rational representations can be expressed in the form

$$\mathbf{x}_t = B(L)\mathbf{u}_t \qquad (*)$$

where $B(L)$ is $n \times q$ and $\mathbf{u}_t$ is q dimensional (tall systems). Representation $(*)$ is fundamental iff rank$(B(z) = q$ for $all\ z$ such that $|z| < 1$. Consider the example

$$\begin{aligned} x_t &= au_t + bu_{t-1} \\ y_t &= cu_t + du_{t-1} \end{aligned}$$

Elementary algebra gives

$$u_t = \frac{1}{ad - bc}(dx_t - by_t)$$

so that u_t is fundamental unless $ad - bc = 0$. This suggests that in the tall rational case $(*)$, $\mathbf{u}_t$ is generically fundamental.

4. Dynamic factor models. Fundamentalness

The common components of the dynamic factor model are

$$\boldsymbol{\chi}_{nt} = A_n(L)\mathbf{u}_t$$

A common restriction is that

$$A_n(L) = B_n N(L)$$

where A_n is $n \times r$, $N(L)$ is $r \times q$, $r > q$, so that

$$\boldsymbol{\chi}_{nt} = B_n \mathbf{F}_t$$

where

$$\mathbf{F}_t = N(L)\mathbf{u}_t$$

is a vector of static factors.

4. Dynamic factor models. Fundamentalness

Rewrite

$$\mathbf{F}_t = N(L)\mathbf{u}_t \qquad (*)$$

Usually we assume that $(*)$ can be approximated by an autoregressive model like

$$\mathbf{F}_t = S\mathbf{F}_{t-1} + \mathbf{u}_t,$$

that is

$$N(L) = (I - SL)^{-1}$$

But if $\mathbf{u}_t$ has structural interpretation this implies that we believe that the structural shocks are fundamental for $\mathbf{F}_t$ and therefore for χ_{nt}. This can be convincingly motivated in a tall system.

4. Dynamic factor models. Aggregation

Back to

$$\begin{pmatrix} \Delta y_t \\ U_t \end{pmatrix} = \begin{pmatrix} a_{11}(L) & a_{12}(L) & b_{11}(L) & b_{12}(L) \\ a_{21}(L) & a_{22}(L) & b_{21}(L) & b_{22}(L) \end{pmatrix} \begin{pmatrix} u_{1t} \\ u_{2t} \\ v_{1t} \\ v_{2t} \end{pmatrix} = \begin{pmatrix} A_{11}(L) & A_{12}(L) \\ A_{21}(L) & A_{22}(L) \end{pmatrix} \begin{pmatrix} W_t \\ Z_t \end{pmatrix}$$

If Δy_t and U_t are just to variable out of a huge macroeconomic dataset, then we can estimate the four shocks of the structural disaggregated model.

4. Doubts, problems, Dahlem report

– What if agents do not use shocks but macrovariables.

– Here we have many possibly heterogeneous agents, not interacting agents. Can we say that part of macroeconomics only needs heterogeneity of agents, not necessarily interacting agents ? I am sure that financial markets cannot be analysed without interacting agents. Is this true for the whole macroeconomics ?

– The Dahlem report has a tone as though everything has to be rebuilt from scratch. But we read these days very persuasive analyses of the crisis that are based on stylized representations of the capitalistic economy. Marx and Keynes, Kalecki, gave impressive analyses of capitalism with poor formalization.

– Microfoundations, stylized representation, narrative,