Testing Exchange Rate Models Based on Rational Expectations versus Imperfect Knowledge Economics: A Scenario Analysis*

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June 29, 2010

Abstract

It is often argued that model based expectations are needed to ensure theoretical consistency of economic models. This paper argues that empirical consistency of basic theoretical assumptions is even more important. This entails carefully matching the basic assumptions underlying the theoretical model with the empirical regularities of the data as structured by a statistically adequate model. Since unit root nonstationarity is endemic in economic data, the paper argues that a correctly specified Cointegrated VAR model is likely to work well as a first statistical approximation. Within this model all basic assumptions on the model's shock structure and steady-state behavior can be formulated as testable hypotheses on common stochastic trends and cointegration in what is called 'a theory-consistent CVAR scenario'. As it allows us to test competing models, its systematic use is likely to enhance our ability to develop empirically relevant models. The scenario idea is illustrated by comparing two types of models for exchange rate determination, one relying on the rational expectations hypothesis and the other on the theory of imperfect knowledge economics hypothesis.

Keywords: Theory-Consistent CVAR, Imperfect Knowledge, Rational Expectations, International Puzzles, Long Swings, Persistence.

JEL Classification: F31, F41, G15, G17

*I am grateful to Mikael Juselius for detailed and insightful comments which have significantly improved this paper. Valuable comments from Roman Frydman, Michael Goldberg, Søren Johansen, and are gratefully acknowledged.

1 Introduction

International macroeconomics is known for its many empirical puzzles: the PPP puzzle, the long swings puzzle, the exchange rate disconnect puzzle, and the forward premium puzzle (Rogoff, 1996). Common for these puzzles is the fact that standard international parity conditions such as Purchasing Power Parity (PPP), Uncovered Interest rate Parity (UIP), and real interest rate differentials deviate from parity with a pronounced persistence. This feature seems hard to reconcile with the assumption of stationary deviations from steady state typical of most exchange rate models based on the Rational Expectations Hypothesis (REH). While it is often argued that model based expectations are needed to ensure theoretical consistency, this paper argues that empirical consistency is even more important, requiring as a minimum that the model can account for the persistence inherent in data. To analyze such persistent movements in the data adequately, an econometric formalization of the concept of persistence is needed.

This paper argues that a measure of persistence could be meaningfully related to the number of (near) unit roots in the characteristic polynomial of a Vector Autoregressive (VAR) model and be used as a structuring device associating variables/relations with a similar persistency profile. The paper also argues that such a procedure is likely to be informative about the underlying causes of the puzzling persistence and, therefore, about a potential explanation.

Real exchange rate dynamics have been modelled theoretically with sticky-price monetary models, such as the over-shooting model of Dornbush (1976) and Dornbush and Frankel (1988) and its new open economy formulations (see Lane, 2001), all of them based on the Rational Expectations Hypothesis. In addition to fairly fast equilibrium adjustment, these models also assume the same speed of adjustment in prices and exchange rates. Cheung, Lai and Bergman (2004) has shown that the latter feature is strongly against the empirical evidence. Benigno's (2004) endogenous money model is able to produce a near unit root persistence in the real exchange rate by allowing the speed of adjustment to be slower for the nominal exchange rate than for relative prices, but the model contains other features which seem empirically questionable. See Frydman and Goldberg (2007) and Frydman et al. (2008).

The question to be addressed here is whether these models have been able to solve the puzzles, i.e. whether they satisfy empirical consistency. The REH-based models are usually taken to the data using calibration and

Bayesian priors, restricting attention to a few specific features of the theoretical model which are then tested. But such tests only make sense if the assumed structure of the economic model is correct, and the results can easily change if tested in the context of a fully specified statistical model. Spanos (2009) forcefully argues that econometric procedures are valid *only* to the extent that the probabilistic assumptions constituting the underlying statistical model are satisfied vis-a-vis the data in question[1].

Hence, a convincing test of the empirical relevance of a theoretical model has to be carried out in the context of a fully specified statistical model that works as an adequate description of the Data Generating Process (DGP). I shall argue that the VAR model is such a statistical model describing a multivariate, dynamic and stochastic DGP. Its probabilistic assumptions are testable and, when correctly specified, the VAR essentially represents the covariance information of the data. The link between the theory model and the statistical model can be established by formulating assumptions of the theoretical model's shock structure and steady-state behavior as testable hypotheses on common stochastic trends and cointegration in a cointegrated VAR (CVAR) model. Such a set of testable assumptions is called a theory-consistent CVAR scenario (Juselius, 2006, Juselius and Franchi, 2007, Møller, 2007).

The idea of a CVAR scenario is to test the empirical consistency of basic underlying assumptions rather than imposing them on the data from the outset. For example, the number of autonomous shocks should be tested rather than assumed, the stationarity of a steady-state relation should be tested rather than assumed, and so on. One could say that a theory-consistent CVAR scenario describes a set of empirical regularities we should expect to see in the data if the basic assumptions of the theoretical model are empirically valid. Checking whether this is the case can be seen as a safeguard against testing internally inconsistent hypotheses. Because a properly done scenario analysis makes it easier to discriminate between competing models, such checking is also likely to enhance our ability to develop empirically relevant models.

For example, the use of scenario analysis would allow us to compare the empirical relevance of REH-based models with other expectational models

[1] When testing REH-based models in the context of a statistically fully specified model, they have often been rejected. See for example the articles in the special issue "Using Econometric for Assessing Economic Models" (Juselius, 2009).

such as imperfect knowledge economics (IKE) based models (Frydman and Goldberg, 2007, 2008, 2009). These models assume that agents are endowed with imperfect knowledge about the correct model to be used when forecasting future outcomes in financial markets. A key implication of such IKE behavior is that asset prices are likely to exhibit persistent movements away from and towards long-run benchmark values. Thus, an IKE-based model for the foreign exchange rate could be potentially relevant for explaining the typical long swings in real and nominal exchange rates characterizing a currency float.

To illustrate the scenario procedure, I shall first translate the basic assumptions underlying some REH-based monetary models for exchange rate determination into a set of testable assumptions on a CVAR model and then do the same for an IKE based monetary model. This exercise shows that one important testable difference is associated with the degree of real exchange rate persistence which is one degree higher under IKE than under REH: the latter models would predict that the persistent swings in real exchange rates are econometrically stationary or (at most) near $I(1)$, due for example to endogenous central bank reactions; whereas the former that it is near $I(2)$, due to financial markets demanding an uncertainty premium for holding foreign currency as a compensation for risky forecasting strategies under imperfect knowledge.

The derived scenarios are tested against a data set describing US - German exchange rate, prices and interest rates in the post Bretton Woods currency float period. The empirical analysis showed that the REH-based theory-consistent scenario was empirically rejected on essentially all counts, whereas the IKE-based scenario obtained a remarkable support for every single testable hypothesis.

The paper is organized as follows. Section 2 discusses persistence based on the notion of $I(1)$ type and $I(2)$ type stochastic trends and Section 3 how to use the CVAR model as a structuring device. Section 4 proposes some rules for associating expectations with observables and suggests a procedure for formulating a theory-consistent CVAR scenario. Section 5 demonstrates how to formulate such a scenario for two types of REH-based monetary models and Section 6 for an IKE-based model. Section 7 introduces the empirical $I(2)$ model and Section 8 presents the empirical results. Section 9 contains a summary and a conclusion and Section 10 discusses further implications of the results.

2 Using unit roots as a structuring device for persistence

The notion of persistence is generally associated with the strength of the time dependence of a shock to a variable. If the effect of a shock dies out quickly it is considered transitory and the corresponding variable is stationary whereas if the shock has a lasting effect it is considered permanent and the variable is called unit root nonstationary. Distinguishing broadly between transitory (stationary) and persistent (nonstationary) behavior is, however, not sufficient for the purpose of classifying data according to their different persistency profiles. For example, stationary processes can be divided into highly erratic $I(-1)$ processes and $I(0)$ processes, both of which describe transitory behavior. Nonstationary unit root processes can be generated from shocks which cumulate once, dubbed $I(1)$; or from shocks which cumulate twice, dubbed $I(2)$.[2]

While such a classification is mathematically unambiguous, it can be more problematic from the point of view of empirical persistence. Depending on the sample size, the degree of permanence, and the relative noise ratio of $I(1)$ and $I(2)$ components, there are often grey zones where data could be said to be near $I(1)$ rather than $I(1)$ or $I(0)$, and near $I(2)$ rather than $I(1)$ or $I(2)$. For example, a random walk process, $x_t = x_{t-1} + \varepsilon_t$, i.e. an $I(1)$ process, and a strongly autoregressive AR(1) process, $x_t = 0.95x_{t-1} + \varepsilon_t$, (mathematically an $I(0)$ process) would often be difficult to distinguish from each other even based on relatively long samples. This is illustrated in Figure 1 where an AR(1) with $\rho = 0.95$ and a random walk are simulated in 200 steps. Both series are seemingly characterized by similar persistence. For a short time series, the difficulty to discriminate between near unit root and unit root processes becomes even more pronounced. This is illustrated in Figure **??** for a stationary AR(1) process with autoregressive parameter $\rho = 0.80$ and a random walk process simulated in 50 steps. In contrast, an AR(1) with $\rho = 0.99$, say, would often be found significantly different from one in a large sample of 5000 observations even though the variable/relation would exhibit very pronounced persistence. Characterizing it as type $I(0)$ would, however, imply that we refrain from using cointegration techniques to find a similar persistent trend in another variables in the VAR model.

[2]See Johansen (1996) for a mathematically precise definition of the order of integration of stochastic processes.

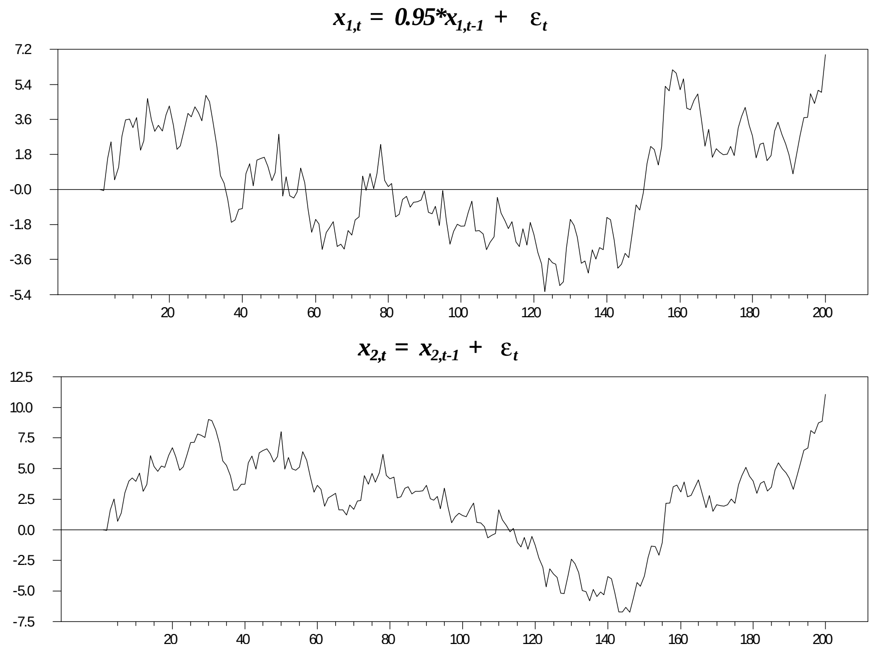

Figure 1: The simulated series of an AR(1) process with $\rho = 0.95$ (upper panel) and a random walk (lower panel).

Because cointegration between two variables implies they are sharing the same persistent shocks, it is a powerful tool to identify causal links. This, of course, should be exploited as much as possible. Thus, statistical significance cannot stand alone as a good organizing principle for classifying data into different persistence profiles. See Hendry and Juselius (2000) for a discussion.

Therefore, I argue in this paper that the notion of persistence can be more meaningfully discussed in terms of the (modulus of) characteristic (eigenvalue) roots of the autoregressive polynomial which for non-explosive models are defined in the interval (-1, 1). Such roots can be given a convenient interpretation as a measure of the speed of adjustment. As an example, consider the simple AR(1) model, $x_t = \rho_1 x_{t-1} + \varepsilon_t$ or equivalently $\Delta x_t = -(1 - \rho_1)x_{t-1} + \varepsilon_t$. Assume a root $\rho_1 = 0.9$ corresponding roughly to an adjustment coefficient $\alpha_1 \simeq -(1- \rho_1) = -0.10$. An adjustment coefficient of -0.10 corresponds to a half life of $ln(2)/0.10 = 7$ periods. With annual data this would imply an average adjustment period of 7 years, with

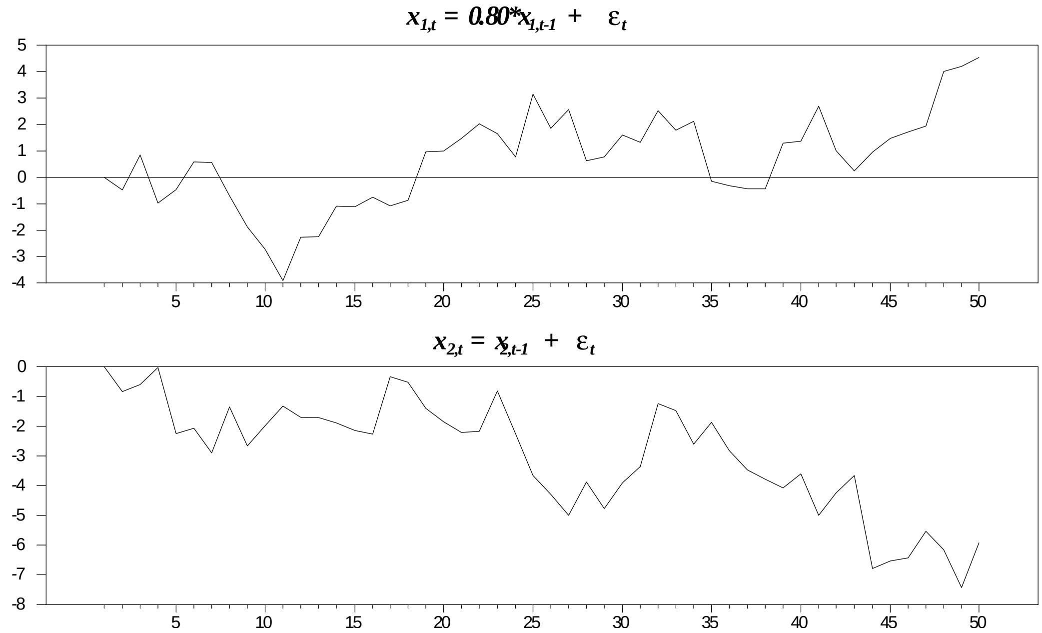

Figure 2: A simulated AR(1) process with $\rho = 0.8$ (upper panel) and a random walk (lower panel).

quarterly data it would imply almost 2 years, with monthly data slightly more than half a year, with weekly data less than 2 months, etc. Whether a characteristic root can be interpreted as evidence of persistent behavior or not depends both on the sample period and the observational frequency.

To illustrate the idea, consider a variable, x_t, which has the following autoregressive representation $(1 - \varphi_1 L - \cdots - \varphi_p L^p)x_t = \varepsilon_t$ where ε_t is $Niid$ and a threshold parameter, ρ^*, above which the process is considered persistent. The choice of ρ^* is to some extent subject to judgement. With high frequency data its value would generally be closer to the unit circle than with low frequency data. In the context of a specific theory, ρ^* could in some cases be thought of as defining the longest adjustment time for which the policy implications of the model are still useful.

The persistence of x_t could for example by defined as:

- $I(0)$ type when the modulus of the largest root, ρ_1, satisfies $\rho_1 < \rho^*$.
- $I(1)$ type when the modulus of the largest root, ρ_1, satisfies $\rho^* < \rho_1 \leq$

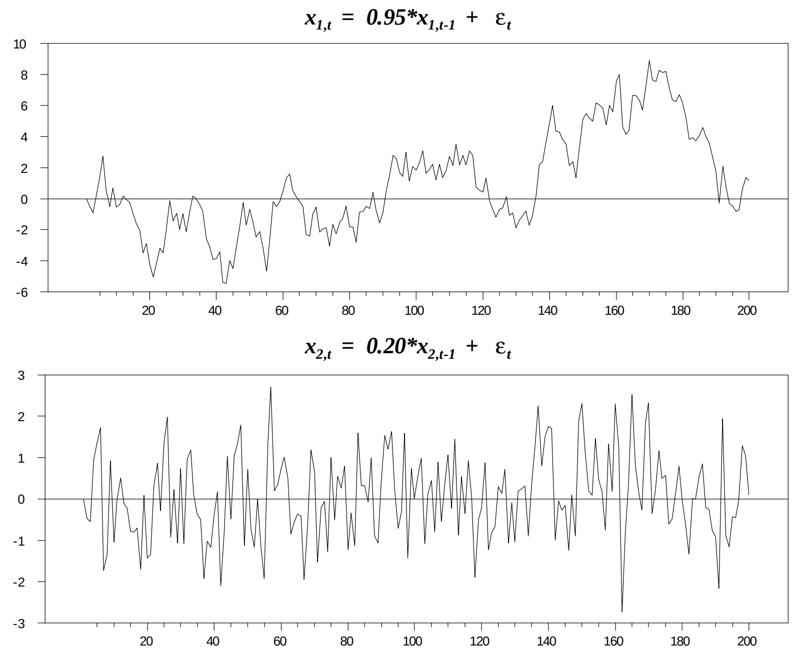

Figure 3: The graphs of an AR(1) process with $\rho = 0.95$ (upper panel) and with $\rho = 0.20$ lower panel.

1.0 and the next root $\rho_2 < \rho^*$.

- $I(2)$ type when the modulus of the largest root, $\rho_1 = 1.0$, and the next one satisfies $\rho^* < \rho_2 \leq 1.0$.

While the above classification is defined for a univariate model, the CVAR analysis is defined for a multivariate model and the persistency classification needs to be extended to this case. There are some important differences: In a univariate model a large characteristic root can be directly associated with the variable in question, $x_{i,t}$, whereas in a p-dimensional VAR model of $x_t' = [x_{1,t}, ..., x_{p,t}]$, the number of large roots in the characteristic polynomial depends on the number of common stochastic trends pushing the system, $p - r$, where r is the number of cointegration relations, and whether they are of first order, s_1, or second order, s_2 where $s_1 + s_2 = p - r$. Consider for example a five-dimensional VAR model in which 3 of the characteristic roots are greater than ρ^*. This can be consistent with three stochastic trends of first order ($p-r=3, s_1=3$), or with two stochastic trends of first order and second order ($p-r=2, s_1=1, s_2=1$). Thus, to be able to determine the order of persistence of the variables and the relations, the order of integration type of the vector process has to be determined as well as the number of stochastic trends of order type $I(1)$ and $I(2)$. The reason for distinguishing between these two types is not just because they are frequently observed in data, but also because the most crucial difference between REH-based and IKE-based models can be formulated in terms of the number of $I(1)$ versus $I(2)$ trends in the VAR. This will be shown subsequently in sections 5 and 6.

While it is not straightforward to distinguish between $I(1)-$ and $I(2)-$type of persistence based on the characteristic roots, a simple procedure based on a combination of counting large roots and testing can be envisaged. For this purpose, a maximum likelihood test procedure is readily available (Nielsen and Rahbek, 2007), though the peril of relying exclusively on significance testing without considering the effect of the sample size is equally relevant for the multivariate as for the univariate case. A simple procedure could for example be to start with the unrestricted VAR model ($r=p, s_1=0, s_2=0$) and determine the number of characteristic roots $\geq \rho^*$[3] and then study those cases (r, s_1, s_2) for which the number of unit roots in the characteristic polynomial is equal to m^*. Test the relevant combinations with the trace test. An empirically relevant candidate is found when the trace test is not rejected, all unrestricted characteristic roots $\rho_i < \rho^*$, and the number of restricted unit roots is m^*.

Another important issue is whether the probability theory for $I(1)$ and $I(2)$ models can be used for near unit root approximations. In this case, Elliot (1998) shows that the asymptotic distribution changes to some extent so that the near unit root distribution falls between the unit root (T consistency) and the stationary ($\sqrt{T}$ consistency) distribution. An important question is whether this effect on the asymptotic inference is large in finite samples. For example, Johansen (2006b) shows with simulations that some inference become very fragile if near unit roots are treated as stationary in moderately sized samples. Up to 5000 observations were needed for the em-

[3] Note that if a large modulus root corresponds to a complex pair with a significant imaginary part it is not possible to force it to become a unit root on the real line. In this case, it will be considered a stationary, albeit persistent, cyclical component. Also, Nielsen and Nielsen (2009) has shown ´that if the VAR model is estimated with many lags (for example adding lags to compensate for a structural break) the number of large, but insignificant, roots will increase. In such a case, the number becomes uninformative.

pirical distribution to converge to Students t when the root was very close to the unit circle.

An even more important issue is whether it at all make sense to associate an $I(1)$-type process (say with a root of 0.95) with an $I(0)$-type process (say with a root less than 0.2). As illustrated in Figure 3 such processes display very different persistency profiles in contrast to the graphs in Figure 1 and **??**. It seems futile from the outset to associate an $I(1)$-type persistent variable with an $I(0)$-type variable, whereas two $I(1)$-type variables may very well share a common trend and, thus, be cointegrated (which is testable). Thus, structuring the data according to their persistence profiles is likely to be helpful for uncovering empirical regularities that originate from the same kind of persistent shocks.

This very intuitive and simple idea will be further exploited and shown to be able to distinguish between different basic assumptions underlying competing theoretical models. The next section discusses how to structure different type of persistence using the CVAR model.

3 Structuring persistence using the CVAR

By its ability to exploit persistency properties in the data, the CVAR model offers a natural way of analyzing economic data as short-run variations around moving long-run equilibria. Long-run forces are themselves divided into the forces that move the equilibria (pushing forces, which give rise to stochastic trends) and forces that correct deviations from equilibrium (pulling forces, which give rise cointegrating relations). Interpreted in this way, the CVAR has a good chance of nesting a multivariate, path-dependent data-generating process and relevant dynamic macroeconomic theories. One could say that the CVAR model gives the data a rich context in which they are allowed to speak freely (Hoover et al., 2008). See also Framroze-Møller (2008) for a detailed exposition of how to translate basic concepts of macroeconomic models into testable concepts of the CVAR model.

To introduce notation and the basic idea of structuring the data into pulling and pushing forces, I shall use a simple 3-dimensional VAR model for $x_t' = [x_1, x_2, x_3]$, where the variables for example could be the nominal exchange rate and domestic and foreign prices. The model distinguishes between $p - r$ pushing and r pulling forces. I assume that $(r = 1, p - r = 2)$ and begin with the $I(1)$ model.

The pulling forces are formulated as the equilibrium error correction model, $\Delta x_t = \alpha\beta' x_{t-1} + \varepsilon_t$, i.e as:

$$\begin{bmatrix} \Delta x_{1,t} \\ \Delta x_{2,t} \\ \Delta x_{3,t} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} \beta' x_{t-1} + \cdots + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix}$$

where $\beta' x_t$ is an equilibrium error and α_i is an adjustment coefficient describing how the system adjusts back to equilibrium when it has been pushed away. The pushing forces are described by the common trends formulation $x_t = \beta_\perp \Sigma u_i + \varepsilon_t$, i.e. by:

$$\begin{bmatrix} x_{1,t} \\ x_{2,t} \\ x_{3,t} \end{bmatrix} = \begin{bmatrix} \beta_{\perp,11} & \beta_{\perp,21} \\ \beta_{\perp,12} & \beta_{\perp,21} \\ \beta_{\perp,13} & \beta_{\perp,21} \end{bmatrix} \begin{bmatrix} \sum_{i=1}^{t} u_{1,i} \\ \sum_{i=1}^{t} u_{2,i} \end{bmatrix} + ... + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix}$$

where $u_{1,t} = \alpha'_{\perp,1}\varepsilon_t$ and $u_{2,t} = \alpha'_{\perp,2}\varepsilon_t$ are two autonomous common shocks that cumulate over time. $\alpha_\perp = [\alpha_{\perp,1}, \alpha_{\perp,2}]$, is a 3×2 matrix, orthogonal to α, defining the two common shocks as linear combination of the VAR residuals $\hat{\varepsilon}_t$. $\beta_\perp = [\beta_{\perp,1}, \beta_{\perp,2}]$, is a 3×2 matrix orthogonal to β measuring how the two stochastic trends load into the variables.

The $I(2)$ model has a more complicated structure. The vector x_t is now integrated of order 2 and the $p - r$ stochastic trends are divided into s_1 first order and s_2 second order stochastic trends, i.e. $p - r = s_1 + s_2$. The r cointegration relations, $\beta' x_t$, are generally integrated of order 1, i.e. they cointegrate from $I(2)$ to $I(1)$, often denoted $CI(2,1)$, and becomes stationary by adding a linear combination of the growth rates, $\delta'\Delta x_t$. In addition there are s_1 linear combinations, $\beta'_{\perp 1} x_t \sim I(1)$, which can become stationary exclusively by differencing, i.e. $\beta'_{\perp 1}\Delta x_t \sim I(0)$. Thus, the $I(2)$ model contains $p - s_2$ $CI(2,1)$ relations, $\tau' x_t$, where $\tau = (\beta, \beta_{\perp 1})$. Furthermore, when $r - s_2 > 0$, it is possible to find $r - s_2$ relations $\beta' x$ which are stationary without adding the growth rates.

Under the assumption $(r = 1, s_1 = 1, s_2 = 1)$, the pulling forces are described by the equilibrium ΣΣerror correcting model $\Delta^2 x_t = \alpha(\beta' x_{t-1} + \delta'\Delta x_{t-1}) + \zeta\tau'\Delta x_{t-1} + \varepsilon_t$, where $\tau = [\beta, \beta_{\perp 1}]$, i.e. as:

$$\begin{bmatrix} \Delta^2 x_{1,t} \\ \Delta^2 x_{2,t} \\ \Delta^2 x_{3,t} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} (\beta' x_{t-1} + \delta'\Delta x_{t-1}) + \begin{bmatrix} \zeta_{11} & \zeta_{21} \\ \zeta_{12} & \zeta_{22} \\ \zeta_{13} & \zeta_{23} \end{bmatrix} \begin{bmatrix} \beta'\Delta x_{t-1} \\ \beta'_{\perp 1}\Delta x_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix}$$

where $\beta' x_{t-1} + \delta' \Delta x_{t-1}$ describes a deviation from a dynamic long-run equilibrium relation, and $\beta' \Delta x_{t-1}$ and $\beta'_{\perp 1} \Delta x_{t-1}$ describe deviations from two medium-run equilibrium relations among growth rates.

The pushing forces are given by the common stochastic trends form $x_t = \beta_{\perp 2} \Sigma\Sigma u_s + B \Sigma u_i + ... \varepsilon_t$, i.e. as:

$$\begin{bmatrix} x_{1,t} \\ x_{2,t} \\ x_{3,t} \end{bmatrix} = \begin{bmatrix} \beta_{\perp 2,1} \\ \beta_{\perp 2,2} \\ \beta_{\perp 2,3} \end{bmatrix} \left[\sum_{i=1}^{t} \sum_{s=1}^{i} u_{1,s} \right] + \begin{bmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \\ b_{13} & b_{23} \end{bmatrix} \begin{bmatrix} \sum_{i=1}^{t} u_{1,i} \\ \sum_{i=1}^{t} u_{2,i} \end{bmatrix} + ...$$

where $u_{1,t} = \alpha'_{\perp 2} \varepsilon_t$ is an autonomous shock that double cumulates over time and $u_{2,t} = \alpha'_{\perp 1} \varepsilon_t$ is an autonomous shocks that cumulates once over time. $\alpha_{\perp} = [\alpha_{\perp,1}, \alpha_{\perp,2}]$, is a 3×2 matrix orthogonal to α, defining the two shocks as linear combination of the VAR residuals $\hat{\varepsilon}_t$. $\beta_{\perp 2}$ is a 3×1 vector orthogonal to $\{\beta, \beta_{\perp 1}\}$ measuring how the $I(2)$ stochastic trend loads into the variables.

The above representations of the $I(1)$ and the $I(2)$ model into the pulling and pushing forces of the vector process is just a convenient formulation of the general unrestricted VAR model subject to (testable) reduced rank conditions. Its usefulness comes from the ability to reduce the number of parameters by sequential testing and to test hypotheses of theoretical interest. For example, most theoretical models are inherently consistent with a given number of exogenous shocks that cumulate to stochastic trends, which can be translated into testable hypotheses on the reduced rank of the CVAR. Most models also assume certain equilibrium relationships to be stationary, or (implicitly or explicitly) basic parity relationships to hold as stationary conditions. The paper demonstrates that such basic assumptions often can be formulated as testable hypotheses on the pulling and pushing forces of the model. For example, the REH-based models discussed in Section 5 are inherently consistent with at most $I(1)$ type of persistence that can be translated into a reduced rank restriction of first order, whereas IKE-based models in Section 6 are consistent with $I(2)$ type of persistence that can be translated into two reduced rank restrictions.

The next sections discuss a systematic procedure for formulating what is called a theory-consistent CVAR scenario and demonstrate that it can be used to test competing theories for exchange rate determination.

4 Bridging theory and evidence: The role of expectations in a CVAR scenario

The idea of a Cointegrated VAR (CVAR) scenario has previously been used in Juselius (2006), Juselius and Franchi (2007) and Møller (2008). It is formulated as a general method for assessing the empirical relevance of most of the *basic assumptions* behind a theoretical model prior to forcing such assumptions onto an empirical model. Thus, as a first step the VAR is used as a general statistical model to examine which basic underlying assumptions are tenable with the economic reality.

Theoretical relationships are mostly formulated in (unobservable) expectations of variables rather than in their realizations, whereas the empirical regularities uncovered by the CVAR analysis are based on the observed data. To be able to derive the implications of the theoretical model and to formulate these in an internally consistent stochastic framework, I need some principles for how to associate expectations with observations. I shall rely on the following assumptions:

Assumption A $(z^e_{t+1} - z_{t+1}) \sim I(0)$, where z^e_{t+1} is the expected value of variable z made at time t for $t+1$.

Assumption B $(z_{t+1} - z_t) \sim I(0)$, where z_t can either be equal to x_t, or Δx_t. Subjecting z_t to Assumption B implies that x_t is either $I(1)$ or $I(2)$. The reason for restricting attention to these two cases is because $x_t \sim I(3)$ is empirically implausible and $x_t \sim I(0)$ defines a non-persistent process for which cointegration and stochastic trends have no added value.[4].

Under Assumption A, agents are rational in the sense that they do not systematically mis-forecast future outcomes and z_{t+1} has the same persistency property as z^e_{t+1}. It can be considered the minimal condition that any expectational assumption should satisfy. Assumption B implies that z_t has the same persistency property as z_{t+1}. This leads to the following corollary:

Corollary Under Assumptions A and B, z_t, z_{t+1} and z^e_{t+1} have the same persistency property (order of integration).

[4] In our typically growing economies the vast majority of our economic variables would quilify as type I(1) or I(2) variables.

Thus, under Assumption A and B, $\beta' z_t$ will have the same persistency property as $\beta' z^e_{t+1}$ or $\beta' z_{t+1}$. It is, therefore, possible to make inference about the long-run steady-state relations of a theoretical model also for the case when the postulated behavior is a function of expected rather than observed outcomes. Relying on these assumptions a theory-consistent CVAR scenario can be formulated as follows:

1. Translate the postulated behavioral equations of a theoretical model into a set of conditions on the persistency properties of the steady-state *relations*. For example, REH-based theoretical models mostly assume that both the purchasing power parity and the uncovered interest rate parity hold as stationary (or at most as a near $I(1)$) conditions, whereas IKE-based models assume that the real exchange rate and the interest rate differential are near $I(2)$ processes and cointegrate to $I(1)$, and by adding the inflation spread that they cointegrated to $I(0)$.

2. Express the expectations variable(s) as a function of observed variables. For example, Uncovered Interest Rate Parity (UIP) assumes that relative interest rates are equal to the expected change in the nominal exchange rate. Thus, provided the parity holds, the observed interest rate spread is a measure of the expected change in nominal exchange rate and its persistency property can, therefore, under Assumptions A and B be *studied empirically*.

3. For a given order of integration of the variable(s) determined outside the model, derive the order of integration of all remaining variables.

4. Translate the stochastically formulated theoretical model into a theory-consistent CVAR scenario by formulating the basic assumptions underlying of the theoretical model as a set of testable hypotheses on cointegration and common trends properties.

5. Estimate a well-specified VAR model and check the empirical adequacy of the theory-consistent CVAR scenario.

In the subsequent discussions I shall distinguish between different types of shocks using the following notation:

a. $\varepsilon_{i,t} \sim Niid(0, \sigma^2_{\varepsilon})$ is a white noise process.

b. $e_{i,t}$ is a stationary deviation from steady-state associated with point 1 above. It can generally be described as an ARMA process, $\rho(L)e_{i,t} = \theta(L)\varepsilon_t$ where $\rho(L)$ and $\theta(L)$ are lag polynomials.

c. $v_{i,t} = v_{i,t}^{(p)} + v_{i,t}^{(tr)}$ where $v_{i,t}$ is a stationary expectational error associated with point 2 above. It can have both a permanent, $v_{i,t}^{(p)}$, and a transitory part, $v_{i,t}^{tr}$. In most cases, $v_{i,t}^{(p)}$ is likely to be small compared to $v_{i,t}^{(tr)}$.

e. $u_{i,t} = f(\varepsilon_{i,t}), i = 1,..,p$ is assumed to be a 'structural' shock being measured as a linear function of unanticipated shocks to the system variables, $\varepsilon_{i,t}$, which are assumed to be $Niid$. The 'structural' shocks are associated with point 4 above.

It is, however, important to note that the VAR model is defined for $Niid$ errors, whereas the postulated steady-state errors of the theory model, $e_{i,t}$, and the expectational errors, $v_{i,t}$, while stationary, are not necessarily white noise. This means that the scenario analysis is only informative on the long-run persistency properties of the vector process and its implications for the postulated steady-state behavior. The idea is first to test whether the basic assumptions underlying a theoretical model's long-run behavior are consistent with the empirical regularities in the data and, if this is the case, continue testing its short-run implications. See M. Juselius (2010) for a further discussion.

5 REH based monetary models for nominal exchange rate

The long swings in real exchange rates under currency floats have puzzled economists for a long time and various models have been proposed to account for this feature. Among the more well-known models are the overshooting models by Dornbush (1976) and Dornbush and Frankel (1988). They are based on the assumption that the nominal exchange rate overshoots because of price rigidities and that the rate of equilibrium adjustment to PPP is identical in relative prices and nominal exchange rates. In contrast, the endogenous money version of the model (Benigno, 2004) loosens up the tight link between exchange rate and price adjustment and can, therefore, explain persistent movements in real exchange rates without having to assume the same slow adjustment in relative prices as in the nominal exchange rate with typical half lives of 4-5 years.

The rational bubble version of the monetary model (Blanchard and Watson, 1982) assumes that the nominal exchange rate is overshooting because at some point agents' forecasting behavior happens to become unrelated to fundamentals. This drives nominal exchange rate away from fundamental values in an explosive way until the market realizes its mistake, the bubble bursts, and the nominal exchange returns rapidly to its fundamental value. While these models differ in various aspects, they share the assumptions that equilibrium in the goods market is characterized by PPP and in the foreign currency market by UIP, and that the international Fisher parity holds as a stationary condition.

5.1 Dornbush/Frankel type of overshooting models

Most monetary models for exchange rate determination are based on the assumption that the following parity relationships hold as stationary conditions:

PPP states that $S = P_d/P_f$, i.e. the nominal exchange rate, S, should reflect relative prices, P_d/P_f. The real exchange rate is defined as the log deviation from PPP:

$$q = s - p_d + p_f \tag{1}$$

where lower cases stand for logarithmic values and a subscript d stands for a domestic and f for a foreign economy. In equilibrium, the real exchange rate, q, is defined by relative prices being equal to the nominal exchange rate, i.e. $q_{ppp} = 0$. When prices are measured by a price index, the equilibrium value, q_{ppp}, is generally undefined and the average of observed q_t may be different from zero. The observed real exchange rate is assumed to deviate from its long-run equilibrium value with the steady-state error e_t, which in the Dornbush/Frankel type of models is assumed to be an AR(1) process:

$$\Delta q_t = -\theta(q_{t-1} - \bar{q}) + \varepsilon_t \tag{2}$$

where $\bar{q}$ is the sample average, $0 < \theta < 1$ measures the speed of adjustment, ε_t is $NIID$, p_t stands for prices, s_t for nominal exchange rates . While (2) is a stationary process some versions of the monetary model allow θ to be very close to zero, so that the real exchange rate can be a persistent near $I(1)$

type of process. Therefore, I shall consider two different cases: one where the real exchange rate is stationary of type $I(0)$ and another where it is a persistent processes of type near $I(1)$.

The Uncovered Interest Rate Parity (UIP):

$$i_{d,t} - i_{f,t} = (s^e_{t+1} - s_t) \tag{3}$$

where i stands for a nominal interest rate and a superscript e denotes an expected value.

The Fisher Parity states that nominal interest rate is equal to expected inflation rate plus a real interest rate provided the two are independent. The real interest rate is assumed to reflect the average profit per capital ratio in the economy, which under certain conditions is associated with the real GDP growth rate. As the latter is generally found to be stationary with a non-zero mean, we assume here that the real interest rate is approximately stationary with a constant mean. Thus:

$$i_{j,t} = \Delta p^e_{j,t+1} + r_{j,t}, \qquad j = d, f \tag{4}$$

where $r_{j,t} = r_j + e_{r_j,t}$ and r_j stands for an average real interest rate.

Finally, the international Fisher Parity:

$$(i_{d,t} - i_{f,t}) = (\Delta p^e_{d,t+1} - \Delta p^e_{f,t+1}) \tag{5}$$

holds as a stationary condition, which follows if (2) and (3) hold.

5.1.1 Anchoring expectations to observables

To be able to formulate a scenario for interest rates, prices and nominal exchange rates in the Dornbush/Frankel type of monetary models, I first need to associate the expected change in nominal exchange rate with some observables. According to (3), the expected change in the nominal exchange rate should be reflected in the nominal interest rate spread. A stationary real exchange rate as in (2) is generally consistent with the nominal exchange rate and the relative prices both being $I(1)$. If nominal exchange rate is $I(1)$ then its difference is $I(0)$ and, under Assumption A, the expected change in nominal exchange rates is $I(0)$. Hence, the nominal interest rate spread is also $I(0)$. This is generally consistent with nominal interest rates being $I(1)$ and cointegrating to $I(0)$. That nominal interest rates are $I(1)$ is consistent with the random walk hypothesis implying that the best predictor of the interest rate next period is the present level of interest rate:

$$i_{j,t} = i_{j,t-1} + e_{j,t} \qquad j = d, f \tag{6}$$

where $e_{j,t}$ is an interest rate shock, which can be white noise or have an ARMA formulation. Integrating (6):

$$i_{j,t} = i_{j,0} + \sum_{s=1}^{t} e_{j,s}, \tag{7}$$

For $(i_{d,t} - i_{f,t}) \sim I(0)$, the permanent part, $\varepsilon_{j,t}$, of the interest rate shock, $e_{j,t}$, which cumulates to a stochastic trend has to be identical, i.e. $\sum_{s=1}^{t}(\varepsilon_{d,s} - \varepsilon_{f,s}) = 0$. Thus, the permanent shocks to the nominal interest rates have no long-run effect on the interest rate spread under the assumption of stationarity.

5.1.2 Persistency properties of the remaining variables

Replacing $(s^e_{t+1} - s_t)$ with $\Delta s_t + v_{1,t}$ in (3), where $v_{1,t}$ is a stationary error, gives:

$$(i_{d,t} - i_{f,t}) = \Delta s_{t+1} + v_{1,t}. \tag{8}$$

Under Assumption A, $\Delta s_t \sim I(0)$, so that $s_t \sim I(1)$.

Eq. (4) can equivalently be expressed as:

$$\Delta p^e_{j,t+1} = i_{j,t} - r_{j,t}, \qquad j = d, f$$

where $r_{j,t} = r_j + e_{r_j,t}$. Under Assumption A, $(\Delta p^e_{t+1} - \Delta p_{t+1}) \sim I(0)$ and under Assumption B $(\Delta p_{t+1} - \Delta p_t) \sim I(0)$ so that $(\Delta p^e_{t+1} - \Delta p_t) \sim I(0)$. Thus the inflation rate can be expressed as:

$$\Delta p_{j,t} = i_{j,t} - r_{j,t} + v_{j,t}, \qquad j = d, f \tag{9}$$

Given the assumption that $i_{j,t} \sim I(1)$ and $r_{j,t} \sim I(0)$, $\Delta p_{j,t} \sim I(1)$ for the Fisher parity to hold. Inserting (7) in (9) gives an expression for the stochastic properties of the inflation rates:

$$\Delta p_{j,t} = i_{j,0} + \sum_{s=1}^{t} e_{j,s} - r_{j,t} + v_{j,t}, \qquad j = d, f \tag{10}$$

Summing over (10) gives us an expression for domestic prices:

$$p_{j,t} = (i_{j,0} - r_j)t + \sum_{s=1}^{t}\sum_{i=1}^{s} \varepsilon_{j,i} + \sum_{s=1}^{t} e_{r_j,s} + \sum_{s=1}^{t} v_{j,s} + p_{j,0}, \qquad j = d, f \quad (11)$$

showing that prices are generally $I(2)$ with a linear trend deriving from the initial value of the nominal interest rate corrected for the average real interest rate, implying that the slope of the linear trend is approximately equal to the initial expected inflation rate.

Replacing $(\Delta p^e_{d,t+1} - \Delta p^e_{f,t+1})$ with $(\Delta p_{d,t} - \Delta p_{f,t}) + v_{2,t}$ in (5) gives:

$$(i_{d,t} - i_{f,t}) = (\Delta p_{d,t} - \Delta p_{f,t}) + v_{2,t} \quad (12)$$

where $v_{2,t} = v_{d,t} - v_{f,t}$ is stationary under Assumption A and B. Given the assumption that $(i_{d,t} - i_{f,t}) \sim I(0)$, $(\Delta p_{d,t} - \Delta p_{f,t}) \sim I(0)$ for the international Fisher parity to hold as a stationary condition. Hence, $(p_{d,t+1} - p_{f,t+1}) \sim I(1)$, i.e. prices being individually$I(2)$are cointegrated (1, -1), implying long-run price homogeneity.

Subtracting (8) from (12) gives:

$$\Delta p_{d,t} - \Delta p_{f,t} = \Delta s_{t+1} + v_{2,t} - v_{1,t}$$

Integrating once gives an expression for q_t:

$$q_t = s_t - (p_{d,t} - p_{f,t}) = \sum_{j=1}^{t}(v_{2,j} - v_{1,j}) + q_0. \quad (13)$$

Thus, stationarity of the real exchange rate is consistent with the case where the permanent part of the unanticipated shocks to UIP and the international Fisher parity are identical and therefore cancel in (13).

5.1.3 A scenario for the Dornbush-Frankel type of monetary models

According to the stochastic properties derived above, prices are $I(2)$, the nominal exchange rate and interest rates are $I(1)$. Based on this stochastic formulation of the vector process, the behavioral steady-state equations underlying the theoretical model can now be translated into a set of testable hypotheses on cointegration and common trends in the CVAR model.

The assumption that $(i_d - i_f) \sim I(0)$, implies that the two interest rates are cointegrated and, therefore, share one common stochastic trend. Because the stochastic properties of the other variables have been derived from the stochastic properties of the interest rates, this is the only stochastic trend in the system. Hence, the theory-consistent CVAR is driven by one common stochastic trend and, the process is, therefore, equilibrium correcting to $p - 1 = 4$ cointegration relations. The common autonomous shock, $u_{1,t}$, cumulates once in the interest rates and twice in the price variables[5] (see Juselius, 2007, Section 2.5). This is because the common stochastic $I(2)$ trend, $\Sigma\Sigma u_1$ (short hand for $\sum_{s=1}^{t}\sum_{j=1}^{s} u_{1,s}$) affects only prices (cf. (11)), whereas Σu_1 (short hand for $\sum_{j=1}^{t} u_{1,j}$) can affect all variables. Thus, the REH based scenario is consistent with $\{r = 4, s_1 = 0, s_2 = 1\}$ as follows:

$$\begin{bmatrix} p_d \\ p_f \\ s \\ i_d \\ i_f \end{bmatrix} = \begin{bmatrix} c_1 \\ c_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \Sigma\Sigma u_1 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_1 - b_2 \\ c_1 \\ c_1 \end{bmatrix} [\Sigma u_1] + \begin{bmatrix} d_1 \\ d_2 \\ d_1 - d_2 \\ 0 \\ 0 \end{bmatrix} t + Z_t. \quad (14)$$

where Z_t is a short hand notation for stationary components and initial values. It acts as a catch-all for the short-run effect on the vector process due to stationary (but not necessary white noise) expectational and steady-state errors, $v_{i,t}$ and $e_{i,t}$. The coefficients c_i, b_i and d_i are not formulated as functions of the parameters of the theory model as this would require the short-run dynamics of the theoretical model to be prespecified. The scenario analysis can however tell us under which conditions the postulated steady-state behavior is empirically correct. For example, the condition for long-run price homogeneity in (14) is that $\Sigma\Sigma u_1$ affects both prices with equal coefficients. If this is the case, the nominal-to-real transformation (Kongsted, 2005) can be applied without loss of information:

[5] Note that the common autonomous shock, $u_{1,t}$, can be associated with a exogenous shock outside the CVAR model. For example, most monetary models assume relative money supply shocks to be the pushing force.

$$\begin{bmatrix} p_d - p_f \\ s \\ \Delta p_d \\ i_d \\ i_f \end{bmatrix} = \begin{bmatrix} b_1 - b_2 \\ b_1 - b_2 \\ c_1 \\ c_1 \\ c_1 \end{bmatrix} [\Sigma u_1] + Z_t. \qquad (15)$$

As $(p_d - p_f) \sim I(1)$, it follows that $(\Delta p_d - \Delta p_f) \sim I(0)$. Thus under long-run price homogeneity, inflation spread is stationary.

The scenario is consistent with $r = 4$ stationary cointegration relations. For example, the following relations are irreducible in the sense of Davidson (1998):

1. $(s - p_d + p_f) \sim I(0)$,
2. $(i_d - i_f) \sim I(0)$
3. $(i_d - \Delta p_d) \sim I(0)$
4. $(i_d - a_1(p_d - p_f)) \sim I(0)$ where $a_1 = c_1/(b_1 - b_2)$,

Of course linear combinations of these relations are also stationary and, hence, would qualify as a cointegration relation.

If relative prices and the nominal exchange rate are homogeneously related in all cointegration relations, then the scenario can also be formulated for the real exchange rate as follows:

$$\begin{bmatrix} s - p_d + p_f \\ \Delta p_d \\ \Delta p_f \\ i_d \\ i_f \end{bmatrix} = \begin{bmatrix} 0 \\ c_1 \\ c_1 \\ c_1 \\ c_1 \end{bmatrix} [\Sigma u_1] + Z_t. \qquad (16)$$

showing that the PPP, the UIP, and the Fisher parities can describe the four irreducible cointegration relations:

1. $p_d - p_f - s \sim I(0)$
2. $i_d - i_f \sim I(0)$
3. $i_d - \Delta p_d \sim I(0)$,

4. $i_f - \Delta p_f \sim I(0)$,

As before, other irreducible relations can be found by linear combinations. For example, $(i_d - i_f) - (i_d - \Delta p_d) + (i_f - \Delta p_f) = (\Delta p_d - \Delta p_f) \sim I(0)$ is a linear combination of 2, 3, and 4.

Note that the $r = 4$ cointegrated relations in the $I(1)$ transformed scenario (15) can be thought of as $r = 4$ polynomially cointegrated relations in the $I(2)$ scenario (14) of which three ($r - s_2 = 3$) can be given as directly stationary relations (1, 2, and 4) and as one ($s_2 = 1$) polynomially cointegrated relation (3). See Johansen (1992) and Juselius (2006, Chapters 16-18).

5.2 The assumption that real exchange rate is I(1)

It is of some interest to do the scenario under the assumption that real exchange rate is $I(1)$, i.e. $\theta = 0$ in (2) without changing the assumptions (32) - (5). For example, the endogenous money, representative agent, DSGE model of Benigno (2004) is consistent with a near $I(1)$ real exchange rate. In the following, I consider this real exchange rate persistence to be a type $I(1)$ process in line with the arguments in Section 2.

The assumption that $(i_{d,t} - i_{f,t}) \sim I(1)$ and that uncovered interest rate parity (8) holds as a stationary condition implies:

$$(i_{d,t} - i_{f,t}) = \Delta s_t + v_{1,t} \tag{17}$$

where $\Delta s^e_{t+1} = \Delta s_{t+1} + v_{1,t}$ and $v_{1,t}$ is a stationary error under Assumption A. Additionally under Assumption B, $(i_{d,t} - i_{f,t})$ and Δs_t share a common stochastic trend, $\Delta s_t \sim I(1)$ and, thus, $s_t \sim I(2)$.

The assumption that *the international Fisher parity condition* holds as a stationary condition implies that:

$$(i_{d,t} - i_{f,t}) = \Delta p_{d,t} - \Delta p_{f,t} + v_{2,t} \tag{18}$$

where $\Delta p^e_{d,t+1} - \Delta p^e_{f,t+1} = \Delta p_{d,t} - \Delta p_{f,t} + v_{2,t}$ and $v_{2,t}$ is a stationary error under Assumption A and B. If $(\Delta p_{d,t} - \Delta p_{f,t}) \sim I(1)$, then $(p_{d,t} - p_{f,t}) \sim I(2)$.

Subtracting (17) from (18) gives:

$$\Delta p_{d,t} - \Delta p_{f,t} = \Delta s_t + (v_{1,t} - v_{2,t})$$

Integrating once gives:

$$p_{d,t} - p_{f,t} - s_t = \sum_{s=1}^{t}(v_{1,s} - v_{2,s}) - q_0 \tag{19}$$

where $(v_{1,s} - v_{2,s}) = (v^p_{1,t} - v^p_{2,t}) + (v^{tr}_{1,t} - v^{tr}_{2,t})$. Thus, the real exchange rate can be $I(1)$ if $(v^p_{1,t} - v^p_{2,t}) \neq 0$, i.e. the permanent part of the unanticipated shocks to UIP and the international Fisher parity differ from each other.

5.2.1 A scenario for nominal exchange rate determination

The assumption that $(i_d - i_f) \sim I(1)$, implies that the interest rates do not cointegrate (1, -1) and, therefore, that two stochastic trends are driving nominal interest rates[6]. Thus the system is pushed by two permanent shocks, $u_{1,t}$ and $u_{2,t}$, one of which cumulates once to an autonomous $I(1)$ trend, whereas the other cumulates twice to the $I(2)$ stochastic trend in price levels. The implication of this assumption for the CVAR scenario is that $\{r = 3, s_1 = 1, s_2 = 1\}$. Given the assumption that interest rates are $I(1)$, the common $I(2)$ trend loads exclusively into prices and the nominal exchange rate.

$$\begin{bmatrix} p_d \\ p_f \\ s \\ i_d \\ i_f \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ 0 \\ 0 \end{bmatrix} [\Sigma\Sigma u_1] + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \\ b_{41} & b_{42} \\ b_{51} & b_{52} \end{bmatrix} \begin{bmatrix} \Sigma u_1 \\ \Sigma u_2 \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \\ 0 \\ 0 \\ 0 \end{bmatrix} t + Z_t. \tag{20}$$

We shall first discuss the cointegration implications for the $I(2)$ variables, p_d, p_f, s.

- Because $s_t \sim I(2)$ (as a result of $\Delta s_t \sim I(1)$), $p_d - p_f$ has to be $I(2)$ for $(p_d - p_f - s)$ to be $I(1)$. Therefore, $c_1 \neq c_2$.
- $(p_d - p_f) \sim I(2)$, implies $(\Delta p_d - \Delta p_f) \sim I(1)$.
- $(p_d - p_f - s) \sim I(1)$ implies that $c_1 - c_2 = c_3$.

[6] This is of course only correct unless $(i_d - a_1 i_f) \sim I(0)$. We disregard this possibility as it would be difficult to explain the reason for $a_1 \neq 1.0$.

- One common $I(2)$ trend driving the three prices implies two $CI(2,1)$ cointegration relations among $\{p_d, p_f, s\}$. These can for example be, $(p_d - a_1 p_f) \sim I(1)$ when $a_1 = c_1/c_2$ and $(p_f - a_2 s) \sim I(1)$ when $a_2 = c_2/c_3$. Any linear combination of the two relations would of course also be $CI(2,1)$.

- The three prices $\{p_d, p_f, s\}$ share one common $I(2)$ trend, $\Sigma\Sigma u_1$, and one common $I(1)$ trend, Σu_2. Hence $(r = 1, s_1 = 1, s_2 = 1)$ and there would be one polynomially cointegrated relation and one medium-run relation in growth rates, but no directly stationary relation between the three variables.

Thus, under the assumptions (17) - (19), long-run price homogeneity between domestic and foreign prices are not likely to hold empirically. Given (20) the CVAR model would be consistent with $r = 3$ stationary polynomially cointegrated relations, $\beta' x_t + \delta' \Delta x_t$ and $s_1 = 1$ stationary relation between nominal growth rates, i.e. between the differenced $I(2)$ variables. The three irreducible polynomially cointegrated relations could, for example, be:

1. $\{(p_d - p_f - s) - a_1(i_d - i_f)\} \sim I(0)$, if $c_1 - c_2 = c_3$ and $b_{11} - b_{21} - b_{31} - a_1(b_{41} - b_{51}) = 0$ and $b_{12} - b_{22} - b_{32} - a_1(b_{42} - b_{52}) = 0$.

2. $\{i_d - \Delta p_d - a_2 p_d - a_3 p_f)\} \sim I(0)$, if $a_2 c_1 = a_3 c_2$ and $b_{41} - c_1 - a_2 b_{11} - a_3 b_{21}$ and $b_{42} - a_2 b_{12} - a_3 b_{22} = 0$

3. $\{i_f - \Delta p_f - a_4 s - a_5 p_f\} \sim I(0)$, if $a_4 c_3 = a_5 c_2$ and $b_{51} - c_2 - a_4 b_{31} - a_5 b_{21}$ and $b_{52} - a_4 b_{32} - a_5 b_{22} = 0$

and the medium-run relation in growth rates:

1. $(\Delta p_d - a_6 \Delta p_f - a_7 \Delta s) \sim I(0)$.

Note that the $I(2)$ model generally contains a linear trend in the variables (see Juselius, 2006, Section 17.2) and in the cointegration relations (as the linear trend may not cancel by cointegration). For example, relation 2, say, could additionally contain a linear trend, so that $\{i_d - \Delta p_d + a_2 \tilde{p}_d - a_3 \tilde{p}_f\} \sim I(0)$ where $\tilde{p}$ denotes a trend-adjusted price $(p - b_1 trend)$.

Thus, a theory-consistent CVAR scenario shows that allowing for near $I(1)$ persistence in REH based monetary models has strong implications for the model:

- i_d and Δp_d can only be cointegrated if $\{b_{42}, b_{52}\} = 0$. The latter condition is, however, in conflict with the assumption that the interest rate differential is $I(1)$ and driven by two stochastic trends. The same applies for i_f and Δp_f. Thus, the domestic Fisher parity does not hold as a stationary condition in this case.

- $\{(i_d - i_f) - (\Delta p_d - \Delta p_f)\} \sim I(0)$, only if $c_1 - c_2 = b_{41} - b_{51}$ and $b_{42} - b_{52} = 0$. The latter condition is again in conflict with the assumption that the interest rate differential is $I(1)$ and driven by two stochastic trends. Thus, the real interest rates differential is $I(1)$ in this case.

The scenario exercise has, thus, pointed out that if the UIP holds as a stationary condition, and the interest rate differential and the PPP hold as a near $I(1)$ conditions, neither the domestic nor the international Fisher parity (18) are likely to hold as stationary conditions.

The overall conclusion seems to be that the REH-based endogenous money model is relying on assumptions which, to some extent, seem internally inconsistent.

6 Imperfect Knowledge Economics and the determination of nominal exchange rate

Imperfect Knowledge Economics (IKE) developed in Frydman and Goldberg (2007) does not postulate an exact mathematical model for how economic agents behave, but instead use the assumption that the process driving outcomes in modern economies changes at times and in ways that cannot be prespecified. Such changes arise in part because individuals's forecasting strategies, which play an important role in driving market outcomes, change in ways that cannot be fully prespecified. But even though IKE speculators have limited knowledge of the factors that drive their future payoff, they are rational in the sense of not passing up profit opportunities. Because they do not know the right model they have diverse forecasting strategies - bulls hold long positions of foreign exchange and bet on appreciation while bears hold short positions and bet on depreciation. IKE speculators change their forecasting strategies, but in a conservative manner, so these revisions are mostly assumed to be moderate.

This implies that the foreign exchange market is assumed to be unstable, but boundedly so. As long as agents base their forecast on trending variables and they revise their forecasting strategies in a moderate manner, the nominal and real exchange rate will trend in one direction, either towards or away from PPP. If the trend movement is in the direction toward the PPP benchmark value the real exchange rate will shoot through the long-run PPP value and continue trending away from this value. Such speculative behavior is shown to lead to a non-constant drift term in the differenced real exchange rate and the differenced interest rate differential. This drift term plays a key role in explaining the long and persistent swings typical of the real exchange rates and interest rates. These basic assumptions imply the following model for the real exchange rates, q_t:

$$\Delta q_t = \zeta_t + w_t \tag{21}$$

where

$$\zeta_t = \bar{\rho}\zeta_{t-1} + \varepsilon_t,$$

and ζ_t is a measure of the change in forecast due to a change of the explanatory variables and a change in agents' forecasting strategies. Frydman et al. (2009) show that the non-constant drift term, ζ_t, (which mostly entail small changes, but with occasional larger changes that involve switches in sign) can be approximated with a near $I(1)$ process so that the real exchange rate is likely to behave like a near $I(2)$ process. The parameter $\bar{\rho}$ may vary over different sample periods but generally within a small band close to the unit circle. It is, therefore, not a structural parameter in the usual sense of the word. A comparison with the REH-based model shows that the differenced real exchange rate in (2) behaves like white noise, if θ is close to zero, or otherwise like a Moving Average (MA) process with a negative parameter. In contrast, the IKE-based model is consistent with more pronounced persistence in the differenced real exchange rate due to ζ_t in (21). Therefore, the swings in real exchange rates will tend to be more persistent and have a longer duration in an IKE world than in a REH world. The length of such swings is, however, not predictable and the near $I(2)$ process will entail swings of shorter and longer duration[7].

[7] Thus, there might be sample periods where the evidence of I(2) is weaker.

Figure 4 illustrates the difference between these two assumptions by showing the graphs of two simulated processes: the random walk process:

$$x_{1,t} - x_{1,t-1} = \varepsilon_t, \qquad \varepsilon_t \sim N(0,1), \qquad t = 1, ..., 500$$

and the near $I(2)$ process:

$$\begin{aligned} x_{2,t} - x_{2,t-1} &= \zeta_t + \varepsilon_{1,t}, \qquad \varepsilon_{1,t} \sim N(0,1) \qquad t = 1, ..., 500 \\ \zeta_t &= 0.95\zeta_{t-1} + \varepsilon_{2,t}, \qquad \varepsilon_{2,t} \sim N(0, 0.15^2) \end{aligned}$$

The near $I(2)$ process can be thought of as a random walk with a small, but very persistent, drift term ζ_t. Because the variance of the ζ_t process is small compared to the variance of $\Delta x_{2,t}$, I have also plotted the graphs of a 12 months moving average (which is how I illustrate the persistence of the actual series in Section 8). The two processes look very similar even though one has been generated as a random walk and the other as a near $I(2)$ process. This is because the variance of the near random walk component, ζ_t, is very small compared to the large variance of $\Delta x_{2,t}$. The 12 months moving average is a time-dependent process in both cases, but the fluctuations are more pronounced in the near $I(2)$ case.

6.1 The Frydman/Goldberg model

In the REH-based models, the expected change in nominal exchange rate was associated with the nominal interest rate differential according to the uncovered interest rate parity. The IKE model replaces the UIP condition with the Uncertainty Adjusted UIP (UAUIP) condition:

$$(i_{d,t} - i_{f,t}) = s^e_{t+1} - s_t + up_t \tag{22}$$

where up_t is an uncertainty premium measuring agents' *loss* averseness. The assumption that agents are loss averse, rather than risk averse, builds on the prospect theory by Kahneman and Tversky (1979). To ensure that the UAUIP condition corresponds to a well-defined equilibrium concept, Frydman and Goldberg (2007) extend the Tversky and Kahneman concept of loss aversion to endogenous loss aversion: the degree of loss aversion increases with the size of the potential loss that might occur. This feature sets limits to speculation and therefore secures that the UAUIP equilibrium exists. The

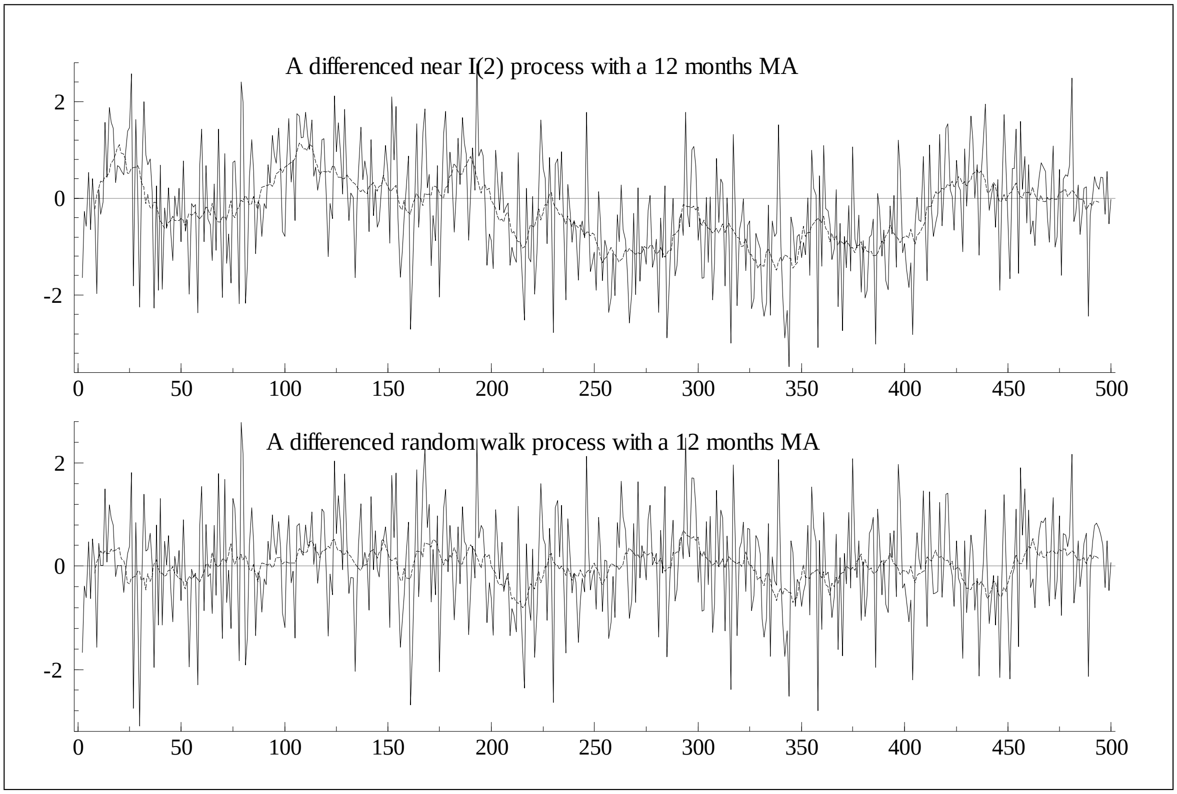

Figure 4: The graphs of a differenced near I(2) process versus that of a random walk together with a 12 period moving average.

latter is consistent with an economy where all speculators require a minimum return - an uncertainty premium - to speculate in the foreign exchange market. A key implication is that the expected change in nominal exchange rate is not directly associated with the observed interest rate differential, but with the interest rate differential corrected for the uncertainty premium.The IKE-based theoretical model assumes that the uncertainty premium is related to the so called 'gap effect', which is measured by the deviation of the real exchange rate from its long-run PPP value. This can be used to associate the expected change in nominal exchange rate with observable variables:

$$s^e_{t+1} - s_t = (i_{d,t} - i_{f,t}) - f(p_{d,t} - p_{f,t} - s_t) \qquad (23)$$

where $f(p_{d,t} - p_{f,t} - s_t) = up_t$.

A further difference between the REH based and the IKE based monetary models is that equilibrium in the goods market under REH is consistent with a stationary real exchange rate and a stationary real interest rate differential, whereas under IKE equilibrium is defined as a stationary cointegration relation between the two:

$$\{(i_{d,t} - i_{f,t}) - (\Delta p_{d,t} - \Delta p_{f,t}) - \omega_1(s_t - p_{d,t} + p_{f,t})\} \sim I(0)$$

6.1.1 Anchoring expectations to observables

To be able to formulate theory-consistent time series properties of exchange rates, I need first to assume a stochastic process describing how nominal interest rates are being generated in an IKE-based model. According to UAUIP, nominal interest rates are affected not only by the expected change in the nominal exchange rate but also by an uncertainty premium, i.e.:

$$i_{j,t} = i_{j,t-1} + \eta_{j,t}, \qquad j = d, f$$

where $\eta_{j,t} = \varepsilon_{j,t} + \omega_{j,t}$, $j = d, f$, consists of an unanticipated error $\varepsilon_{j,t} \sim Niid(0, \sigma^2_{\varepsilon,j})$ and an IKE error, $\omega_{j,t}$, measuring the uncertainty premium $\omega_{j,t} \sim (0, \sigma^2_{\omega,j})$ which is stationary but near $I(1)$. Thus:

$$\Delta i_{j,t} = \omega_{j,t} + \varepsilon_{j,t} \tag{24}$$

where

$$\omega_{j,t} = \bar{\rho}_j \omega_{j,t-1} + \varepsilon_{\omega j,t}, \qquad j = d, f$$

and $\bar{\rho}_j$ is less than but fairly close to 1.0 and corresponds to the IKE parameter in (21). is an average of time-varying coefficients $\rho_{t,j} \approx 1.0$ in periods when q_t moves persistently away or towards its long-run benchmark value, and $\rho_{t,j} \ll 1.0$ when q_t is sufficiently far from this long-run value..Frydman et al. (2008) show that such a process is consistent with persistent swings in nominal interest rates of shorter and longer durations and that $\bar{\rho}_j$ and therefore can be approximated with a near $I(2)$ process.

Integrating (24) over t gives:

$$i_{j,t} = i_{j,0} + \sum_{s=1}^{t} \varepsilon_{j,s} + \sum_{s=1}^{t} \omega_{j,s}, \qquad j = d, f \tag{25}$$

Under the near $I(1)$ assumption of $\omega_{j,t}$, $\sum_{s=1}^{t} \omega_{j,s}$ is near $I(2)$ which implies that nominal interest rates are near $I(2)$.

Thus, under IKE the best predictor of the interest rate next period is not just the interest rate level but also the rate of change has predictive content:

$$E_t(i_{t+1} \mid X_t) = i_t + \Delta i_t \tag{26}$$

where X_t stands for the information available at time t. In contrast, REH-based model assume that the best predictor is the present level of interest rate:

$$E_t(i_{t+1} \mid X_t) = i_t \tag{27}$$

i.e. the direction of change has no predictive content.

6.1.2 The UAUIP condition

Based on (25) the interest rate differential can be expressed as:

$$(i_{d,t} - i_{f,t}) = (i_{d,0} - i_{f,0}) + \sum_{s=1}^{t}(\varepsilon_{d,s} - \varepsilon_{f,s}) + \sum_{s=1}^{t}(\omega_{d,s} - \omega_{f,s}). \tag{28}$$

As the uncertainty premium $\omega_{j,t}$ in interest rates is assumed to be near $I(1)$, the cumulation $\sum_{j=1}^{t}(\omega_{d,j} - \omega_{f,j})$ is near $I(2)$, implying that the interest rate differential is also near $I(2)$ unless $\omega_{d,j} - \omega_{f,j} = 0$. Because equality implies no uncertainty premium in the market, a test of the hypothesis that $(i_{d,t} - i_{f,t})$ is near $I(2)$ is a test of whether exchange rate determination in speculative currency markets is based on IKE versus REH. Replacing $\sum_{j=1}^{t}(\omega_{d,j} - \omega_{f,j})$ with up_t in (28) gives:

$$(i_{d,t} - i_{f,t}) - up_t = (i_{d,0} - i_{f,0}) + \sum_{s=1}^{t}(\varepsilon_{d,s} - \varepsilon_{f,s}),$$

showing that the interest rate differential corrected for the uncertainty premium is $I(1)$. Thus, under Assumption A, Δs_t is also $I(1)$ in (22) and, hence, s_t is $I(2)$. The IKE-based model assumes that the uncertainty premium is a function of the gap effect, $up_t = \sigma(s_t - p_{d,t} + p_{f,t})$, measuring the deviation from long-run PPP based benchmark values. Thus, the expected change in nominal exchange rates is a function of the interest rate differential corrected for the IKE risk premium. Replacing up_t with $\sigma(s_t - p_{d,t} + p_{f,t})$ in (22) gives the expression:

$$\begin{aligned} (i_{d,t} - i_{f,t}) - \sigma(s_t - p_d - p_f)_t &= s^e_{t+1} - s_t \\ &\approx \Delta s_{t+1} + v_{3,t} \end{aligned} \tag{29}$$

where $v_{3,t} \sim I(0)$ under Assumption A. The implication of $\Delta s_{t+1} \sim I(1)$ in (29) is that $\{(i_{d,t} - i_{f,t}) - \sigma(s_t - p_d - p_f)_t\} \sim I(1)$ and the near $I(2)$ trend in $(i_{d,t} - i_{f,t})$ and $(s_t - p_{d,t} + p_{f,t})$ must be the same. Thus the nominal interest rate spread and the real exchange rate are cointegrated $CI(2,1)$. Furthermore, the assumption that $v_{3,t} \sim I(0)$ implies that Δs_{t+1} and $\{(i_{d,t} - i_{f,t}) - \sigma(s_t - p_{d,t} - p_{f,t})\}$ share the same $I(1)$ trend and thus cointegrate to $I(0)$.

6.1.3 The Fisher parity condition

Under Assumption A and B, $\Delta p^e_{j,t+1} = \Delta p_{j,t} + v_{j,t}$, and the real interest rate can be formulated as:

$$r_{j,t} = i_{j,t} - \Delta p_{j,t} + v_{j,t}, \qquad j = d, f \tag{30}$$

where $v_{j,t} \sim I(0)$.

We first investigate the time-series properties of $\Delta p_{j,t}$ under the near $I(2)$ assumption of nominal interest rates:

$$\Delta p_{j,t} = i_{j,t} - r_{j,t} + v_{j,t}, \qquad j = d, f \tag{31}$$

Inserting (25) in (31) gives:

$$\Delta p_{j,t} = i_{j,0} + \sum_{s=1}^{t} \varepsilon_{j,s} + \sum_{s=1}^{t} \omega_{j,s} - r_{j,t}, \qquad j = d, f \tag{32}$$

Summing over (32) gives us an expression for prices:

$$p_{j,t} = (i_{j,0})t + \sum_{s=1}^{t}\sum_{i=1}^{s} \varepsilon_{j,i} + \sum_{s=1}^{t}\sum_{i=1}^{s} \omega_{j,i} - \sum_{s=1}^{t} r_{j,s} + \sum_{s=1}^{t} v_{j,s} + p_{j,0}, \qquad j = d, f \tag{33}$$

Thus, when $\omega_{j,t}$ is near $I(1)$, prices would be near $I(3)$ unless $\sum_{s=1}^{t}\sum_{i=1}^{s}\omega_{j,i}$ and $\sum_{s=1}^{t} r_{j,s}$ are cointegrated $CI(3,1)$ or $CI(3,2)$. Assuming that the financial market uncertainty premium, $\omega_{j,t}$, affects nominal interest rates, but not goods prices (which are determined by demand and supply in the international goods market and not by speculation) then $r_{j,t} = \sum_{s=1}^{t}\omega_{j,s}$ so that $\left\{\sum_{s=1}^{t}\sum_{i=1}^{s}\omega_{j,i} - \sum_{s=1}^{t} r_{j,s}\right\} = 0$ in (33). In this case, prices would be $I(2)$, inflation rates $I(1)$, whereas both nominal and real interest rates would be near $I(2)$. Thus, IKE predicts that nominal and real interest rates are integrated of the same order and that the Fisher parity does not hold as a stationary condition[8]. Such a result would be untenable with Fisher's original idea of the real interest rate as measure of the difference between the present and expected production/consumption possibilities. As the real GDP is usually found to be $I(1)$ around a linear trend, so that real growth is $I(0)$ around a constant mean, Fisher's original idea would in general be consistent with a stationary real interest rate.

Discussion: Finding real interest rates to be very persistent, even near $I(2)$, would suggest that speculative IKE-based behavior in the financial market may have strong implications for macroeconomic modelling. Thus, IKE speculation implies a de-linking of the real interest rate from the expected real productive growth in the economy.

To summarize: IKE is consistent with the following testable hypotheses:

- $(p_{d,t} - p_{f,t}) \sim I(2)$,
- $s_t \sim I(2)$,
- $(i_{d,t} - i_{f,t}) \sim I(2)$,
- $(s_t - p_d - p_f)_t \sim I(2)$,
- $\{(i_{d,t} - i_{f,t}) - b_1(s_t - p_d - p_f)_t\} \sim I(1)$
- $\{\Delta s_t - (i_{d,t} - i_{f,t}) + b_1(s_t - p_{d,t} - p_{f,t})\} \sim I(0)$
- $\{(\Delta p_{d,t} - \Delta p_{f,t}) - b_2(i_{d,t} - i_{f,t}) + b_1(s_t - p_{d,t} - p_{f,t})\} \sim I(0)$
- $(i_{j,t} - \Delta p_{j,t}) \sim I(2)$.

[8] References to work where the stationarity of the real interest rate is strongly rejected.

6.2 IKE based model scenarios

The IKE assumption that the interest rate spread and the real exchange rate are near $I(2)$ would imply two stochastic trends, one is $I(2)$ and originates from the twice cumulated interest rate shocks, $\sum_{s=1}^{t}\sum_{i=1}^{s}\varepsilon_{j,i}$ in (33), the other is near $I(2)$ and originates from the once cumulated near $I(1)$ IKE risk premium, $\sum_{s=1}^{t}(\omega_{d,s}-\omega_{f,s})$ in (28). Two stochastic $I(2)$ trends pushing five variables implies three cointegrated $CI(2,1)$ relations. Stationarity can either be obtained by polynomial cointegration $(\beta' x_t+\delta\Delta x_t)$ or by differencing $(\beta'_{\perp,1}\Delta x_t)$, implying that two$I(2)$trends can be consistent with different choices of r and s_1 as long as $r+s_1=p-s_2=3$. The IKE theory predicts that $(i_{d,t}-i_{f,t})$ and $(s_t-p_{d,t}+p_{f,t})$ are cointegrated so $r\geq 1$. Section 9.1 shows that the case $\{r=1,s_1=2,s_2=2\}$ is not tenable with the information in the data, whereas the following two cases, $\{r=3,s_1=0,s_2=2\}$ and $\{r=2,s_1=1,s_2=2\}$, are acceptable and almost identical in terms of likelihood values. Therefore, an IKE theory-consistent scenario will be formulated for both cases.

6.2.1 Case 1: Two common stochastic shocks

The CVAR for $\{r=3,s_1=0,s_2=2\}$ is consistent with $r=3$ stationary polynomially cointegrated relations but, as $s_1=0$, no medium-run relation in growth rates. It corresponds to 4 unit roots in the characteristic polynomial. The IKE scenario can be given the following general formulation:

$$\begin{bmatrix} p_d \\ p_f \\ s \\ i_d \\ i_f \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \\ 0 & c_{42} \\ 0 & c_{52} \end{bmatrix} \begin{bmatrix} \Sigma\Sigma u_1 \\ \Sigma\Sigma u_2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \\ b_{41} & b_{42} \\ b_{51} & b_{52} \end{bmatrix} \begin{bmatrix} \Sigma u_1 \\ \Sigma u_2 \end{bmatrix} + Z_t, \qquad (34)$$

where u_1 is assumed to describe a relative price shock and u_2 a speculative IKE shock.

Based on the derivations in the previous section it is possible to impose testable restrictions on some of the coefficients. For example, $(c_{12},c_{22})=0$ if the uncertainty premium is only relevant in the speculative market for foreign currency and $(c_{11}-c_{21})=c_{31}$ if the long-run stochastic trend in relative prices and nominal exchange rate cancel in (p_d-p_f-s). These are theoretically reasonable restrictions and will be assumed to hold:

$$\begin{bmatrix} p_d \\ p_f \\ s \\ i_d \\ i_f \end{bmatrix} = \begin{bmatrix} c_{11} & 0 \\ c_{21} & 0 \\ c_{11}-c_{21} & c_{32} \\ 0 & c_{42} \\ 0 & c_{52} \end{bmatrix} \begin{bmatrix} \Sigma\Sigma u_1 \\ \Sigma\Sigma u_2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \\ b_{41} & b_{42} \\ b_{51} & b_{52} \end{bmatrix} \begin{bmatrix} \Sigma u_1 \\ \Sigma u_2 \end{bmatrix} + Z_t, \quad (35)$$

All variables are $I(2)$ consistent with the derivations in Section 6.1.1. The real exchange rate and the interest rate differential are both $I(2)$ and can therefore cointegrate to produce (29). Furthermore, as the two prices and the exchange rate share two stochastic $I(2)$ trends, there exists just one relation, $(p_d - b_1 p_f - b_2 s) \sim I(1)$ with $(b_1, b_2) \neq 1.0$. The following three $CI(2,1)$ cointegration relations, $\beta' x_t$, are consistent with (35):

1. $\{p_d - p_f - s - a_1(i_d - i_f)\} \sim I(1)$ if $c_{32} - a_1(c_{42} - c_{52}) = 0$
2. $(i_d - a_2 p_d - a_3 s) \sim I(1)$, if $c_{42} - a_2 c_{32} = 0$ and $a_2 c_{11} - a_3(c_{11} - c_{21}) = 0$
3. $(i_f - a_4 p_f - a_5 s) \sim I(1)$ if $c_{52} - a_4 c_{32} = 0$ and $a_4 c_{21} - a_5(c_{11} - c_{21}) = 0$

Of course, any linear combination of the three relations would also be $I(1)$. To obtain stationarity, the above relations need to be combined with the growth rates. This is illustrated below for the first relation, by spelling out the necessary restrictions on the parameters that secure stationarity. The conditions for the other relations can be similarly derived.

1. $(p_d - p_f - s) - a_1(i_d - i_f) - d_1(\Delta p_1 - \Delta p_2) \sim I(0)$ if $c_{32} - a_1(c_{42} - c_{52}) = 0$ $\{(b_{11} - b_{21} - b_{31}) - a_1(b_{41} - b_{51}) - d_1(c_{11} - c_{21})\} = 0$ and $\{(b_{11} - b_{21} - b_{31}) - a_1(b_{42} - b_{52})\} = 0$,
2. $(i_d - d_2 \Delta p_d - a_2 p_d - a_3 s) \sim I(0)$,
3. $(i_f - d_3 \Delta p_f - a_4 p_f - a_5 s) \sim I(0)$.

Again, any linear combination of the three relations is also $I(0)$. The fact that there is a linear trend in the model implies that prices and the nominal exchange rate in the last two relations can be replaced by their trend-adjusted values, such as $\{p_d - b_1 trend\}$.

6.2.2 Case 2: Three common stochastic shocks

The CVAR for $\{r = 2, s_1 = 1, s_2 = 2\}$ is consistent with $r = 2$ stationary polynomially cointegrated relations, and one medium-run relation exclusively between growth rates. The latter is consistent with the existence of an additional $I(1)$ stochastic trend. This case corresponds to 5 unit roots in the characteristic polynomial. The IKE scenario is expressed as:

$$\begin{bmatrix} p_d \\ p_f \\ s \\ i_d \\ i_f \end{bmatrix} = \begin{bmatrix} c_{11} & 0 \\ c_{12} & 0 \\ c_{11} - c_{12} & c_{32} \\ 0 & c_{42} \\ 0 & c_{52} \end{bmatrix} \begin{bmatrix} \Sigma\Sigma u_1 \\ \Sigma\Sigma u_2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \\ b_{41} & b_{42} & b_{43} \\ b_{51} & b_{52} & b_{53} \end{bmatrix} \begin{bmatrix} \Sigma u_1 \\ \Sigma u_2 \\ \Sigma u_3 \end{bmatrix} + Z_t, \tag{36}$$

where u_1 as before describes an expected relative price shock and u_2 a speculative IKE shock. In addition there is a third shock, $u_{3,}$ which can be interpreted as a medium-run price shock. As the two prices and the exchange rate are affected by the two stochastic $I(2)$ trends in the same way as in the first scenario, the conditions for cointegration from $I(2)$ to $I(1)$ are identical and will not be repeated.

Two of the three polynomially cointegrated stationary relations of the previous scenario can be found also in the present case, whereas the third stationary relation is a medium run relation exclusively between growth rates.

In terms of cointegration this case is consistent with for example the following two stationary polynomially cointegrated relations, $\beta' x_t + \delta' \Delta x_t$:

1. $ppp - a_1(i_d - i_f) - a_2(\Delta p_d - \Delta p_f) \sim I(0)$,
2. $(i_d - a_3 \Delta p_d - a_4 p_d - a_5 s) \sim I(0)$,

and one medium-run relation, $\beta'_{\perp 1} \Delta x_t$:

1. $(\Delta p_d + d_1 \Delta p_f + d_2 \Delta s + d_3 \Delta i_f) \sim I(0)$.

Of course, linear combinations between the above stationary relations are also stationary.

7 Introducing the empirical $I(2)$ model

The empirical analysis will be based on a VAR model with two lags. For convenience of interpretation, the unrestricted VAR is formulated in acceleration rates, changes and levels:

$$\Delta^2 x_t = -\Gamma_2 \Delta^2 x_{t-1} + \Gamma \Delta x_{t-1} + \Pi x_{t-2} + \mu_0 + \mu_{01} Ds_{91.1,t} + \mu_1 t + \mu_1 t_{91.1} + \phi D_{tax,t} + \varepsilon_t, \tag{37}$$

where $x_t = [p_{1,t} - p_{2,t}, s_{12,t}, \Delta p_{1,t}, b_{1,t} - b_{2,t}, b_{1,t}]$, p_t stands for CPI prices, s_t for nominal exchange rate, b_t for long-term bond rates, a subscript 1 for US and a subscript 2 for German, $t_{91:1,t}$ is a linear trend starting in 1991:1, $Ds_{91:1,t}$ is a step dummy starting in 1991:1, and $D_{tax,t}$ is a dummy accounting for three different excise taxes levied to pay for the German reunification. All parameters are unrestricted in (37).

The hypothesis that x_t is $I(1)$ is formulated as a reduced rank hypothesis on Π:

$$\Pi = \alpha\beta' \text{ , where } \alpha, \beta \text{ are } p \times r \tag{38}$$

implicitly assuming that Γ is unrestricted, i.e. full rank. The hypothesis that x_t is $I(2)$ is formulated as an additional reduced rank hypotheses

$$\alpha'_\perp \Gamma \beta_\perp = \xi\eta', \text{ where } \xi, \eta \text{ are } (p-r) \times s_1. \tag{39}$$

where $\alpha_\perp, \beta_\perp$ are the orthogonal complements of α, β respectively. The first reduced rank condition (38) is associated with the levels of the variables and the second (39) with the differenced variables. The intuition is that the differenced process also contains unit roots when data are $I(2)$.

The moving average representation of (37) subject to (38) and (39) expresses the variables x_t as a function of once and twice cumulated errors and deterministic components given by:

$$\begin{aligned} x_t \;=\; & C_2 \sum_{j=1}^{t} \sum_{i=1}^{j} (\varepsilon_i + \Phi_s D_{s,i} + \Phi_p D_{p,i} + \Phi_{tr} D_{tr,i} + \mu_0 + \mu_1 i + \mu_2 t_{91:1,i}) \\ & + C_1 \sum_{j=1}^{t} (\varepsilon_j + \Phi_s D_{s,j} + \Phi_p D_{p,j} + \Phi_{tr} D_{tr,j} + \mu_0 + \mu_1 j + \mu_2 t_{91:1,j}) \\ & + C^*(L)(\varepsilon_t + \Phi_s D_{s,t} + \Phi_p D_{p,t} + \Phi_{tr} D_{tr,t} + \mu_0 + \mu_1 t + \mu_2 t_{91:1,t}) \\ & + A + Bt, \end{aligned} \tag{40}$$

where $D_{s,t}$ is a step dummy, $D_{p,t}$ is a permanent impulse dummy, $D_{tr,t}$ is a transitory impulse dummy, $t_{91:1,t}$ is a linear trend which is 0 before 1991:1, and

$$\begin{aligned} C_2 &= \beta_{\perp 2}(\alpha'_{\perp 2}\Psi\beta_{\perp 2})^{-1}\alpha'_{\perp 2}, & (41)\\ \beta' C_1 + \overline{\alpha}'\Gamma C_2 &= 0, \quad C_1\alpha + C_2\Gamma\bar{\beta} = 0, & (42)\\ \beta'_{\perp 1}C_1 &= -\overline{\alpha_{\perp 1}}'(I - \Psi C_2), & (43)\\ (\beta, \beta_{\perp 1})'B &= 0, \quad \beta' A + \bar{\alpha}'\Gamma B = 0, & (44) \end{aligned}$$

where $\beta, \beta_{\perp 1}, \beta_{\perp 2}$ are orthogonal decompositions of dimensions r, s_1, and s_2 respectively and $\Psi = \Gamma\overline{\beta}\overline{\alpha}'\Gamma + I_p - \Gamma_1)$ is a function of the parameters of the VAR model. See Johansen (1992) and Johansen et al. (2009) for further detail.

To facilitate the interpretation of the $I(2)$ trends and how they load into the variables, I denote $\breve{\beta}_{\perp 2} = \beta_{\perp 2}(\alpha'_{\perp 2}\Psi\beta_{\perp 2})^{-1}$, so that

$$C_2 = \breve{\beta}_{\perp 2}\alpha'_{\perp 2}. \tag{45}$$

It appears that C_2 has a similar reduced rank representation as C in the $I(1)$ model, so that it is straightforward to interpret $\alpha'_{\perp 2}\sum_{j=1}^{t}\sum_{i=1}^{j}\varepsilon_i$ as an estimate of the s_2 second order stochastic trends which load into the variables x_t with the weights $\breve{\beta}_{\perp 2}$.

From (40) it follows that an unrestricted constant will cumulate twice to a quadratic trend, and an unrestricted trend to a cubic trend and similarly for the step dummy and the broken trend. Thus, the coefficients of the deterministic components need to be appropriately restricted in the model equations to avoid undesirable effects in the process. The subsequent empirical model will be estimated subject to the restriction that all quadratic and cubic trends are zero.

Because the second rank condition is formulated as a reduced rank on the transformed $\mathbf{\Gamma}$ matrix its coefficients in (37) are no longer unrestricted as in the $I(1)$ model. This is is the reason why the ML estimation procedure needs a different parameterization given by:

$$\Delta^2 x_t = \alpha \left[\rho' \begin{pmatrix} \tau \\ \tau_{01} \\ \tau_0 \end{pmatrix}' \begin{pmatrix} x_{t-1} \\ t_{91:1,t-1} \\ t-1 \end{pmatrix} + \begin{pmatrix} \delta \\ \delta_{01} \\ \delta_0 \end{pmatrix}' \begin{pmatrix} \Delta x_{t-1} \\ D_{s91:1,t-1} \\ 1 \end{pmatrix} \right]$$

$$+\alpha_{\perp\Omega}\kappa' \begin{pmatrix} \tau \\ \tau_{01} \\ \tau_0 \end{pmatrix}' \begin{pmatrix} \Delta x_{t-1} \\ D_{s91:1,t-1} \\ 1 \end{pmatrix} + \Phi_p D_{tax,t} + \varepsilon_t, \tag{46}$$

$$t = 1975.09 - 1998.12$$

where $\tau = [\beta, \beta_{\perp 1}]$ and $\alpha_{\perp\Omega} = \Omega\alpha_{\perp}(\alpha'_{\perp}\Omega\alpha_{\perp})^{-1}$.

Finally, $\rho' = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$ is a $(p - s_2) \times p_d$ matrix, where I_r is a $r \times r$ unit matrix and 0 stands for a zero matrix. Its main purpose is to pick up the β relations that belong to the r polynomially cointegrating relations.

8 Empirical illustration: US-German prices, interest rates and the \$/Dmk rate

Figure 5, upper panel shows the graphs of relative prices and the nominal exchange rate. The data seem to exhibit two stochastic trends: an upward sloping trend in US-German prices, which is also visible in nominal exchange rates, though it is somewhat diffused by the long swings movements typical of the latter. The Dornbush overshooting type of models based on RE would assume that relative prices and nominal exchange share a common $I(1)$ trend, whereas the endogenous money model versions would be consistent with the trend to be near $I(2)$. In both cases, the model assume that the real exchange rate is stationary or at most a near $I(1)$ process. In the lower panel the real exchange rate is graphed together with the real interest rate differential. The REH-based models assume that both of them are stationary or at most near $I(1)$. Similar to the endogenous money models, the IKE based models assume that relative prices and nominal exchange rates are (near) $I(2)$, but contrary to the REH-based models, they assume that the real exchange rate and the nominal interest rate differential are also near $I(2)$, but cointegrate to $I(1)$ and further down to $I(0)$ when adding the inflation spread.

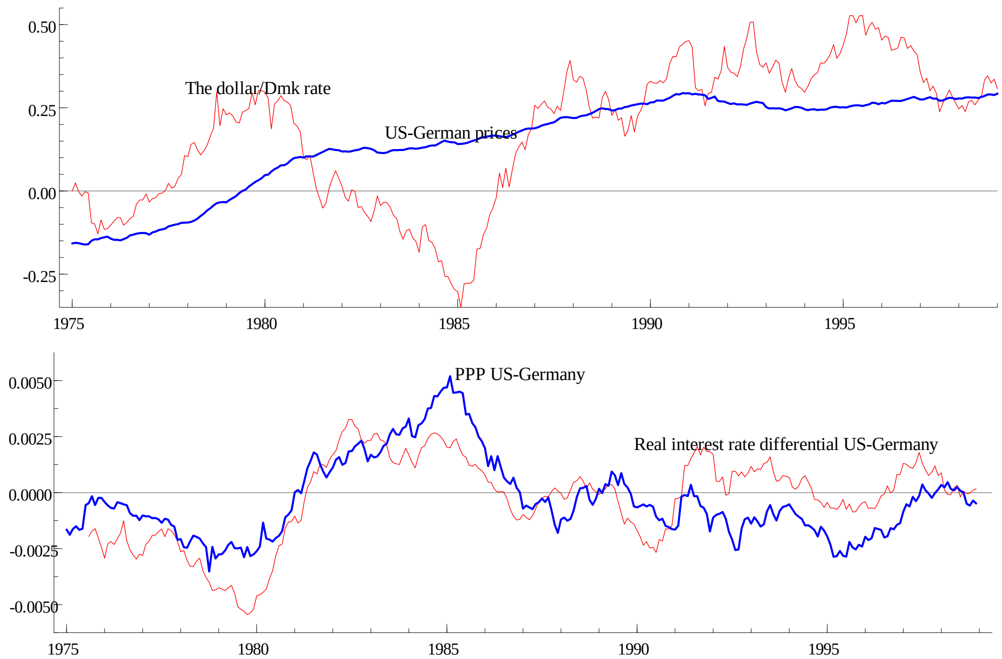

Figure 5: The graphs of the (mean and range adjusted) German-US price differential, pp, and the nominal exchange rate, s_{12} (upper panel), and the $ppp = pp - s_{12}$ and the real bond rate differential(lower panel).

Figure 6, upper panel, shows the graph of the bond differential together with its 12 month moving average and lower panel its difference.

8.1 Specification tests and rank determination

Table 1 reports various model specification tests which show that the interest differential and the US bond rate model do not pass the ARCH and the residual normality test. Non-normality and ARCH are typical features of financial variables, but adding more dummies is not necessarily a good solution. As the non-normality is primarily due to excess kurtosis but not skewness and the VAR results are reasonably robust to moderate ARCH and excess kurtosis I continue with this model.

Table 2 reports the $I(2)$ trace tests for the choice $r = 2, 3$ as well as

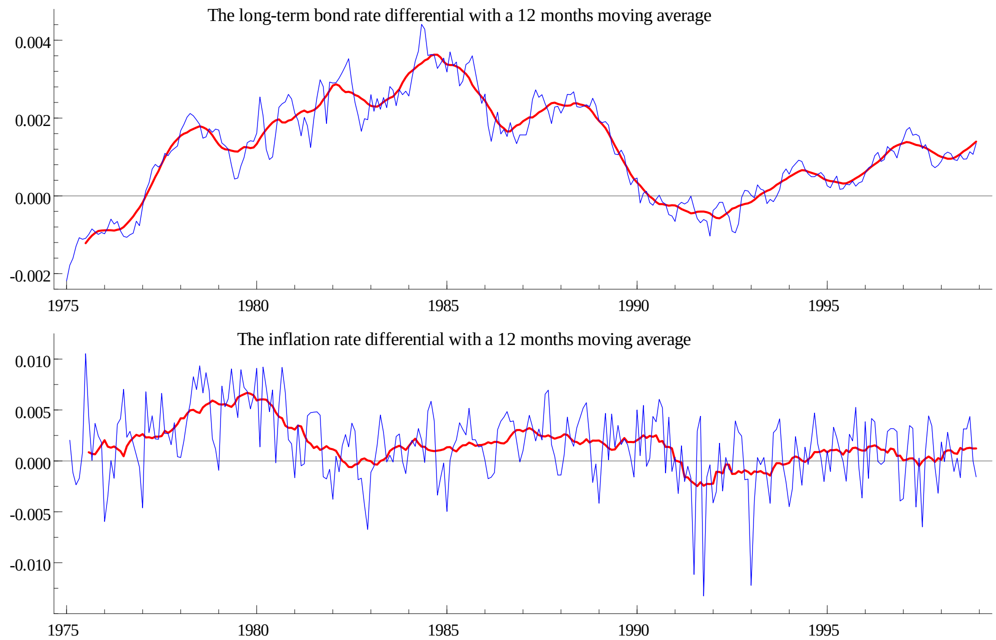

Figure 6: The graph of the long-term bond rate differential (upper panel) and inflation rate differential (lower panel. Actual values together with a 12 months moving average.

the characteristic roots of the model. For the unrestricted VAR model there are five large roots, four of which are almost exactly on the unit circle (0.98) while the fifth is large (0.93) but not equally close to one. Thus, the choice of rank indices should be consistent with four or five unit roots. The trace test suggests $\{r = 2, s_1 = 1, s_2 = 2\}$ with a p-value of 0.65. This choice restricts five of the characteristic roots to be on the unit circle and reports the largest unrestricted root to be 0.49. As a sensitivity check I report also the case $\{r = 3, s_1 = 0, s_2 = 2\}$ which forces four of the characteristic roots to be on the unit circle. This choice leaves, however, a fairly large root of 0.87 in the model.

Table 2 also reports the roots under the assumption that data are $I(1)$. The choice of $\{r = 2, s_1 = 3, s_2 = 0\}$ would leave two large characteristic roots (0.96, 0.96) and the choice $\{r = 3, s_1 = 2, s_2 = 0\}$ three large char-

Table 1: Misspecification tests

Multivariate tests:					
Autocorrelation:	Lag 1: $\chi^2(25) = 33.1$ [0.13]				
	Lag 2: $\chi^2(25) = 27.0$ [0.36]				
ARCH:	Lag 1: $\chi^2(225) =$ **368.1** [0.00]				
	Lag 1: $\chi^2(450) =$ **644.5** [0.00]				
Normality:	$\chi^2(10) =$ **40.7** [0.00]				
Univariate tests:					
	$\Delta^2 pp_t$	$\Delta^2 s_t$	$\Delta^2(b_1 - b_2)$	$\Delta^2 p_1$	$\Delta^2 b_1$
ARCH	0.06 [0.97]	4.72 [0.09]	**15.46** [0.00]	2.10 [0.35]	**29.00** [0.00]
Skew.	-0.10	-0.07	-0.12	-0.17	-0.10
Kurt.	3.08	3.56	4.97	4.75	5.32
Norm.	0.74 [0.69]	4.78 [0.09]	**34.20** [0.00]	**27.93** [0.00]	**43.90** [0.00]
R^2	0.60	0.12	0.13	0.70	0.09

acteristic roots (0.97,0.97,0.93) in the model. In both cases, such large root would render any inference on stationarity completely unreliable. I conclude, therefore, that only the case $\{r = 2, s_1 = 1, s_2 = 2\}$ is able to account for all five large roots in the unrestricted VAR and continue with this choice.

Based on the tests of the reduced rank conditions it seems straightforward to conclude that the Dornbush (1976) and the Dornbush-Frankel (1976) overshooting model cannot explain the variation in the data. These models would suggest $\{r = 4, s_1 = 0, s_2 = 1\}$ for which there is little support. This leaves the Benigno endogenous money model to compete with the IKE based model. The REH based model with endogenous money was consistent with the case $\{r = 3, s_1 = 1, s_2 = 1\}$ and the IKE based models with $\{r = 3, s_1 = 0, s_2 = 2\}$ or $\{r = 2, s_1 = 1, s_2 = 2\}$. Thus, the REH based model seems unable to account for the second $I(2)$ trend that is associated with the long and persistent movements of real and nominal exchange rates away from fundamental PPP values. In contrast, the IKE based model predicts two near $I(2)$ trends one of which should be associated with the long swings in the nominal exchange rates. Thus, the reduced rank tests and the characteristic roots seem to favor the IKE-based theoretical model compared to the REH-based models.

Table 2: Determination of the two rank indices

Rank Test Statistics								
$p-r$	r	$s_2=5$	$s_2=4$	$s_2=3$	$s_2=2$	$s_2=1$	$s_2=0$	
3	2			132.58	**50.41**	**36.81**	**38.09**	
				[0.00]	[0.65]	[0.66]	[0.14]	
2	3				**30.91**	**12.08**	**15.62**	
					[0.76]	[0.99]	[0.53]	
Six largest characteristic roots:								
Unrestricted VAR			**0.98**	**0.98**	**0.98**	**0.98**	**0.93**	0.48
Case 1:								
$r=2, p-r=3$			1.0	1.0	1.0	**0.96**	**0.96**	0.49
$r=2, s_1=1, s_2=2$			1.0	1.0	1.0	1.0	1.00	0.49
Case 2:								
$r=3, p-r=2$			1.0	1.0	**0.97**	**0.97**	**0.93**	0.48
$r=3, s_1=0, s_2=2$			1.0	1.0	1.0	1.0	**0.87**	0.50

8.2 Testable Hypotheses

The empirical implications formulated in the CVAR scenarios of the REH and IKE based models differ regarding the persistence properties characterizing data and relations. The trace tests of the previous section showed that $x_t \sim I(2)$ could not be rejected and the more specific hypotheses should be formulated and tested within the $I(2)$ VAR model (46). All tests discussed below are described in (Johansen et al. 2009).

8.2.1 General tests of model specification

The first type of hypotheses is expressed as $\tau = H\varphi$ or alternatively $R'\tau = 0$ and imposes the same restriction on all τ. Three hypotheses are of interest: (i) The hypothesis that a linear trend is needed in the cointegration relations was rejected based on $\chi(3) = 92.95[0.00]$. (ii) The hypothesis that the trend slope in the cointegration relations changed after the reunification could be rejected based on $\chi(3) = 3.71[0.29]$. (iii) The hypothesis of long-run price-homogeneity was rejected based on $\chi(3) = 13.01[0.00]$. Thus, the test results indicated that the change in the slope of the linear trend after the reunification is not highly significant in the cointegration relations. As a sensitivity check, the CVAR analysis was performed with and without this trend, t_{91}. While the conclusions were altogether very robust to this change in specification, the broken trend seemed nevertheless to improve the model specification to some extent and was, therefore, left in the model.

8.2.2 Specific tests of the order of integration

Testing the hypothesis that a variable/relation is $I(1)$ in the $I(2)$ model can be formulated as a known vector b_1 in τ, i.e. $\tau = (b_1, b_{1\perp}\varphi)$ where $b_{1\perp}\varphi$ defines the other vector to be unrestricted and lie in the orthogonal space of b_1. For example $b_1 = [1,0,0,0,0,0,0,0,0]$ is a test whether relative prices is a unit vector in τ. If accepted, it would imply that pp_t is $I(1)$. However, it might be reasonable to allow a deterministic trend to enter the variable/relation, i.e. to test whether the trend-adjusted variable/relation is $I(1)$. Therefore, prices, and nominal and real exchange rates which will be tested with and without a (broken) trend. In the latter case, the test is formulated as $\beta = (H_1\varphi_1, H_2\varphi_2)$. For example, $H_1' = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ would be a test of trend-adjusted relative prices.

Table 3 reports the test results. Except for the German bond rate, all hypotheses were strongly rejected, implying that the differenced processes have exhibited sufficiently pronounced persistence to reject the $I(1)$ hypothesis. I interpret this to mean that variables/relations exhibit such pronounced persistence to be classified as type (near) $I(2)$ according to the discussion in Section 2. The fact that the German bond rate could be rejected as $I(2)$ with a p-value of 0.20 is an indication that the German bond rate has moved in a slightly less persistent manner than the other variables.

The above results support the IKE-based models, whereas the rejection of the $I(1)$ hypotheses $\mathcal{H}_8 - \mathcal{H}_9$ and $\mathcal{H}_{11} - \mathcal{H}_{15}$ is inconsistent with the REH-based models. As essentially all hypotheses so far have supported the IKE-based model, the remaining results will, therefore, exclusively be discussed in terms of the former.

8.3 The pushing forces

Equation (40) provided a general expression for how the first and second order stochastic trends load into the variables. Because the meaning of the first order stochastic trends is less clear in the $I(2)$ model, I shall here primarily focus on the (near) $I(2)$ trends and how they load into the data as given by $C_2 = \beta_{\perp 2}(\alpha'_{\perp 2}\Psi\beta_{\perp 2})^{-1}\alpha'_{\perp 2}$ in (41).

Table 3: Testing hypotheses of I(1) versus I(2)

		pp	s	$b_1 - b_2$	b_1	p_1	t_{91}	t	$\chi^2(v)$	$p-val$
Are relative prices $I(1)$?										
$\mathcal{H}_1$	τ_1'	1.0	-	-	-	-	-	-	79.9 (4)	0.00
$\mathcal{H}_2$	β_1'	1.0	-	-	-	-	-	*	41.7 (4)	0.00
$\mathcal{H}_3$	β_1'	1.0	-	-	-	-	*	*	24.0(3)	0.00
Is the nominal exchange rate $I(1)$?										
$\mathcal{H}_4$	τ_1'	-	1.0	-	-	-	-	-	23.9 (4)	0.00
$\mathcal{H}_5$	β_1'	-	1.0	-	-	-	-	*	23.9 (4)	0.00
$\mathcal{H}_6$	β_1'	-	1.0	-	-	-	*	*	13.2 (3)	0.00
Is the US trend-adjusted price $I(1)$?										
$\mathcal{H}_7$	β_1'	-	-	-	-	1.0	*	*	40.7 (3)	0.00
Is the bond rate differential $I(1)$?										
$\mathcal{H}_8$	τ_1'	-	-	1.0	-	-	-	-	13.9 (4)	0.00
Is the US bond rate $I(1)$?										
$\mathcal{H}_9$	τ_1'	-	-	-	1.0	-	-	-	16.3 (4)	0.00
Is the German bond rate $I(1)$?										
$\mathcal{H}_{10}$	β_1'	-	-	-1.0	1.0	-	-	-	5.9 (4)	0.21
Is the real exchange rate $I(1)$?										
$\mathcal{H}_{11}$	τ_1'	1.0	-1.0	-	-	-	-	-	12.0 (4)	0.02
$\mathcal{H}_{12}$	β_1'	1.0	-1.0	-	-	-	-	*	19.5 (4)	0.00
$\mathcal{H}_{13}$	β_1'	1.0	-1.0	-	-	-	*	*	14.6 (3)	0.00
Are prices cointegrated CI(2,1)?										
$\mathcal{H}_{14}$	β_1'	1	-	-	-	*	-	-	30.5 (4)	0.00
$\mathcal{H}_{15}$	β_1'	-	1	-	-	*	-	-	22.9 (4)	0.00

The unrestricted estimates of $\beta_{\perp 2}$ and $\alpha_{\perp 2}$ are reported in Table 4. The estimates of $\alpha_{\perp 2}$ show that the twice cumulated shocks to the bond rate differential and to the US bond rate seem to dominate the two $I(2)$ stochastic trends in this period. The estimate of $\beta_{\perp 2,1}$ to the first trend, $\alpha'_{\perp 2,1}\Sigma\Sigma\varepsilon$, seems to load into relative prices, nominal exchange rate and US prices with coefficients of similar magnitude and signs, whereas the second trend seems to load into relative prices and US prices with coefficients of opposite sign of the nominal exchange rate. The intuition of this can be seen from Figure **??**, which suggests that one of the stochastic long-run trends is likely to describe the common long-run upward trending behavior visible in both relative prices

Table 4: The unrestricted I(2) matrices $\beta_{\perp 2}$ and $\alpha_{\perp 2}$

	pp_t	s_t	$b_{1,t} - b_{2,t}$	$p_{1,t}$	$b_{1,t}$
$\beta_{\perp 2,1}$	**0.50**	**0.73**	-0.00	**0.46**	0.00
$\beta_{\perp 2,2}$	**-0.25**	**0.66**	-0.01	**-0.66**	-0.01
$\alpha_{\perp 2,1}$	0.00	-0.00	**1.00**	0.00	**0.18**
$\alpha_{\perp 2,1}$	.0.00	-0.00	**0.10**	-0.00	**1.00**

and nominal exchange rate, and the other the long and persistent swings away from long-run PPP benchmark values. Therefore, the first $I(2)$ trend is likely to describe the smooth upward trending behavior in relative prices and nominal exchange rates and the second one the long swings behavior in nominal and real exchange rates.

When $s_2 = 1$, i.e. when there is just one $I(2)$ trend, the inverted matrix $(\alpha'_{\perp 2}\Psi\beta_{\perp 2})^{-1}$ in the expression for the C_2 matrix (41) is just a scalar and $\beta_{\perp 2}$ and $\alpha_{\perp 2}$ are identified up to this scalar. When $s_2 = 2$, as in our case, the inverse is a 2×2 matrix and it is possible to choose different identification schemes. Table 5 reports a just identified $I(2)$ representation using the decomposition in (41) where $\tilde{\beta}_{\perp 2}$ is derived subject to two just-identifying restrictions on $\alpha_{\perp 2}$. As these restrictions entail a sharp distinction between shocks to the interest rate differential and the level of US interest rate, they change the loadings, $\tilde{\beta}_{\perp 2,1}$,to some extent, in particular the loadings to the nominal exchange rate. The estimates of $\alpha_{\perp 2}$ and $\tilde{\beta}_{\perp 2}$ suggest that the *long-run impact* of positive permanent shocks to the US-German *interest differential* is to increase relative prices between US and Germany and to increase the dollar/Dmk rate and that the *long-run impact* of positive shocks to *the level* of US bond rate is a lowering of relative prices and an appreciation of the dollar/Dmk rate.

How can this be understood? First, the interest rate spread trend seems to describe the standard mechanism of cost push pressure: A positive permanent shock to the domestic-foreign interest rate differential, tends to increase the relative cost of capital in production and, in the long-run, cause relative prices to increase and the exchange rate to depreciate. The loadings suggest that the long-run impact is strongest for nominal exchange rates. However, the results in Table 4 suggest a more similar long-run impact when the stochastic trend is defined as shocks to both the interest rate spread and the level of long-term interest rate, suggesting that the cost-push mechanism is triggered

Table 5: Just identified common stochastic trends and their loadings

$$\begin{bmatrix} pp_t \\ s_t \\ b_{1,t}-b_{2,t} \\ p_{1,t} \\ b_{1,t} \end{bmatrix} = \begin{bmatrix} \mathbf{0.99} & \mathbf{-0.47} \\ \mathbf{2.71} & \mathbf{-4.89} \\ -0.02 & \mathbf{0.05} \\ \mathbf{0.54} & \mathbf{0.78} \\ -0.01 & \mathbf{0.04} \end{bmatrix} \begin{bmatrix} \sum_{\mathbf{s=1}}^{\mathbf{t}} \sum_{\mathbf{j=1}}^{\mathbf{s}} u_{1,s} \\ \sum_{\mathbf{s=1}}^{\mathbf{t}} \sum_{\mathbf{j=1}}^{\mathbf{s}} u_{2,s} \end{bmatrix} + \begin{bmatrix} * & * \\ * & * \\ * & * \\ * & * \\ * & * \\ * & * \\ * & * \end{bmatrix} \begin{bmatrix} t_{91.1} \\ t \end{bmatrix} + ...$$

where $u_{1,t} = \alpha'_{\perp 2,1}\varepsilon_t$ and $u_{2,t} = \alpha'_{\perp 2,2}\varepsilon_t$ with

$$\begin{bmatrix} \alpha'_{\perp 2,1} \\ \alpha'_{\perp 2,2} \end{bmatrix} = \begin{bmatrix} \underset{[0.67]}{0.02} & \underset{[1.30]}{0.01} & \mathbf{1.00} & \underset{[-0.01]}{-0.00} & \mathbf{0} \\ \underset{[0.83]}{0.01} & \underset{[0.82]}{0.00} & \mathbf{0} & \underset{[1.30]}{0.02} & \mathbf{1.00} \end{bmatrix},$$

and the residual standard errors for the variables are

$\hat{\sigma}(pp_t)$	$\hat{\sigma}(s_t)$	$\hat{\sigma}(b_{1,t}-b_{2,t})$	$\hat{\sigma}(p_{1,t})$	$\hat{\sigma}(b_{1,t})$
0.00218	0.0311	0.00027	0.00174	0.00029

t-ratios are given in [ ] and standard errors are calculated using Paruolo (2002).

off when there is a positive/negative shock to both the interest rate level and the differential.

The second identified trend shows that the twice cumulated shocks to the level of long-term interest rate has a *negative* long-run impact on relative prices and the nominal exchange rate. This seems to be more consistent with speculative behavior in the currency exchange markets and suggests that shocks to the long-term bond rate is the main driver of the long swings.

An increase in the long-term bond rate (for example, as a result of government deficit due to high unemployment, say, or a large trade deficit) tends to increase the amount of speculative capital moving into the economy thereby causing an appreciation of the exchange rate, which is likely to worsen the domestic imbalance (whether due to government deficit or trade balance deficit) and cause further increases in the long-term interest rate, which again will tend to increase the speculative demand for domestic currency. Thus, as long as there is a growing structural imbalance in the economy the long-term interest rate is likely to increase as a consequence of the increased financing need. This is likely to generate persistent movements away from parity in domestic interest rates and real exchange rates. As domestic competitiveness worsens, enterprises might be forced to lower their prices in domestic currency to maintain an international competitive level. This can, for example, be achieved by improving labor productivity and/or by squeezing profit shares. This process will continue until the real exchange rate has been driven into such 'far from equilibrium' regions that loss averse speculators increase their risk premium for holding domestic currency, thereby causing a reversal of the exchange rate movements now toward the long-run benchmark values. As the currency depreciates, enterprises are likely to compensate for previous low profits rather than changing prices. This is essentially what the pricing-to-market theory in Krugman (1993) would suggest. It also seems consistent with Phelps (1994) customer markets theory.

To summarize: two forces seem to be at work: on one hand, the standard forces based on competition between two trading partners, on the other, the speculative forces based on speculation in the currency market.

8.4 The pulling forces

The case $\{r = 2, s_1 = 1, s_2 = 2\}$ defines two stationary polynomially cointegrating relations, $\beta_i^{'} x_t + \delta_i' \Delta x_t$, $i = 1, 2$ and one stationary medium-run relation in growth rates, $\beta_{\perp 1}' \Delta x_t$. The former can be thought of as a dynamic equilibrium relations in the following sense: when data are $I(2)$, $\beta^{'} x_t$ is generally $I(1)$ and can be given an interpretation as equilibrium errors that exhibit pronounced persistence. In such a case, it is relevant to ask how the growth rates, Δx_t, dynamically react to these deviations. $\delta' \Delta x_t$ provides an answer to this question. Thus, when discussing the adjustment dynamics in the $I(2)$ model, it is useful to interpret the coefficients α and δ as two levels of equilibrium correction: the δ adjustment describes how the growth rates, Δx_t, adjust to the long-run equilibrium errors, $\beta^{'} x_t$, the α adjustment describes how the acceleration rates, $\Delta^2 x_t$, adjust to the dynamic equilibrium relations, $\beta^{'} x_t + \delta' \Delta x_t$. This is illustrated below for the variable $x_{i,t}$:

$$\Delta^2 x_{i,t} = \cdots \sum_{i=1}^{r} \alpha_{ij} (\delta_i' \Delta x_{t-1} + \beta_i' x_{t-2}) + \cdots, j = 1, ..., p \qquad (47)$$

where $\delta_i' = [\delta_{i1,...,}\delta_{ij,...,}\delta_{ip}]$ and β_i' is similarly defined. If $\alpha_{ij}\delta_{ij} < 0$ then the acceleration rates, $\Delta^2 x_{i,t}$, are equilibrium correcting to the changes, $\Delta x_{i,t}$,

Table 6: An identified long-run structure in β

	$\tilde{\beta} = (h_1 + H_1\varphi_1, \ldots, h_r + H_r\varphi_r)$, $\chi^2(5) = 3.08[0.69]$						
	$p_{1,t} - p_{2,t}$	s_t	$b_{1,t} - b_{2,t}$	$p_{1,t}$	$b_{1,t}$	$t_{91.1}$	t
$\tilde{\beta}_1'$	**−0.01** [−28.15]	**0.01** [28.15]	**1.00** [NA]	−	−	0.00 [2.02]	−
$\tilde{\delta}_1'$	**−0.53**	**−0.27**	−0.00	**−0.66**	−0.01	−0.01	0.01
α_1'	**0.54** [16.18]	**1.21** [2.56]	**−0.02** [−5.54]	**0.17** [6.45]	**−0.01** [−2.19]		
$\tilde{\beta}_2'$	−	**0.01** [12.87]	−	**−0.01** [−96.69]	**1.00** [NA]	−	**3.37**[1)] [14.48]
$\tilde{\delta}_2'$	**−0.27**	**0.13**	**−0.01**	**−0.42**	**−0.01**	0.00	0.05
α_2'	**−0.28** [−2.72]	**−4.30** [−2.96]	**0.05** [3.71]	**0.51** [6.18]	0.01 [0.49]	−	−
$\tilde{\beta}_{\perp,1}'$	**1.00**	**−0.23**	0.02	**−0.71**	−0.01	−0.00	0.00[1)]
$\alpha_{\perp,1}'$	**0.30** [2.26]	−0.07 [−1.63]	0.00 [0.28]	**−0.44** [−4.10]	0.01 [0.53]		

1) The trend has been multiplied by 10000. t-values> 2.0 in bold face.

and if $\delta_{ij}\beta_{ij} > 0$, then the changes, $\Delta x_{i,t}$, are equilibrium correcting to the levels, $x_{i,t}$. Whether a variable is equilibrium error correcting (equilibrium error correcting) or equilibrium error increasing is an important feature of a dynamic system. In particular, one would expect that the long and persistent swings away from fundamental PPP values implies equilibrium error increasing behavior somewhere in the system. This is also what I find.

As discussed in Johansen et al. (2008), it is straightforward to impose and test (over)identifying restrictions on $\beta' x_t$, whereas not yet on $\delta'\Delta x_t$ and $\beta'_{\perp 1}\Delta x_t$. Table 6 report an overidentified structure on β, and the corresponding unrestricted estimates of δ and $\beta'_{\perp 1}$.

The first polynomially cointegrated relation, $\tilde{\beta}_1'\tilde{x}_t + \tilde{\delta}_1'\Delta\tilde{x}_t$, is given by:

$$(b_1 - b_2)_t - 0.01(p_{1,t} - p_{2,t} - s_t) - 1.2\Delta p_{1,t} + 0.53\Delta p_{2,t} - 0.27\Delta s_t + ...$$

corresponds closely to the relation between real exchange rates and the real interest rate differential predicted by the IKE model. In the medium-run, both inflation rates adjust to the long-run PPP but very slowly so.

The speed of adjustment of US inflation rate is approximately 0.01 and of the German inflation rate 0.02. In both cases, it would take on average 6-8 years for the real exchange rate to return to its long-run value if inflation rates alone were to adjust. However, the IKE relation tells us that real exchange rate can deviate from its long-run value as long as the interest rate differential moves in a compensating manner. The adjustment coefficient $\alpha_{11} = 0.54$ shows that relative prices adjust quickly (in two months on average) when the interest rate differential exceeds the 0.01 fraction of the real exchange rate. Both the α and the δ adjustment is equilibrium error correcting in prices, but not in the nominal exchange rate, for which the δ adjustment is equilibrium error increasing but the α adjustment is equilibrium error correcting. This supports the IKE hypothesis that over the medium run the nominal exchange rates will have a tendency to move away from long-run benchmark values while over the long run they will move back towards equilibrium.

For the bond rates, the δ and the α adjustment are both equilibrium error increasing, suggesting that it is the behavior of the long-term interest rate that is key to understanding the long swings movements in the currency market. This conclusion was also reached from the analysis of the pushing forces in the previous section.

The second polynomially cointegrated relation, $\tilde{\beta}_1' \tilde{x}_t + \tilde{\delta}_1' \Delta \tilde{x}_t$, is given by:

$$b_{1,t} - 0.7\Delta p_{1.t} - 0.01(p_{1,t} - s_t) + 0.00000 trend + 0.27\Delta p_{2,t} + 0.13\Delta s_t + ... \quad (48)$$

and corresponds closely to the second cointegration relation predicted by the scenario analysis. It shows that an increase in nominal/real interest rates is associated with an increase in the domestic price nominated in the foreign currency. US prices and the Dollar/Dmk rate are equilibrium error correcting in both δ and α. The US bond rate is equilibrium error increasing in δ but error correcting in α. Thus, the second relation seems to describe standard mechanisms for prices and exchange rates in the market for tradeables. The bond rate is equilibrium increasing over the medium run, while equilibrium correcting in the long run.

The medium-run stationary relation between growth rates can be formulated as:

$$\Delta p_{2,t} \simeq 0.3(\Delta p_{1,t} - \Delta s_t).$$

This corresponds closely to the medium-run relation predicted by the IKE scenario analysis. It suggests that over the medium-run the German inflation rate has been affected by the US inflation rate measured in Dmk with a coefficient of approximately 0.3, possibly measuring the effect of imported inflation. That the coefficient differs from 1.0 is consistent with the IKE scenario predicting that $(p_1 - p_2 - s) \sim I(2)$ and, hence, $(\Delta p_1 - \Delta p_2 - \Delta s) \sim I(1)$. It suggests that German inflation rate has generally been lower than the US inflation rate even when accounting for the exchange rate. The $I(1)$ trend, given by $\alpha'_{\perp,1} \sum \varepsilon_{i,s}$, seems to be primarily be related to US and German inflationary shocks.

9 Summary and conclusions

In this paper I argue that properly accounting for unit roots (near unit roots) in the model provides a powerful way of classifying data into persistent and less persistent directions and that this can be used for testing and comparing different competing models. I also argue that the order of integration of a variable/relation should not necessarily be thought of as a structural parameter, but rather as a useful way of classifying data and relations according to their persistence. This, however, does not exclude the possibility that structural hypotheses can be translated into theory-consistent scenario analyses based on theoretical assumptions of persistence. For example, the following basic implications of IKE and REH based models for exchange rate determination were translated into testable hypotheses in the CVAR model:

1. Under IKE, speculative behavior in the currency market is likely to drive prices away from long-run PPP benchmark values for extended periods of time. Such persistent movements away from equilibrium PPP values are likely to have the property of a near $I(2)$ process, i.e. real exchange rates are likely to be near $I(2)$. REH-based models assume that movements away from long-run PPP values are stationary, or at most near $I(1)$.

2. Such persistent swings in real exchange rates have to be offset by something else. The IKE theory tells us that it should be the real interest rate differential. Hence, real interest rate differentials should exhibit a similar persistence as real exchange rates, i.e. be near $I(2)$. The REH-based theory assume that the real interest rate differential should be stationary or at most near $I(1)$.

3. According to IKE, real exchange rates and real interest rate differentials should cointegrate to a stationary relation, whereas according to REH,

they should be individually stationary albeit allowed to exhibit some persistence.

4. According to IKE, prices need not be rigid to produce the long swings in real exchange rates, but the speed of adjustment has to differ between relative prices and nominal exchange rate, i.e. to explain the long swings a de-linking of prices and nominal exchange rate is needed. REH-based overshooting models with price rigidities do not assume de-linking whereas endogenous money versions do.

The tests strongly supported the empirical relevance of IKE-based economic mechanisms as opposed to REH-based. In particular, the Dornbush-Frankel overshooting model with price rigidities was not able to explain the features of the data. This was partly due to the assumption of identical speed of adjustment coefficients for relative prices and nominal exchange rate that has to be loosened up to understand exchange rate movements. But this delinking (a feature of both the endogenous money and the IKE based model) is not sufficient to explain the near $I(2)$ properties of the data that have generated the long swings in real exchange rates. Of the two type of models, only the IKE model could explain these features of the data.

10 Discussing the results

The advantage of the scenario analysis is that it forces us to formulate *all* testable implications of the hypotheses underlying a theoretical model rather than to focus on single hypotheses, which might make sense in isolation but not in the full context of the model. A fully specified scenario can, therefore, be seen as a safeguard against testing internally inconsistent hypotheses. It can also help us to modify untenable parts of the theoretical model and to choose between competing models. Therefore, it is likely to enhance our ability to select and develop empirically relevant models in contrast to the common practice of forcing a theoretical model onto the data with its numerous untested assumptions. In the latter case there is an obvious risk that such signals in the data which suggest a different set of economic mechanisms will be overlooked. The failure of extant models to foresee the recent financial and economic crisis suggests that important signals in the data were overlooked as a result of relying on untested basic assumptions.

While testing basic assumptions is important as a starting point, to be useful, models need also to be specific about the underlying economic mechanisms. The data analyses seemed to tell a story of strong speculative behavior, heterogeneous agents, imperfect knowledge, long swings, and strong reflexivity between the financial and the real economy. Hence, empirically relevant models should be based on these basic assumptions. But an empirically relevant model should also be informative about factors likely to trigger off a long swings cycle. Even though the purpose of the present paper was to discuss the basic assumptions, the empirical results provide some tentative hints as to which specific features such a model should contain.

What is initiating a long swings cycle?

The empirical results suggested that shocks to the long-term bond rate is the main driver of the long swings. Such persistent shocks would, for example, be likely to hit the domestic economy in periods of severe structural imbalances (compared to a baseline economy). In Europe such structural imbalances have typically been associated with high unemployment rates often due to a political reluctance to adequately address painful structural reforms. In USA, structural imbalances are typically associated with trade balance problems, possibly because of an overvalued dollar (due to the role of the dollar as a reserve currency). Whatever the reason for the structural imbalance, the long-term interest rate is likely to increase relative to a global benchmark as a consequence of the increased financing need.

Why speculation?

In a world where the Fisher parity holds as a stationary condition, high long-term interest rates due to structural imbalances would be associated with high inflation rates. Therefore, speculators would not have much incentive for moving their long-term capital to an economy with increasing nominal interest rates. In an IKE world the Fisher parity does not hold as a stationary condition. This is because nominal interest rates exhibit pronounced persistence due to an uncertainty premium, whereas inflation rates are much more stable. Thus, real domestic interest rates are likely to behave as a very persistent process, offering an opportunity for financial markets to increase speculative positions in this economy. When the demand for the domestic currency increases, the domestic interest rate tends to increase, which again will increase the real domestic interest rate, which in turn leads to an increased demand for currency and a real appreciation of the exchange rate.

What is the likely impact of speculation on the real economy?

When the nominal long-term interest rate increases but consumer price inflation does not, the real interest rates will raise. Increasing real interest rates are likely to increase the speculative demand for domestic currency, hence increasing its price. Thus, there will be a tendency for the domestic real interest rate to increase and the real exchange rate to appreciate at the same time, aggravating domestic competitiveness. The equilibrium increasing behavior in the nominal exchange rate over the medium run and the persistent swings in real exchange rates support such an interpretation. Enterprises cannot in general count on nominal exchange rates to restore competitiveness after a price increase as due to increasing costs. As enterprises struggle to survive, they will tend to improve labor productivity by laying off the least productive part of the labor force. The struggle for market shares implies that profits have to adjust and pricing to market is likely to replace constant mark-up pricing. If this mechanism is at work the profit share would be co-moving with the real exchange rate (see Phelps, 1994). Evidence of this can for example be found in Juselius (2006).Enterprises will be forced to adjust profits rather than prices and profits are likely to be squeezed in periods of persistent appreciation and increased during periods of depreciation. Thus, pricing-to-market (Krugman, 1993) or Phelps customer markets (Phelps, 1994) is likely to work as a pricing mechanism in an IKE world with strong global competition and a currency float.

This vicious spiral of increasing real interest rates and real appreciation rates (which was empirically manifested in the equilibrium error increasing behavior of the δ adjustment) is likely to continue until the gap effect takes over. The fact that financial markets, being aware of the accruing imbalances, are likely to require increasingly large risk premiums for holding the currency will sooner or later cause a reversal in the exchange rate movement. In the empirical analysis this was manifested in the equilibrium error correcting behavior of the α adjustment.

If this scenario is correct, I would expect unemployment to rise/decrease and prices to stay unchanged or even decrease/increase during an appreciation/depreciation period and I would expect the real long-term interest rate, unemployment rate and inflation rate to be co-moving in a relationship that could be dubbed a modified Phillips Curve: $\Delta p = -b_1(u - u^*)$ where $u^* = f(i)$ describes the (non-constant) natural rate as a function of the interest rate. This relation plays an important role in Phelps Structural Slumps Theory (1994). The inflation rate is usually found to be equilibrium correcting to this relation.

What causes a reversal of the long swings?

The IKE theory suggests that the uncertainty premium increases with the gap effect. But to be able to use it constructively, it has to be measurable. As discussed above, the deviation from a long-run fundamental PPP value, the *ppp* term, is one important measure of the gap-effect, but there are likely to be other measures. As speculative behavior in the currency market, drives real exchange rates and real interest rates away from long-run benchmark values, domestic competitiveness comes under increasingly strong pressure causing profit shares and unemployment rates to adjust. Therefore, one can think of the *ppp* gap, the unemployment gap and the profit share gap as different but related effects that eventually are likely to put an end to the long swings movements in nominal exchange rates. Strong evidence of this can be found in Juselius (2006) and Juselius and Javier (2009) suggesting that reflexivity between the financial and the real sector of the economy is an important feature that needs to be understood.

11 References

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