Speaker

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(joint work with Theis Lange, both University of Copenhagen)

Title

A probability analysis of the Blanchard–Watson bubble model.

Abstract

Blanchard and Watson (1982) suggested a stochastic model for bubbles, and a simple example of this is the two regime $AR(1)$ model given by

$$y_t = s_t \rho y_{t-1} + \varepsilon_t,\ t = 1, \ldots, T,$$

where ε_t are i.i.d. $(0, \sigma^2)$ and independent of s_t which is i.i.d. $0, 1$ with $p = P(s_t = 1)$. It has been suggested that such a bubble model could generate persistence in the process y_t, and this is what we shall investigate.

We first prove that y_t is stationary and that the variance is infinite if $p\rho^2 > 1$.

We define persistence as a large coefficient in the regression of y_t on y_{t-1}.

The main result so far is that

$$\sum_{t=1}^{T} y_{t-1} y_t / \sum_{t=1}^{T} y_{t-1}^2 \xrightarrow{P} \min(\rho, \rho^{-2}), T \to \infty$$

Another result is that there are no coefficients c and c_T such that $c_T \sum_{t=1}^{T} y_{t-1}^2 \xrightarrow{P} c > 0$.

We also investigate the asymptotics for $p \to 1, T \to \infty$, and finally the convergence of the process to the continuous process

$$z_u = \sum_{i=0}^{N_u} \int_{T_{i-1}}^{u - T_{i-1}} e^{a(u-s)} dW(s), 0 \leq u < \infty,$$

where N_t is a Poisson process with jump points $0 = T_0 < T_1 < \cdots < T_{N_t}$

Reference

Blanchard, Olivier and Mark Watson. (1982). “Bubbles, Rational Expectations, and Financial Markets,” in Paul Wachter (ed.) *Crises in the Economic and Financial Structure.* Lexington, MA: Lexington Books, pp. 295–315.