

# Some Reflections on Agent-Based Models, Bounded Rationality and Heterogeneous Expectations

Cars Hommes

CeNDEF, Amsterdam School of Economics  
University of Amsterdam

**Workshop "Toward an alternative macroeconomic analysis of  
microfoundations, finance-real economy dynamics and crises",  
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# Questions

- a. Is **simulation** an effective tool to go from microfoundations with heterogeneous agents to macro conclusions?
- b. How to handle the problem of **individual expectations**?
- c. How to handle **price determination** and **market clearing** (or not)?

# Is Simulation an Effective Tool?

Simulations alone are **not** enough, because

- ▶ "wilderness of ABMs", too many degrees of freedom;
- ▶ often simulation results are **not** (easily) reproducible;
- ▶ **limited insights**: what exactly causes the (macro) outcome?

Einstein: A scientific theory should be **as simple as possible**, but **no simpler**.

Strong need for the **simplest ABM** explaining macro phenomena through micro interactions

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# How to model Heterogeneous Expectations of Boundedly Rational Individuals?

Brock and Hommes (1997): as an **evolutionary selection process** among different rules ranging from simple to sophisticated

- ▶ agents choose from a (small) list of simple forecasting heuristics and more complicated (costly) rules
- ▶ **adaptive learning**: some parameters of the heuristics are updated over time, e.g. anchor  $\equiv$  time average
- ▶ **performance based reinforcement learning**: agents evaluate the performances of all heuristics, and tend to switch to more successful rules; impacts are evolving over time

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- ▶ small list of forecasting heuristics with fixed parameters or some adaptive learning
- ▶ evolutionary selection of better performing rules, with 2 or 3 parameters for speed of switching (details not important)
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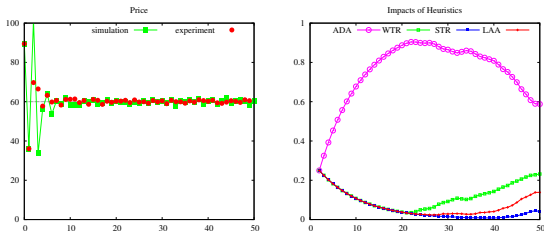
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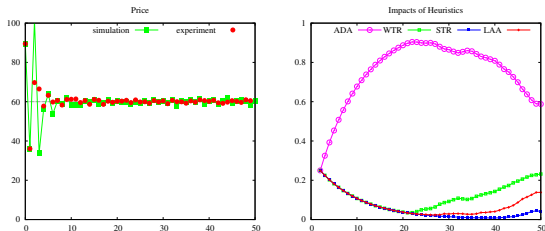
## Lab Experiments and Heuristics Switching Model: (Hommes, 2010)



for Muth's cobweb model with **negative expectations feedback**  
quick convergence to RE

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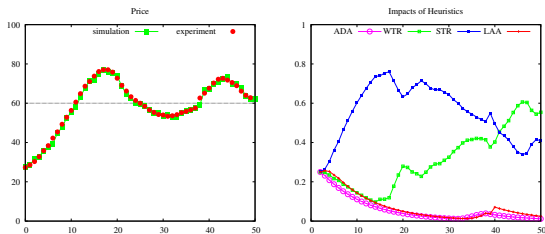
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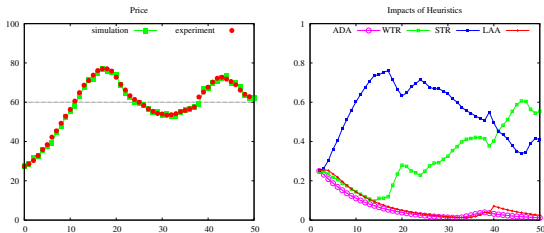
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# What about **price determination** and **market clearing** (or not)?

Two observations:

- ▶ price adjustment rule through excess demand/supply has **speed of adjustment parameter** which critically affects dynamics.  
How large is it??
- ▶ equilibrium model with **heterogeneous expectations** can explain important macro phenomena, such as persistence in inflation and output, price stickiness, misalignments, excess volatility, etc.

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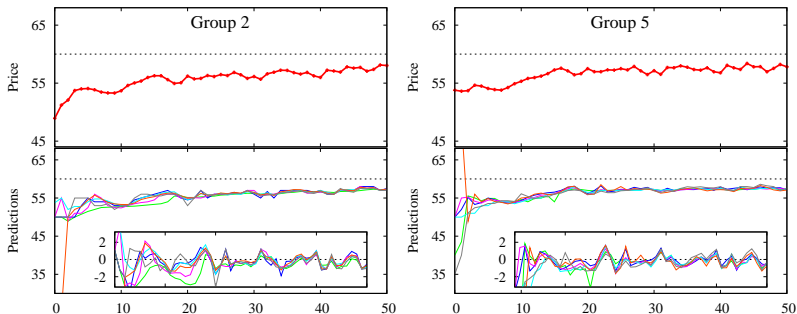
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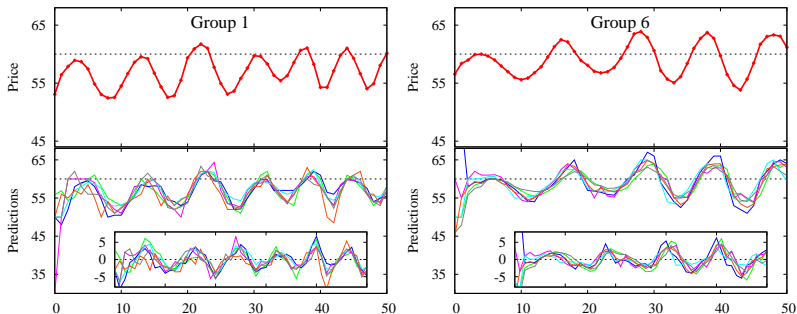
## 2 Groups with (Almost) Monotonic Convergence

prices, individual predictions and individual errors



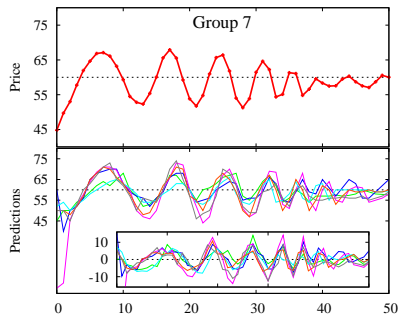
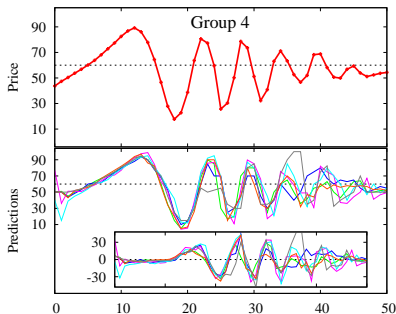
## 2 Groups with Perpetual Oscillations

prices, individual predictions and individual errors



## 2 Groups with Damping Oscillations

prices, individual predictions and individual errors



## Four forecasting heuristics

- ▶ adaptive rule

$$\text{ADA} \quad p_{1,t+1}^e = 0.65 p_{t-1} + 0.35 p_{1,t}^e$$

- ▶ weak trend-following rule

$$\text{WTR} \quad p_{2,t+1}^e = p_{t-1} + 0.4 (p_{t-1} - p_{t-2})$$

- ▶ strong trend-following rule

$$\text{STR} \quad p_{3,t+1}^e = p_{t-1} + 1.3 (p_{t-1} - p_{t-2})$$

- ▶ anchoring and adjustment heuristics with learnable anchor

$$\text{LAA} \quad p_{4,t+1}^e = 0.5 p_{t-1}^{av} + 0.5 p_{t-1} + (p_{t-1} - p_{t-2})$$

# Evolutionary Switching

Brock and Hommes (1997), Anufriev and Hommes (2009)

- ▶ **performance measure** of heuristic  $i$  is

$$U_{i,t-1} = -\left(p_{t-1} - p_{i,t-1}^e\right)^2 + \eta U_{i,t-2}$$

parameter  $\eta \in [0, 1]$  – the **strength** of the agents' **memory**

- ▶ **discrete choice** model with **asynchronous updating**

$$n_{i,t} = \delta n_{i,t-1} + (1 - \delta) \frac{\exp(\beta U_{i,t-1})}{\sum_{i=1}^4 \exp(\beta U_{i,t-1})}$$

parameter  $\delta \in [0, 1]$  – the **inertia** of the traders

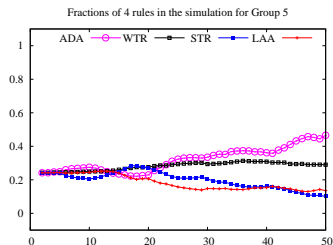
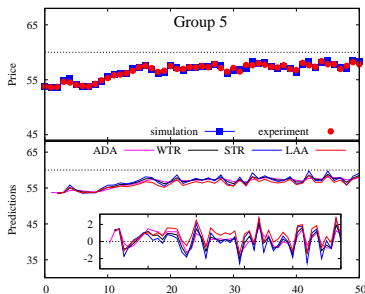
parameter  $\beta \geq 0$  – the **intensity of choice**

## Group 5 (Convergence)

experimental prices

**simulated** prices, predictions and errors

**Parameters:**  $\beta = 0.4, \eta = 0.7, \delta = 0.9$

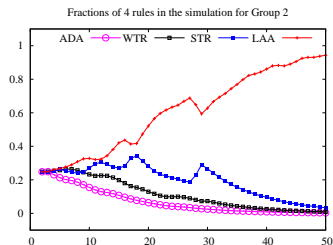
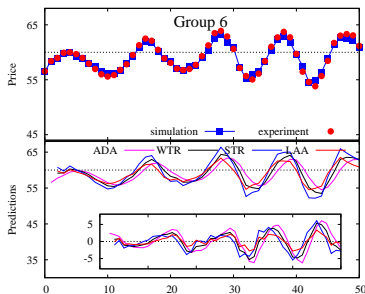


## Group 6 (Constant Oscillations)

experimental prices

**simulated** prices, predictions and errors

**Parameters:**  $\beta = 0.4, \eta = 0.7, \delta = 0.9$

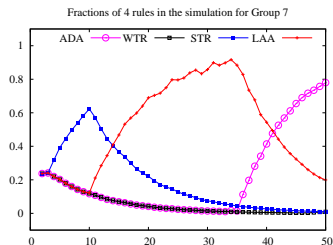
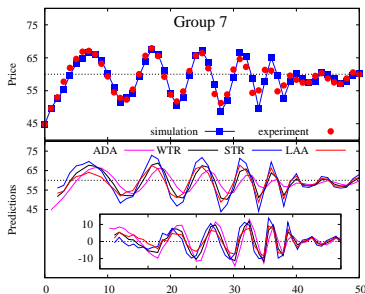


## Group 7 (Damping Oscillations)

experimental prices

**simulated** prices, predictions and errors

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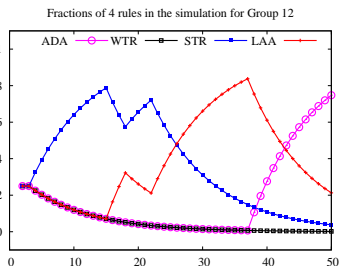
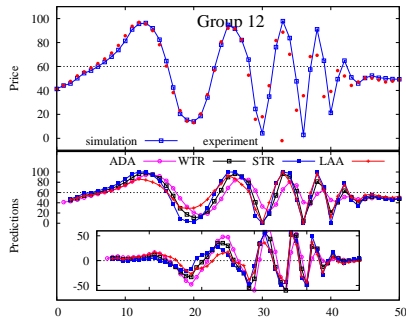


# Asset Pricing Experiments without Fundamental Trader

experimental prices

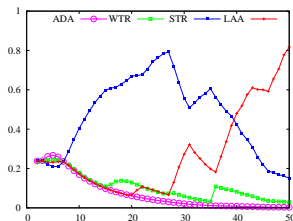
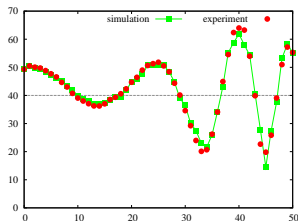
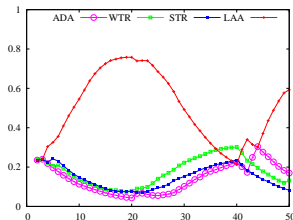
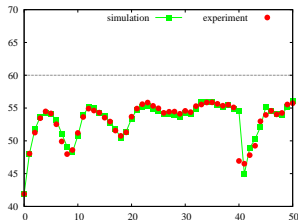
**simulated** prices, predictions and errors

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# More Asset Pricing Experiments

## Group 3 (Typing Error) and Fundamental $p^* = 40$



# Positive versus Negative Feedback Experiments

Heemeijer et al. (JEDC 2009); Te Bao, MPhil thesis, 2009

- ▶ **negative feedback** (strategic substitute environment)

$$p_t = 60 - \frac{20}{21} \left[ \sum_{h=1}^6 \frac{1}{6} p_{ht}^e \right] - 60] + \epsilon_t$$

- ▶ **positive feedback** (strategic complementarity environment)

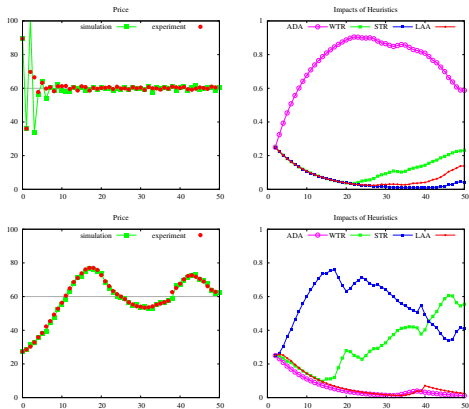
$$p_t = 60 + \frac{20}{21} \left[ \sum_{h=1}^6 \frac{1}{6} p_{ht}^e - 60 \right] + \epsilon_t$$

- ▶ **different types of shocks**  $\epsilon_t$ : small resp. large permanent shocks
- ▶ **common feature**: same RE equilibrium
- ▶ **only difference**: sign in the slope of linear map  $+0.95$  vs  $-0.95$

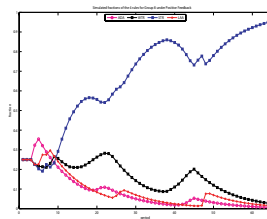
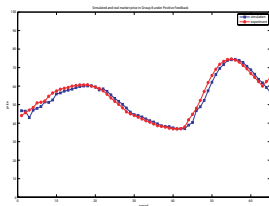
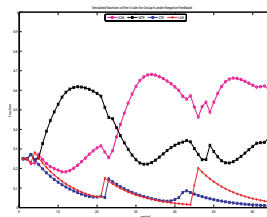
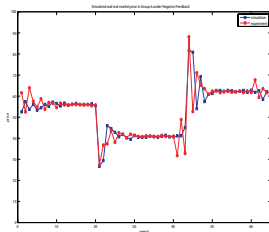
# Positive vs Negative Feedback; Small Shocks

## Heuristics Switching Model Simulations

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# Positive/Negative Feedback; Large Shocks

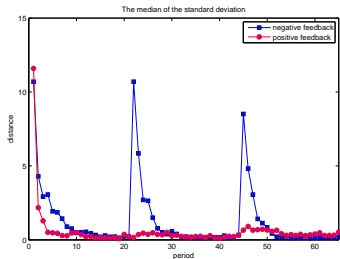
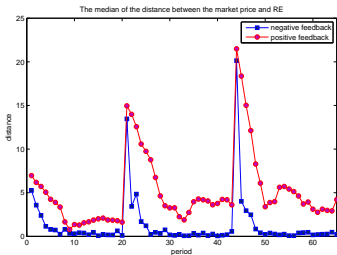


# Positive/Negative Feedback; Large Shocks

## Coordination & Price Discovery

median absolute distance to RE fundamental price;

median standard deviation of individual predictions



# New Keynesian Macro Model; Expectations on Inflation & Output Gap

Assenza et al. (2010)

$$y_t = y_{t+1}^e - \varphi(i_t - \pi_{t+1}^e) + g_t, \quad \text{output} \quad (1)$$

$$\pi_t = \lambda y_t + \beta \pi_{t+1}^e + u_t, \quad \text{inflation} \quad (2)$$

$$i_t = \phi_\pi(\pi_t - \bar{\pi}) + \bar{\pi}, \quad \text{interest rate rule} \quad (3)$$

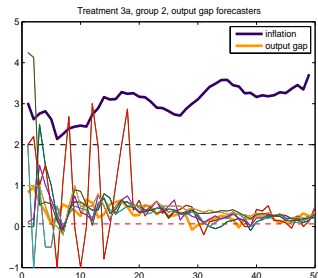
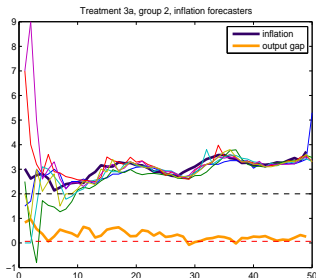
Two treatments:

- ▶ **passive** monetary policy ( $\phi_\pi = 1$ )
- ▶ **aggressive** monetary policy ( $\phi_\pi = 1.5$ )

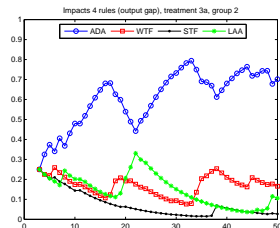
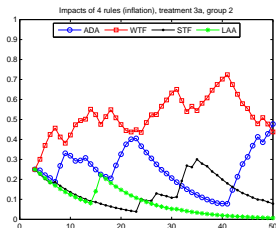
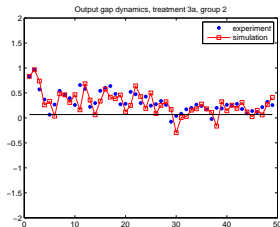
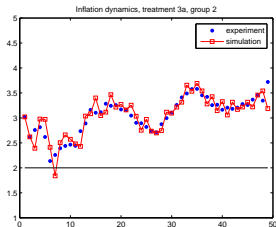
# New Keynesian Macro Model; Expectations on Inflation & Output Gap

Assenza et al. (2010)

passive monetary policy (i.e.  $\phi_\pi = 1$ )



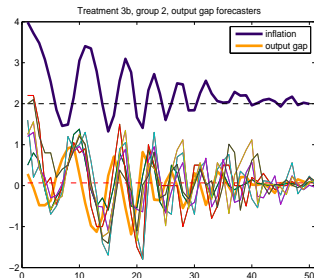
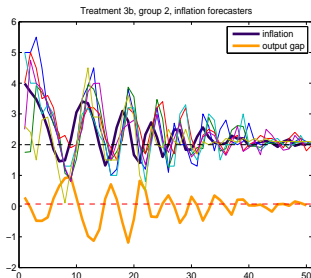
# New Keynesian Macro Model: Simulations (Domenico Massaro)



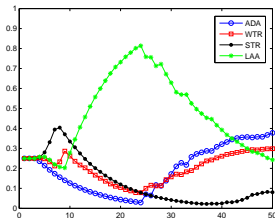
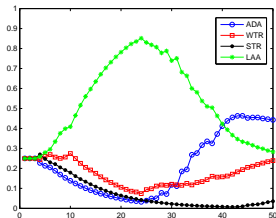
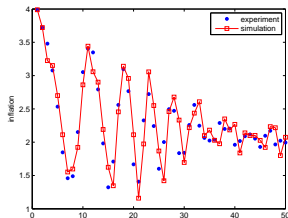
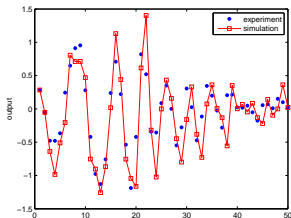
# New Keynesian Macro Model; Expectations on Inflation & Output Gap

Assenza et al. (2010)

aggressive monetary policy (i.e.  $\phi_\pi = 1.5$ )



# New Keynesian Macro Model: Simulations (Domenico Massaro)



# Concluding Remarks on Heterogeneous Expectations

- ▶ **no homogeneous** expectations model fits **all** experiments
- ▶ **only** in stable cobweb/negative feedback quick **convergence** to REE
- ▶ **heterogeneity** in expectations is crucial, because **one model** explains observed
  - ▶ **path dependence** in **same** market environment
  - ▶ **different** aggregate outcomes in **different** markets
  - ▶ **different** forecasting behavior for different variables in one macro economy
- ▶ **challenge:** universal theory of heterogeneous expectations