LEM | Laboratory of Economics and Management

Institute of Economics
Scuola Superiore Sant'Anna

Piazza Martiri della Libertà, 33 - 56127 Pisa, Italy
ph. +39 050 88.33.43
institute.economics@sssup.it

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Giulio Bottazzi [a]
Daniele Giachini [a]
Eleonora Priori [b]

[a] Institute of Economics and Department L'EMbeDS, Scuola Superiore Sant'Anna, Pisa, Italy
[b] Dipartimento di Scienze Economiche ed Aziendali, Università di Pavia, Italy

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Production Networks, Capital Dynamics, and Heterogeneous Agents

Giulio Bottazzi[1], Daniele Giachini[1], and Eleonora Priori[2]

[1]Institute of Economics and Department L'EMbeDS, Scuola Superiore Sant'Anna, Pisa, Italy
[2]Dipartimento di Scienze Economiche ed Aziendali, Università di Pavia, Italy

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Abstract

We develop a general equilibrium production network model that spans two periods and incorporates heterogeneous households, firm-specific Cobb-Douglas production technologies, and a time-to-build mechanism for capital formation. Within this dynamic framework, we establish the existence and uniqueness of a competitive equilibrium and provide explicit analytical solutions for key economic variables. In particular, we derive closed-form expressions for the welfare and real interest rate effects of supply-side shocks occurring at different points in time. We calibrate the model using input-output data from the Italian economy, identifying key structural features such as the prominent roles of the real estate, food, and tourism-related sectors. We then extend the calibration to incorporate household heterogeneity by skill level and examine the consequences in terms of welfare and real interest rates of a climate-related productivity shock. This shock is sector-specific, time-dependent, and scaled according to differential exposure to climate risks. Our results show that climate-induced negative supply-side shocks generate disproportionate welfare losses for low-skilled households and induce nontrivial adjustments in real interest rates across sectors.

Keywords: Production Network, Capital Formation, Heterogeneous Agents, Two-date Model.
JEL Codes: C68, D51, D57, D58, E43.

1 Introduction

The propagation of microeconomic shocks through sectoral production networks has become a central topic in contemporary macroeconomic research (see, e.g., the review by Carvalho and Tahbaz-Salehi, 2019). Production network models, formalizing input-output relationships between firms, have been widely used to investigate the transmission of idiosyncratic shocks to aggregate outcomes (e.g., Gabaix, 2011; Acemoglu et al., 2012, 2015; Baqaee, 2018; Baqaee and Farhi, 2024). While this literature has yielded important insights within static, one-period settings or under representative agent assumptions, relatively less attention has been given to frameworks that jointly incorporate intertemporal dynamics, capital formation, and heterogeneous agents. Recent contributions have begun to fill this gap by extending the production network paradigm to dynamic general equilibrium settings, featuring heterogeneity or investment (see, for example, Baqaee and Farhi, 2018; Ding, 2022; Baqaee and Malmberg, 2024).

In this paper, we develop a general equilibrium production network model that spans two periods and features heterogeneous households, firm-specific Cobb-Douglas production technologies, several production factors, and a time-to-build mechanism for capital accumulation with heterogeneous composition. Although our contribution shares some elements with the framework proposed by Baqaee and Malmberg (2024), our approach is more directly grounded in the Arrow-Debreu tradition and relies on a distinct mechanism of capital formation. By explicitly modeling intertemporal choices, our framework captures the propagation of sector-specific productivity shocks both across sectors and over time, through input-output linkages and capital investment dynamics. We formally characterize the competitive equilibrium and prove the existence and uniqueness of equilibrium prices and allocations under general conditions. In addition, we derive closed-form expressions for the welfare and real interest rate effects of supply-side shocks occurring at either point in time. We show that the capital formation mechanism is key in generating heterogeneous time effects of supply side shocks both in terms of welfare and real interest rates.

To assess the empirical relevance of our model, we calibrate it using input-output data from the Italian economy. Our baseline exercise identifies structurally significant sectors (such as real estate, food, and tourism) and quantifies the sector-specific welfare and interest rate responses to productivity shocks. We then extend the calibration to distinguish between high-skilled and low-skilled households and introduce a climate-related supply-side shock scenario. In this extension, we model heterogeneous sectoral exposure to environmental risks and simulate a shock to total factor productivity in time 2, scaled by each sector's climate vulnerability. This allows us to quantify the distributional impacts of climate-induced disruptions. Our findings show that climate-related supply-side negative shocks generate asymmetric welfare losses, with low-skilled households experiencing significantly greater utility reductions. In addition, we document a systemic downward adjustment in real interest rates, whose magnitude is heterogeneously distributed across sectors depending on their direct exposure and positioning in the production network. These results highlight the importance of jointly considering temporal structure, capital allocation, and heterogeneous vulnerability when evaluating climate risks and policies.

2 Literature Review

The literature on production networks grounds its roots in the 1940s in Leontief's studies, who first proposed a networked view of the productive processes (Leontief, 1941). This intuition brought to the development of a conceptual frame of reference encoding and measuring the role played by the interconnectedness among the constitutive units of a network, particularly focusing on how this determines macroeconomic behaviors. This framework, often integrated with the tools of the general equilibrium theory, served to analyze sector comovements over business cycles (*e.g.*, Long and Plosser, 1983; Shea, 2002), but also, in a more recent stream of literature —which we extensively discuss in this section—, to outline shock propagation patterns from localized micro disturbances to systemic fluctuations. Moreover, the availability of novel datasets treating production at a more granular level has recently added some significant information to the traditional Leontief's architecture.

Carvalho and Tahbaz-Salehi (2019) offer an exhaustive overview of the recent theoretical and empirical literature that discusses the role of production networks in propagating shocks and transforming microeconomic shocks into macroeconomic fluctuations. What makes this contribution crucial is that it tracks all the main issues related to the topic emerged across the literature, discussing the evolution of the debate, the common points over which some consensus has been reached and the open questions laying the foundation for further research. Their work moves its first steps from a milestone contribution, namely the multi-sector general equilibrium model of real business cycles by Long and Plosser (1983), and then proposes some modifications and generalizations to demonstrate the role of input–output linkages as a shock propagation channel throughout the economy.

The main pillars on the production network theoretical literature cover demand-side shocks, questions about how to generalize the modelization of production technology, the role of frictions and market imperfections, the effects of changes in the endogenous structure of production networks. The common thread among all these topics is the research on how shocks propagate through the networks, and in particular the relation between micro-shocks and aggregate macroeconomic fluctuations.

One of the most influential contributions is that by Acemoglu et al. (2012), who build on the Long and Plosser (1983) model to develop a general mathematical framework providing an empirical exploration of the linkages between networks and macroeconomic behaviors. They frame the debate between Horvath (1998, 2000) and Dupor (1999) about the opportunity of translating sectoral shocks into aggregate fluctuations, and answer some of the questions raised by this debate through their model, which presents many touching points with that of Gabaix (2011). The conclusion they find is that, as the economy becomes more disaggregated, the rate at which aggregate volatility decays is determined by the structure of the network capturing such linkages, highlighting the possibility of "cascade effects" due to the higher-order interconnections present in the economy.

It is again the work by Acemoglu et al. (2015) that clarifies the differences on how demand-side shocks and supply-side productivity shocks yield propagation patterns, concluding that the former propagate upstream from one industry to its direct and indirect suppliers, whereas the latter do it downstream. The observation of downstream propagation of supply shocks to direct and indirect customers confirms this result in a working paper from 2016, then published as Carvalho et al. (2021): here, a generalization of the production technology employed in the model is proposed by introducing a nested constant elasticity of substitution (CES) structure. Baqaee and Farhi (2018) resume this result, by proposing a broad class of disaggregated general equilibrium models with heterogeneous agents and input-output networks to overcome two pillars of the traditional macroeconomic models: namely, the representative agent and the aggregate production function.

Contributions about frictions and market imperfections stress on the idea that the production network interacts with productivities and markups affecting aggregate behaviors in a standard Cobb-Douglas economy (Jones, 2011; **?**; Bigio and La'o, 2020). However, the introduction of reduced-form exogenous wedges or markups do not capture shape propagation dynamics of specific market imperfections as this analysis requires a micro-founded model of the interaction between shocks and wedges, as Grassi et al. (2017) points out. A similar attempt is that of Baqaee (2018), who shows the existence of an amplification channel in the form of upstream and downstream cascades of firms' exits. More recently, Baqaee and Farhi (2024) propose a flexible class of trade models with international production networks and arbitrary wedge-like distortions like markups, tariffs, or nominal rigidities, which can be used as a toolbox to study large-scale trade models.

Some publications developed an integrated theory of production and endogenous network formation to consider how the structure of the production network reacts to shocks (*e.g.*, Atalay et al., 2011; Carvalho and Voigtländer, 2014, and more recently, Oberfield 2018; Acemoglu and Azar 2020; Taschereau-Dumouchel

2020). In particular, Acemoglu and Azar (2020) develop a tractable model of endogenous production networks providing comparative static results on the response of prices and endogenous technology/input choices to changes in parameters. These results show that the endogenous evolution of the production network could be a powerful force towards sustained economic growth.

Several empirical works seem to confirm such theoretical findings about the production networks properties: *e.g.*, Acemoglu et al. (2015) propose some evidence emerged at industry level, whereas Barrot and Sauvagnat (2016), Ozdagli and Weber (2017), Carvalho et al. (2021) provide a firm-level analysis. In a different slightly different nuance, Auer et al. (2019) explain how linkages across country–sector pairs affect inflation comovements across countries.

After the Covid-19 breakdown, several contributions on the effects of negative supply shocks and shocks to the composition of final demand on aggregate output appeared. The intuition about cascade effects due to the interconnections in production networks developed by Acemoglu et al. (2012) roots itself in the study of the financial crisis of 2007–2008, and may regain popularity after the Covid-19 crisis outbreak. To consider the relevance of network structures in propagating shocks, Baqaee and Farhi (2021) prove which assumptions must be broken if the network is to matter, finding that either one of the following must hold: (i) TFP shocks, (ii) sector-specific demand shocks, (iii) variable elasticities of substitution or (iv) sticky prices. Pichler and Farmer (2022) reach a similar conclusion, establishing a strong dependency of economic impacts on the emergence of input bottlenecks, and further concluding that the magnitude of initial shocks and network density heavily influence model predictions. Bizzarri (2024) offers an original perspective to exlpore the topic, as he proposes an inter-temporal model on the dynamic diffusion of production networks, with some relevant evidences about the influence of the time dimension. In particular, he shows that, with a model considering time to build, the direction of the diffusion of productivity shocks is the opposite, thus demand shocks also diffuse downstream. Secondly, he proves that time to build yields smaller comovements across sectors. Finally, the study also discusses the bounds of recovery time after a shock.

Our model lays at the intersection of two streams of literature, as it keeps the same conceptual framework of Baqaee and Farhi (2018), which considers heterogeneous agents as a main novelty, and combines it with that of Bizzarri (2024), which introduces explicit time dynamics. Focusing on the propagation of shocks from one producer to another, the former work discusses the effects of microeconomic shocks on all prices and quantities in a general equilibrium environment providing some results that allow for the analysis of any neoclassical production structure and patterns of distorting wedges. Our work, explicitly addressing the role of investments and disaggregated capital in production, is also close to the contributions of Ding (2022) and Baqaee and Malmberg (2024). Ding (2022) builds a dynamic model where capital services (such as machinery use) are tradable across borders as intermediate inputs. Unlike standard models, this approach shows that trade raises welfare by 8% to 36% and helps explain global trends like the declining labor share. Baqaee and Malmberg (2024) develop a dynamic general equilibrium model with heterogeneous households and producers across countries. They show that the long-run equilibrium of such an economy can be represented as a static economy with wedges. In particular, a "Golden Rule" wedge captures how capital is priced above its marginal cost. Thus, if the return to capital exceeds the growth rate, reallocating resources toward capital-intensive sectors can increase long-run consumption. Moreover, shocks can have large long-run effects not just through productivity changes but via endogenous reallocation of capital.

3 The Model

We consider an economy characterized by M factors and two dates (indicated as 1 and 2), and populated by N firms and L households.

Each firm $i =\in \{1, \dots, N\}$ represents a different sector and produces a single good subject to a (firm-specific) Cobb-Douglas technology. Goods can be used as intermediate inputs for contemporary production, as consumption goods for households, and as a capital goods for future production. In particular, call $x^1_{i,j}$ the amount of good produced by firm j used for production by firm i at time 1, $y^1_{i,m}$ the amount of factor m used for production by firm i at date 1, and z^1_i the total factor productivity of firm i at time 1. Defining $\forall i \in \{1, \dots, N\}$ the vectors $\mathbf{x}^1_i = (x^1_{i,1}, \dots, x^1_{i,N})$ and $\mathbf{y}^1_i = (y^1_{i,1}, \dots, y^1_{i,M})$, the total output of firm i at date 1 is

$$\chi^1_i = \chi^1_i(\mathbf{x}^1_i, \mathbf{y}^1_i) = z^1_i \left(\prod_{m=1}^{M} \left(\frac{y^1_{i,m}}{e_{i,m}} \right)^{e_{i,m}} \prod_{j=1}^{N} \left(\frac{x^1_{i,j}}{a_{i,j}} \right)^{a_{i,j}} \right)^{(1-\delta_i)}$$

with $\delta_i \in [0, 1)$, $a_{i,j} \geq 0$, $e_{i,m} \geq 0$, $\sum_j a_{i,j} > 0$, $\sum_m e_{i,m} > 0$, and $\sum_m e_{i,m} + \sum_j a_{i,j} = 1$. Concerning date 2, call $x^2_{i,j}$ the amount of the good produced by firm j used in the production of the good of firm i at time 2, $y^2_{i,m}$ the amount of factor m used for producing the good of firm i at time 2, and z^2_i the total factor productivity of firm i at time 2. Each firm i can purchase an amount $\kappa_{i,j}$ of the good of firm j at time 1 that enters production at time 2 as capital good. The assumption here is that capital goods need some time to be technically adapted to the use in production by the firm. Hence, define $\forall i \in \{1, \dots, N\}$ the vectors $\mathbf{x}^2_i = (x^2_{i,1}, \dots, x^2_{i,N})$, $\mathbf{y}^2_i = (y^2_{i,1}, \dots, y^2_{i,M})$, and $\boldsymbol{\kappa}_i = (\kappa_{i,1}, \dots, \kappa_{i,N})$; the date 2 production of firm i is

$$\chi_i^2 = \chi_i^2(\mathbf{x}_i^2, \mathbf{y}_i^2, \boldsymbol{\kappa}_i) = z_i^2 \left(\prod_{m=1}^{M} \left(\frac{y_{i,m}^2}{e_{i,m}} \right)^{e_{i,m}} \prod_{j=1}^{N} \left(\frac{x_{i,j}^2}{a_{i,j}} \right)^{a_{i,j}} \right)^{(1-\delta_i)} \left(\prod_{j=1}^{N} \left(\frac{\kappa_{i,j}}{\gamma_{i,j}} \right)^{\gamma_{i,j}} \right)^{\delta_i}$$

with $\delta_i \in (0,1)$, $a_{i,j} \geq 0$, $e_{i,m} \geq 0$, $\gamma_{i,j} \geq 0$, $\sum_m e_{i,m} + \sum_j a_{i,j} = 1$, and $\sum_j \gamma_{i,j} = 1$. Notice that capital goods not appearing in the production function at date 1 should not be understood as firms using a different technology but, simply, to the assumption that the amount of capital to be used for production at date 1 cannot be revised and it is fully depreciated.

Consider the vectors of good prices, $\mathbf{p}^1 = (p_1^1, \ldots, p_N^1)$ and $\mathbf{p}^2 = (p_1^2, \ldots, p_N^2)$, where p_j^1 represents the spot price of the good of firm j at time 1 and p_j^2 is the present value price at date 1 of the good produced by firm j at date 2. Consider also the vectors of factor prices, $\mathbf{q}^1 = (q_1^1, \ldots, q_M^1)$ and $\mathbf{q}^2 = (q_1^2, \ldots, q_M^2)$, with q_m^1 the spot price of factor m at time 1 and q_m^2 the present value price at date 1 of factor m available for production at date 2. Then, the profit of firm i reads

$$\begin{aligned} \pi_i &= \pi_i(\mathbf{x}_i^1, \mathbf{x}_i^2, \mathbf{y}_i^1, \mathbf{y}_i^2, \boldsymbol{\kappa}_i; \mathbf{p}^1, \mathbf{p}^2, \mathbf{q}^1, \mathbf{q}^2) \\ &= \chi_i^1(\mathbf{x}_i^1, \mathbf{y}_i^1) p_i^1 + \chi_i^2(\mathbf{x}_i^2, \mathbf{y}_i^2, \boldsymbol{\kappa}_i) p_i^2 - (\mathbf{x}_i^1 + \boldsymbol{\kappa}_i) \cdot \mathbf{p}^1 - \mathbf{x}_i^2 \cdot \mathbf{p}^2 - \mathbf{y}_i^1 \cdot \mathbf{q}^1 - \mathbf{y}_i^2 \cdot \mathbf{q}^2, \end{aligned} \tag{1}$$

where $\cdot$ indicates the standard dot product.

Households are exogenously endowed with factors and possess the shares of firms' equity. That is, each household l has an amount of factor m equal to $E_{m,l}^1$ at date 1 and $E_{m,l}^2$ at date 2. Thus, the total amount of each factor m is $E_m^1 = \sum_l E_{m,l}^1$ at date 1 and $E_m^2 = \sum_l E_{m,l}^2$ at date 2. Moreover, each household l owns a share $s_{i,l} \geq 0$ (with $\sum_l s_{i,l} = 1$) of firm i's equity, hence, it can claim a fraction $s_{i,l}$ of firm i's profit. Then, at date 1, each household $l \in \{1, \ldots, L\}$ has wealth

$$w_l = \sum_i \pi_i s_{i,l} + \sum_m (q_m^1 E_{m,l}^1 + q_m^2 E_{m,l}^2) \,.$$

Define the vectors $\mathbf{c}_l^1 = (c_{1,l}^1, \ldots, c_{N,l}^1)$ and $\mathbf{c}_l^2 = (c_{1,l}^2, \ldots, c_{N,l}^2)$ $\forall l \in \{1, \ldots, L\}$; the preferences of household l are represented by the utility function

$$U_l(\mathbf{c}_l^1, \mathbf{c}_l^2) = \sum_i b_{l,i} (\log c_{l,i}^1 + \beta_l \log c_{l,i}^2)$$

with $b_{l,i} \geq 0$, $\sum_i b_{l,i} = 1$, $\beta_l \in (0,1]$, and where $c_{l,i}^1$ and $c_{l,i}^2$ represent, respectively, the consumption of good i by household l at dates 1 and 2.

4 Competitive Equilibrium

Definition 4.1 (Competitive Equilibrium)**.** The allocation

$$(\mathbf{x}_1^{1*}, \dots, \mathbf{x}_N^{1*}, \mathbf{x}_1^{2*}, \dots, \mathbf{x}_N^{2*}, \mathbf{y}_1^{1*}, \dots, \mathbf{y}_M^{1*}, \mathbf{y}_1^{2*}, \dots, \mathbf{y}_M^{2*}, \\ \boldsymbol{\kappa}_1^{*}, \dots, \boldsymbol{\kappa}_N^{*}, \mathbf{c}_1^{1*}, \dots, \mathbf{c}_N^{1*}, \mathbf{c}_1^{2*}, \dots, \mathbf{c}_N^{2*}) \tag{2}$$

and the price vector $(\mathbf{p}^{1*}, \mathbf{p}^{2*}, \mathbf{q}^{1*}, \mathbf{q}^{2*})$ constitute a competitive equilibrium if the following conditions hold.

1. Each firm $i \in \{1, \dots, N\}$ maximizes its profit,

$$\max_{\mathbf{x}_i^1, \mathbf{x}_i^2, \mathbf{y}_i^1, \mathbf{y}_i^2, \boldsymbol{\kappa}_i \geq 0} \pi_i(\mathbf{x}_i^1, \mathbf{x}_i^2, \mathbf{y}_i^1, \mathbf{y}_i^2, \boldsymbol{\kappa}_i; \mathbf{p}^{1*}, \mathbf{p}^{2*}, \mathbf{q}^{1*}, \mathbf{q}^{2*}).$$

2. Each household $l \in \{1, \dots, L\}$ solves

$$\begin{aligned} &\max_{\mathbf{c}_l^1, \mathbf{c}_l^2 \geq 0} U_l(\mathbf{c}_l^1, \mathbf{c}_l^2) \text{ s. t.} \\ &\mathbf{c}_l^1 \cdot \mathbf{p}^{1*} + \mathbf{c}_l^2 \cdot \mathbf{p}^{2*} \leq \sum_i s_{i,l}\, \pi_i(\mathbf{x}_i^{1*}, \mathbf{x}_i^{2*}, \mathbf{y}_i^{1*}, \mathbf{y}_i^{2*}, \boldsymbol{\kappa}_i^*; \mathbf{p}^{1*}, \mathbf{p}^{2*}, \mathbf{q}^{1*}, \mathbf{q}^{2*}) \\ &\qquad\qquad + \sum_m (q_m^{1*} E_{m,l}^1 + q_m^{2*} E_{m,l}^2). \end{aligned}$$

3. Markets clear, that is

$$\begin{cases} \chi_i^1(\mathbf{x}_i^{1*}, \mathbf{y}_i^{1*}) = \sum_j x_{j,i}^{1*} + \sum_l c_{l,i}^{1*} + \sum_j \kappa_{j,i}^* & \forall i \in \{1, \dots, N\}, \\ \chi_i^2(\mathbf{x}_i^{2*}, \mathbf{y}_i^{2*}, \boldsymbol{\kappa}_i^*) = \sum_j x_{j,i}^{2*} + \sum_l c_{l,i}^{2*} & \forall i \in \{1, \dots, N\}, \\ \sum_l E_{m,l}^1 = \sum_i y_{i,m}^{1*} & \forall m \in \{1, \dots, M\}, \\ \sum_l E_{m,l}^2 = \sum_i y_{i,m}^{2*} & \forall m \in \{1, \dots, M\}. \end{cases}$$

Theorem 4.1 (Existence and Uniqueness of Competitive Equilibrium)**.** *Given a level $W > 0$ of total wealth in the economy, there exists a unique competitive equilibrium with positive prices.*

Proof. Consider the profit maximization problem of firm i, the necessary and sufficient first order conditions read

$$\begin{cases} (1-\delta_i) a_{i,j} p_i^1 \chi_i^1 = p_j^1 x_{i,j}^1 & \forall j \in \{1, \dots, N\}, \\ (1-\delta_i) a_{i,j} p_i^2 \chi_i^2 = p_j^2 x_{i,j}^2 & \forall j \in \{1, \dots, N\}, \\ \delta_i \gamma_{i,j} p_i^2 \chi_i^2 = p_j^2 \kappa_{i,j} & \forall j \in \{1, \dots, N\}, \\ (1-\delta_i) e_{i,m} p_i^1 \chi_i^1 = q_m^1 y_{i,m}^1 & \forall m \in \{1, \dots, M\}, \\ (1-\delta_i) e_{i,m} p_i^2 \chi_i^2 = q_m^2 y_{i,m}^2 & \forall m \in \{1, \dots, M\}. \end{cases}$$

Hence, substituting in (1), the profit of firm i becomes $\pi_i = \delta_i \chi_i^1 p_i^1$.

The necessary and sufficient first order conditions of the utility maximization problem of household l are

$$\begin{cases} p_i^1 c_{l,i}^i = \dfrac{b_{l,i}}{\mu_l} & \forall j \in \{1,\dots,N\}, \\ p_i^2 c_{l,i}^2 = \dfrac{\beta_l b_{l,i}}{\mu_l} & \forall j \in \{1,\dots,N\}, \\ w_l = \sum_i p_i^1 c_{l,i}^1 + p_i^2 c_{l,i}^2 = \dfrac{1+\beta_l}{\mu_l}, \end{cases}$$

where μ_l is the Lagrange multiplier of the utility maximization problem of household l. From these conditions, one has $p_i^1 c_{l,i}^1 = b_{l,i} w_l/(1+\beta_l)$ and $p_i^2 c_{l,i}^2 = b_{l,i}\beta_l w_l/(1+\beta_l)$. Define the value of production in sector i as $v_i^1 = p_i^1 \chi_i^1$ at time 1 and $v_i^2 = p_i^2 \chi_i^2$ at time 2, using the profit, the first order conditions of the firms' problems, and the market clearing conditions for factors; the wealth of household l can be rewritten as

$$w_l = \sum_j s_{j,l}\delta_j v_j^1 + \sum_m \left(\frac{E_{m,l}^1}{E_m^1} \sum_j v_j^1 e_{j,m}(1-\delta_j) + \frac{E_{m,l}^2}{E_m^2} \sum_j v_j^2 e_{j,m}(1-\delta_j) \right) .$$

Then, substituting in the market clearing conditions for production at the two dates, one obtains

$$\begin{cases} v_i^1 = \sum_j v_j^1 \Theta_{j,i} + \sum_j v_j^2 \Omega_{j,i} & \forall i \in \{1,\dots,N\}, \\ v_i^2 = \sum_j v_j^2 \Phi_{j,i} + \sum_j v_j^1 \Psi_{j,i} & \forall i \in \{1,\dots,N\}, \end{cases} \tag{3}$$

with

$$\begin{aligned}
\Theta_{j,i} =& (1-\delta_j) a_{j,i} + \sum_l \frac{b_{i,l}}{1+\beta_l} \left(\delta_j s_{j,l} + (1-\delta_j) \sum_m \frac{E_{m,l}^1}{E_m^1} e_{j,m} \right) \geq 0 \, , \\
\Omega_{j,i} =& \delta_j \gamma_{j,i} + (1-\delta_j) \sum_l \frac{b_{l,i}}{1+\beta_l} \sum_m \frac{E_{m,l}^2}{E_m^2} e_{j,m} \geq 0 \, , \\
\Phi_{j,i} =& (1-\delta_j) \left(a_{j,i} + \sum_l \frac{b_{i,l}\beta_l}{1+\beta_l} \sum_m \frac{E_{m,l}^2}{E_m^2} e_{j,m} \right) \geq 0 \, , \\
\Psi_{j,i} =& \sum_l \frac{b_{l,i}\beta_l}{1+\beta_l} \left(\delta_j s_{j,l} + (1-\delta_j) \sum_m \frac{E_{m,l}^1}{E_m^1} e_{j,m} \right) \geq 0 \, .
\end{aligned}$$

Hence, one can build the matrices Θ, Ω, Φ, $\Psi \in \mathbb{R}_+^{N\times N}$ whose entries are the elements reported above and, given the vectors of value of production $\mathbf{v}^1 =$

$(v_1^1, \ldots, v_N^1)$ and $\mathbf{v}^2 = (v_1^2, \ldots, v_N^2)$, one can define $\mathbf{v} = (\mathbf{v}^1, \mathbf{v}^2) \in \mathbb{R}_+^{2N}$. Thus, define the matrix

$$\Gamma = \begin{pmatrix} \Theta & \Psi \\ \Omega & \Phi \end{pmatrix} \in \mathbb{R}_+^{2N \times 2N},$$

such that the system in (3) can be written as $\mathbf{v} = \mathbf{v}\Gamma$. Notice that, $\forall j \in \{1, \ldots, N\}$, it is

$$\sum_i \Theta_{j,i} + \Psi_{j,i} = 1 \quad \text{and} \quad \sum_i \Omega_{j,i} + \Phi_{j,i} = 1\,,$$

hence, Γ is a row-stochastic and irreducible matrix. Then, by the Perron-Frobenius theorem, $\exists!\, \boldsymbol{\lambda} \in \mathbb{R}_+^{2N}$ such that $\boldsymbol{\lambda} = \boldsymbol{\lambda}\Gamma$ and $\boldsymbol{\lambda} \cdot \mathbf{1} = 1$, with $\mathbf{1}$ a vector of ones. Let $\mathbf{v} = \alpha\boldsymbol{\lambda}$ with $\alpha \in \mathbb{R}$ and notice that the total wealth in the economy reads

$$W = \sum_\ell w_\ell = \sum_j v_j^1 \left(\delta_j + (1 - \delta_j) \sum_m e_{j,m} \right) + \sum_j v_j^2 (1 - \delta_j) \sum_m e_{j,m}\,.$$

Thus, defining the column vector

$$\mathbf{d} = \begin{pmatrix} \delta_1 + (1 - \delta_1) \sum_m e_{1,m} \\ \vdots \\ \delta_N + (1 - \delta_N) \sum_m e_{N,m} \\ (1 - \delta_1) \sum_m e_{1,m} \\ \vdots \\ (1 - \delta_N) \sum_m e_{N,m} \end{pmatrix} \in \mathbb{R}_+^{2N}$$

one has $W = \alpha\boldsymbol{\lambda} \cdot \mathbf{d}$. Hence, given W, one has

$$\mathbf{v}^* = \frac{W}{\boldsymbol{\lambda} \cdot \mathbf{d}} \boldsymbol{\lambda}\,.$$

The next steps consist in determining prices. Let us start from factors, from the market clearing conditions and the firms' first order conditions, using $\mathbf{v}^*$ one directly obtains the equilibrium prices

$$\begin{cases} q_m^{1*} = \dfrac{\sum_i v_i^{1*} e_{i,m} (1 - \delta_i)}{E_m^1} & \forall m \in \{1, \ldots, M\}\,, \\ q_m^{2*} = \dfrac{\sum_i v_i^{2*} e_{i,m} (1 - \delta_i)}{E_m^2} & \forall m \in \{1, \ldots, M\}\,. \end{cases}$$

For the prices of the goods, substitute the first order conditions in the date 1 production function of sector i and consider the equilibrium factor prices and value of production, after few algebraic manipulations, one obtains

$$p_i^1 = \frac{(v_i^{1*})^{\delta_i} \left(\prod_j (p_j^1)^{a_{i,j}} \prod_m (q_m^{1*})^{e_{i,m}} \right)^{1 - \delta_i}}{z_i^1 (1 - \delta_i)^{1 - \delta_i}}\,.$$

Taking the logarithm on both sides and defining $\tilde{\mathbf{p}}^{1*} = (\log p_1^{1*} \ldots, \log p_N^{1*})$, $\tilde{\mathbf{z}}^1 = (\log z_1^1 + (1-\delta_1)\log(1-\delta_1), \ldots, \log z_N^1 + (1-\delta_N)\log(1-\delta_N))$, $\tilde{\mathbf{v}}^{1*} = (\delta_1 \log v_1^{1*}, \ldots, \delta_N \log v_N^{1*})$, $\tilde{\mathbf{q}}^{1*} = (\log q_1^{1*}, \ldots, \log q_M^{1*})$, $\mathcal{E} \in \mathbb{R}_+^{N\times M}$ with $\mathcal{E}_{i,m} = (1-\delta_i)e_{i,m}$, and $\mathcal{A} \in \mathbb{R}_+^{N\times N}$ with $\mathcal{A}_{i,j} = (1-\delta_i)a_{i,j}$, one obtains

$$\tilde{\mathbf{p}}^{1*} = (I_N - \mathcal{A})^{-1}(\tilde{\mathbf{v}}^{1*} + \mathcal{E}\tilde{\mathbf{q}}^{1*} - \tilde{\mathbf{z}}^1)\,, \tag{4}$$

Where I_N indicates the identity matrix in $\mathbb{R}^{N\times N}$ and the non-singularity of $I_N - \mathcal{A}$ is ensured by the fact that it is strictly diagonally dominant. The equilibrium prices of date 1 goods composing $\mathbf{p}^{1*}$ can be recovered taking the exponential of the elements of the vector in (4).

For the present value prices of the goods at date 2, consider the date 2 production function of sector i, using the first order conditions and the equilibrium quantities, it can be rewritten as[1]

$$p_i^2 = \frac{\left(\prod_j (p_j^2)^{a_{i,j}} \prod_m (q_m^{2*})^{e_{i,m}}\right)^{1-\delta_i} \left(\prod_j (p^{1*})^{\gamma_{i,j}}\right)^{\delta_i}}{z_i^2 (\delta_i)^{\delta_i}(1-\delta_i)^{1-\delta_i}}\,.$$

Define $\tilde{\mathbf{p}}^{2*} = (\log p_1^{1*} \ldots, \log p_N^{1*})$, $\tilde{\mathbf{z}}^2 = (\log z_1^2 + \log((\delta_1)^{\delta_1}(1-\delta_1)^{1-\delta_1}), \ldots, \log z_N^2 + \log((\delta_N)^{\delta_N}(1-\delta_N)^{1-\delta_N}))$, $\tilde{\mathbf{q}}^{2*} = (\log q_1^{2*}, \ldots, \log q_M^{2*})$, and $\mathcal{G} \in \mathbb{R}^{N\times N}$ with $\mathcal{G}_{i,j} = \delta_i \gamma_{i,j}$, taking the logarithm of both sides of the previous equation, considering it $\forall i \in \{1, \ldots, N\}$ and after some algebraic manipulations, one obtains

$$\tilde{\mathbf{p}}^{2*} = (I_N - \mathcal{A})^{-1}\left(\mathcal{G}\tilde{\mathbf{p}}^{1*} + \mathcal{E}\tilde{\mathbf{q}}^{2*} - \tilde{\mathbf{z}}^2\right)\,. \tag{5}$$

Again, the equilibrium values of present value prices of date 2 goods appearing in $\mathbf{p}^{2*}$ can be recovered taking the exponential of the entries of the vector in (5).

Finally, the unique values for the entries of the equilibrium allocation in (2) can be computed using $\mathbf{v}^*$, the equilibrium prices, and the first-order conditions of firms and consumers. Moreover, since product prices are the result of an exponentiation operation and the factor prices depend only upon positive quantities, all prices are positive. □

5 Supply Side Shocks on Welfare and Real Interest Rates

Our general equilibrium model allows us to evaluate the equilibrium welfare effect of a supply side shock occurring in firm $i \in \{1, \ldots, N\}$ at time $t \in \{1, 2\}$. In particular, we model it as an infinitesimal change in total factor productivity, such

[1] We impose the convention $0^0 = 1$, such that p_i^2 remains well-defined even for $\delta_i = 0$.

that we can measure the welfare effect using the gradient of the vector of household utilities. Thus, define the utility gradient generated by a variation in total factor productivity of firm i at time t as $\nabla \mathbf{U}(z_i^t) = (\partial U_1/\partial z_i^t, \ldots, \partial U_L/\partial z_i^t)$. Moreover, consider the matrices $B^1, B^2 \in \mathbb{R}_+^{L\times N}$ with $B_{l,i}^1 = b_{l,i}$ and $B_{l,i}^2 = \beta_l b_{l,i}$, the matrix $\mathcal{A} \in \mathbb{R}_+^{N\times N}$ with $\mathcal{A}_{i,j} = (1-\delta_i)a_{i,j}$, the matrix $\mathcal{G} \in \mathbb{R}^{N\times N}$ with $\mathcal{G}_{i,j} = \delta_i \gamma_{i,j}$, and the vectors $\nabla \mathbf{z}^\tau(z_i^t) = (\partial \tilde{\mathbf{z}}_1^\tau/\partial z_i^t, \ldots, \partial \tilde{\mathbf{z}}_N^\tau/\partial z_i^t)$, $\tau = 1, 2$, with $\partial \tilde{\mathbf{z}}_j^\tau/\partial z_i^t = 1/z_j^\tau$ if $\tau = t$ and $j = i$, and $\partial \tilde{\mathbf{z}}_j^\tau/\partial z_i^t = 0$ otherwise. Then, indicating with I_N the identity matrix in $\mathbb{R}^{N\times N}$, the following proposition holds.

Proposition 5.1. *The equilibrium welfare effect of a supply side shock occurring in firm i at time t is*

$$\nabla \mathbf{U}(z_i^t) = \left(B^1 + B^2(I_N - \mathcal{A})^{-1}\mathcal{G}\right)(I_N - \mathcal{A})^{-1}\nabla \mathbf{z}^1(z_i^t) + B^2(I_N - \mathcal{A})^{-1}\nabla \mathbf{z}^2(z_i^t)\,. \quad (6)$$

Proof. Notice, first of all, that z_i^t does not appear in the matrix Γ or in the vector $\mathbf{d}$. Thus, it is $\partial v_j^\tau/\partial z_i^t = 0$ $\forall i \in \{1, \ldots, N\}$ and $\forall \tau \in \{1, 2\}$. As a consequence, w_l does not depend on z_i^t and, substituting the first order conditions of household l in its objective function and differentiating, one obtains

$$\frac{\partial U_l}{\partial z_i^t} = -\sum_i b_{l,i}\left(\frac{\partial \log p_i^1}{\partial z_i^t} + \beta_l \frac{\partial \log p_i^2}{\partial z_i^t}\right)\,.$$

This implies

$$\nabla \mathbf{U}(z_i^t) = -B^1\nabla \tilde{\mathbf{p}}^1(z_i^t) - B^2\nabla \tilde{\mathbf{p}}^2(z_i^t)\,, \quad (7)$$

where $\nabla \tilde{\mathbf{p}}^\tau(z_i^t) = (\partial \log p_1^\tau/\partial z_i^t, \ldots, \partial \log p_N^\tau/\partial z_i^t)$, $\tau = 1, 2$. Since factor prices do not depend on z_i^t, differentiating equations (4) and (5), one has

$$\begin{aligned} \nabla \tilde{\mathbf{p}}^1(z_i^t) &= -\left(I_N - \mathcal{A}\right)^{-1}\nabla \mathbf{z}^1(z_i^t)\,, \\ \nabla \tilde{\mathbf{p}}^2(z_i^t) &= -\left(I_N - \mathcal{A}\right)^{-1}\left(\nabla \mathbf{z}^2(z_i^t) - \mathcal{G}\nabla \tilde{\mathbf{p}}^1(z_i^t)\right)\,. \end{aligned} \quad (8)$$

Substituting in equation (7) and rearranging terms, equation (6) and the statement follows. □

As one can notice, supply shocks have an asymmetric effect on welfare depending on the time they occur. In particular, a supply shock at time $t = 2$ affects welfare only through time 2 consumption, while a supply shock at time $t = 1$ affects welfare through both date 1 and date 2 consumption. To understand the underlying mechanisms, consider that supply shocks have no effect on the value of production. That is, in line with other models assuming Cobb-Douglas technologies (see, e.g., Carvalho and Tahbaz-Salehi, 2019), one has $\partial \log \chi_j^\tau/\partial z_i^t = -\partial \log p_j^\tau/\partial z_i^t$

$\forall i, j \in \{1, \dots, N\}$ and $\forall t, \tau \in \{1, 2\}$. This has a direct implication for factor prices. Their equilibrium values depend only upon the value of production, the endowments, and their (respective) technological parameters, hence, supply shocks do not affect them. As a consequence, the effect on equilibrium consumption of a variation in the total factor productivity of a firm is completely driven by the variation of the log-prices of goods. Then, notice that, in equilibrium, the logarithm of present value prices of goods available at date 2 depend upon the log-prices of goods available at date 1, while the opposite does not hold. Thus, since log-prices are a (linear) function of the logarithm of the total factor productivity parameters at the respective dates, the asymmetric effect of shocks follows.

The source of such an asymmetry is twofold. On the one hand there is the trivial effect of the intertemporal preference parameters $\{\beta_1, \dots, \beta_L\}$, generating the differences between the matrices B^1 and B^2. On the other hand, there is the capital structure, generating the effect of time 1 supply shocks on time 2 consumption mediated by the term $B^2(I_N - \mathcal{A})^{-1}\mathcal{G}$. The two sources can be easily identified sterilizing the intertemporal preference for date 1 consumption (i.e., setting $\beta_l = 1 \; \forall l \in \{1, \dots, L\}$) and eliminating capital from production (i.e., setting $\delta_i = 0 \; \forall i \in \{1, \dots, N\}$). Indeed, in such a scenario, it is

$$\nabla \mathbf{U}(z_i^t) = B^1(I_N - A)^{-1}(\nabla \mathbf{z}^1(z_i^t) + \nabla \mathbf{z}^2(z_i^t)),$$

with $A \in \mathbb{R}_+^{N \times N}$ and $A_{i,j} = a_{i,j}$. Hence, a shock in the supply of a good produces the same effect on welfare no matter the time in which it occurs. Moreover, provided a constant total factor productivity over time, the symmetric case represents the situation in which a date 2 shock reaches its maximum welfare effect with respect to an equivalent date 1 production shock. That is, in the generic case, assuming $z_i^1 = z_t^2 = z$ and defining the vector $\nabla \mathbf{z} \in \mathbb{R}^N$ with $\nabla \mathbf{z}_j = 1/z$ if $j = i$ and zero otherwise, one has

$$\nabla \mathbf{U}(z_i^1 = z) - \nabla \mathbf{U}(z_i^2 = z) = \left(B^1 - B^2\left(I_N - (I_N - \mathcal{A})^{-1}\mathcal{G}\right)\right)(I_N - \mathcal{A})^{-1}\nabla \mathbf{z}.$$

Since $||\mathcal{A}||_\infty = \max_i \sum_j (1 - \delta_i) a_{i,j} < 1$, the spectral radius of the positive matrix $\mathcal{A}$ is smaller than one and this implies $(I_N - \mathcal{A})^{-1} = \sum_{n=0}^\infty \mathcal{A}^n$. Thus,

$$\begin{aligned}\nabla \mathbf{U}(z_i^1 = z) - \nabla \mathbf{U}(z_i^2 = z) &\geq \left(B^1 - B^2\right)\sum_{n=0}^\infty \mathcal{A}^n \nabla \mathbf{z} \\ &\geq \left(1 - \max_{l \in \{1, \dots, L\}} \beta_l\right) B^1 \sum_{n=0}^\infty \mathcal{A}^n \nabla \mathbf{z} \geq \mathbf{0}.\end{aligned}$$

Finally, the previous results can be used to compute the total effect on welfare of multiple supply shocks of different magnitudes hitting different firms at different periods. That is, consider Z shocks, each generating a variation $\zeta_i^t \in \mathbb{R}$, where i indicates the firm and t the time in which the variation of the total factor productivity occurs. Then, defining the vectors $\Delta\mathbf{z}^t = (\zeta_1^t/z_1^t, \ldots, \zeta_N^t/z_N^t),\ t = 1, 2$, the total effect on welfare can be approximated to the first order by the utility differential

$$\Delta\mathbf{U}\left(\Delta\mathbf{z}^1, \Delta\mathbf{z}^2\right) = \left(B^1 + B^2(I_N - \mathcal{A})^{-1}\mathcal{G}\right)(I_N - \mathcal{A})^{-1}\Delta\mathbf{z}^1 + B^2(I_N - \mathcal{A})^{-1}\Delta\mathbf{z}^2\,.$$

Next, we analyze the interest rate structure emerging from the economy and its reaction to supply side shocks. Notice that the assumption that agents can make their production and consumption decisions using present value prices is equivalent, on the financial side, to market completeness. In our economy without uncertainty, this reduces to assume that a single security is traded at time 1 such that, for any unit of value (i.e., money) invested in it at time 1, $1 + g$ units of value are delivered at date 2. Thus, calling P_i^2 the spot price of good i at time 2, according to the *Law of One Price*, for any good i the following equality must hold

$$p_i^2 = \frac{P_i^2}{1+g}\,.$$

That is, the price at date 1 of one unit of a consumption good available at date 2 must be equal to the cost of the amount of security needed to purchase one unit of the consumption good on the spot market at date 2. Rearranging terms and multiplying and dividing by p_i^1, one obtains $\forall i \in \{1, \ldots, N\}$ the standard Fisher's equation

$$1 + g = \frac{p_i^1}{p_i^2}\frac{P_i^2}{p_i^1} = (1 + r_i)(1 + \iota_i)\,,$$

where $1 + r_i = p_i^1/p_i^2$ is the gross real interest rate of good i and $i + \iota_i = P_i^2/p_i^1$ is the gross inflation rate of good i. Moreover, as expected, $1 + g$ is the gross nominal interest rate of the economy. Define the vector of net real interest rates as $\mathbf{r} = (r_1, \ldots, r_N)$ and its gradient generated by an infinitesimal variation of z_i^t as $\nabla\mathbf{r}(z_i^t) = (\partial r_1/\partial z_i^t, \ldots, \partial r_N/\partial z_i^t)$, then the following proposition holds.

Proposition 5.2. *The equilibrium effect on the net real interest rates of a supply side shock occurring in firm i at time t is*

$$\nabla\mathbf{r}(z_i^t) \simeq \left((I_N - \mathcal{A})^{-1}\mathcal{G} - I_N\right)(I_N - \mathcal{A})^{-1}\nabla\mathbf{z}^1(z_i^t) + (I_N - \mathcal{A})^{-1}\nabla\mathbf{z}^2(z_i^t)\,.$$

Proof. From the definition of the gross real interest rate and the first-order Taylor approximation around zero of $\log(1 + r_i)$, one directly obtains $r_i \simeq \log(1 + r_i) = \log p_i^1 - \log p_i^2$, hence

$$\mathbf{r} \simeq \tilde{\mathbf{p}}^1 - \tilde{\mathbf{p}}^2\,.$$

Differentiating both sides and using the equations in (8), the statement follows. □

It is interesting to notice that the same shock to the same firm may have different effects depending on the period in which it occurs. Indeed, recalling $(I_N - \mathcal{A})^{-1} = \sum_{n=1}^{\infty} \mathcal{A}^n$, a time 2 positive shock has an effect of the same sign on net real interest rates. The effect of a shock at time 1, instead, is ambiguous, since it depends on the components of $\mathcal{G}(I_N - \mathcal{A})^{-1} - I_N$. With respect to what observed for welfare before, the capital structure remains key for generating the asymmetry, while intertemporal preferences of households do not play any role. Indeed, it is enough to set $\delta_i = 0 \ \forall i \in \{1, \dots, n\}$ to obtain

$$\nabla \mathbf{r}(z_i^t) \simeq (I_N - A)^{-1} \left(-\nabla \mathbf{z}^1(z_i^t) + \nabla \mathbf{z}^2(z_i^t)\right)$$

and this indicates that, without the capital structure, the same shock occurring in one period instead of another has a symmetric but opposite effect on net real interest rates.

As done in advance, one can compute the (approximated) total effect on net real interest rates of Z shocks hitting different firms in different periods by means of the total differential

$$\Delta \mathbf{r}(\Delta \mathbf{z}^1, \Delta \mathbf{z}^2) \simeq \left((I_N - \mathcal{A})^{-1}\mathcal{G} - I_N\right)(I_N - \mathcal{A})^{-1}\Delta \mathbf{z}^1 + (I_N - \mathcal{A})^{-1}\Delta \mathbf{z}^2 \,,$$

where $\Delta \mathbf{z}^1$ and $\Delta \mathbf{z}^2$ are defined as in advance.

6 Empirical Application to the Italian Economy

In this section we calibrate our model on the data of the Italian economy and study the effects on welfare and real interest rates of supply shocks. In particular, after a general investigation of how the economy reacts to shocking one sector at a time, we will focus on multiple shocks hitting the economy simultaneously. These simultaneous (negative) shocks shall represent the effect of some climate-related event that affect multiple sectors in a differentiated manner depending on their exposure to climate risk. We shall also differentiate between skilled and unskilled households, such that to understand the effects of the shocks on welfare and real interest rates.

6.1 Input-Output Data and Baseline Calibration

The key data source for our analysis is the Input-Output table system provided by the Italian national institute of statistics (ISTAT). In particular, we focus on the Input-Output table (product-by-product) for the year 2019.[2] To adapt the

[2] Data retrieved in February 2025 and available at the following url: https://www.istat.it/tavole-di-dati/il-sistema-di-tavole-input-output-anni-2015-2020/.

data to our model, we proceed in the following way. First, we eliminate the row and column relative to the sector "Activities of households and cohabitations as employers of domestic personnel; production of undifferentiated goods and services for own use by households and cohabitations", since it does not buy from or sells to any other sector. Then, we consider three factors of production: labor (whose remuneration is the gross wage), a public factor (whose remuneration is the total amount of taxes payed by the sector), and a foreign factor (whose remuneration is the value of imports). If negative values occur in any of these elements, then zero is assigned to element and the total value added is reduced accordingly. Three agents are assumed to populated the economy. The first is a private household and its value of consumption is computed as the sum of "household final consumption expenditure" and "final consumption expenditure of non-profit social institutions serving households (NPSI)". The second is a public agent whose consumption is assumed to be the "final consumption expenditure of public administration". The third is a foreign agent whose value of consumption matches the value of exports. Finally, we take care of investment. In particular, negative values may occur in the column relative to "gross investments". Whenever a negative value occurs, we assign a zero to the relative entry and accordingly correct the sector's row sum. We also correct the value added of the relative sector by adding the absolute value of the investment entry, such that the equality between sectors' row and column sums is respected.

Next, we use first order conditions to directly estimate parameters. In particular, we take the ratio between the value of inputs bought by sector i from sector j over the total value of production of sector i to estimate each $a_{i,j}$. Along the same lines, we estimate $e_{i,m}$ by taking the ratio between the value of factor m bought by sector i over the total value of production of sector i. Each parameter δ_i is estimated for each sector i as one minus the sum of the values of factors and intermediate inputs over the total value of production. Each parameter $b_{l,i}$ is estimated by dividing the value of agent l's consumption of the product of sector i by the total value of consumption of such an agent.

Concerning intertemporal preferences and endowments, we proceed as follows. We assume $\beta_l = 0.95 \ \forall l$ and $E_m^t = 1 \ \forall t, m$, assigning the whole endowment of labor to the private household, the whole endowment of the public factor to the public agent, and the whole endowment of the foreign factor to the foreign agent. With respect to the technological parameters of capital, we assume $\gamma_{i,j} \propto a_{i,j}$, that is, the technological structure of capital mimics the one of intermediate production goods. Finally, ownership shares are fully assigned to the private household and all the total factor productivity parameters are set to one.

In such a baseline calibration, we investigate the effect of a +1% supply side shock in one sector and one period at a time on both the private household welfare and the real interest rates. We can do that in two ways. The first one consists in computing equilibrium quantities twice, once with baseline settings and one with the +1% supply side shock and then taking differences. The second one, instead, consists in applying the approximated formulas derived in the previous section. For space reasons, however, in Figure 1 we show the results obtained applying the first procedure while we report in Figure 6 in Appendix C the results of the second procedure. Comparing the two pictures, even if some differences at the quantitative level emerge, they do not change the qualitative interpretation of results.

As one can notice, consistently with what we proved in the previous section, a time 1 shock generates a larger variation than a time 2 shock in the same sector. Moreover, the heterogeneity of the effects is striking. For instance, the sector where a time 1 shock generates the largest effect is "RL - Real estate services", followed by "R10_12 - Food products, beverages, and tobacco products", "RI - Accommodation and food service activities", and "R46 - Wholesale trade services, excluding motor vehicles and motorcycles". Concerning time 2 shocks, the only difference in terms of sectors generating the largest variation is that "R47 - Retail trade services, excluding motor vehicles and motorcycles" presents a larger value than R46. On the other hand, the sectors generating the smallest effect for a shock at time 1 are "R02 - Products of forestry, logging operations, and related services", "R03 - Fish and other fishery products; aquaculture products; support ser- vices for fishing", and "R95 - Repair services of computers and personal and household goods". with respect to time 2, these three sectors are confirmed together with "R53 - Postal and courier services" and "R72 - Scientific research and development services". These results indicates that historically prominent activities in the Italian economy such as real estate, food, and tourism are still key in generating welfare improvements for private households. More worrying, however, is the small effect on welfare of a positive productivity shock in an important sector such as scientific research and development services.

We repeat the exercise considering net real interest rates. As in advance, we show here the results obtained computing equilibrium quantities twice, both for a time 1 (Figure 2) and for a time 2 (Figure 3) +1% supply side shock. The results obtained from the approximations provided in the previous section are in Appendix C, Figure 7 and Figure 8. As one can notice, the approximation is worst in such a case, probably because of the double first order approximation applied.

Consistently with our analytical results, the effect on net real interest rates of a time 2 shock is always positive, while the effect of a time 1 shock is ambiguous. Both for time 1 and time 2 shocks, the largest variation is observed in the net real interest rate of the sector in which the shock occurs, with time 1 +1% shocks having a negative effect and time 2 +1% shocks having a positive effect. Cross-

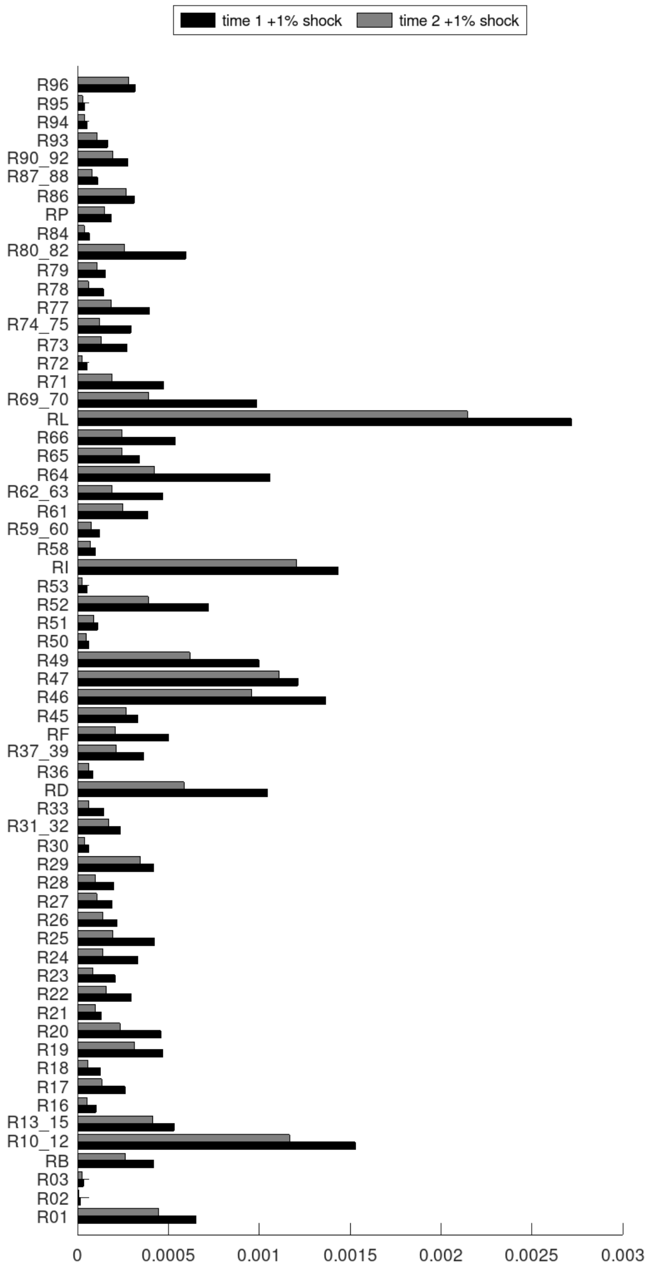

Figure 1: Variation of private household utility as a consequence of a +1% shock in a given sector at a given period obtained computing equilibrium quantities twice. Labels on the vertical axis refer to code sectors, the full list of sector names is provided in appendix A.

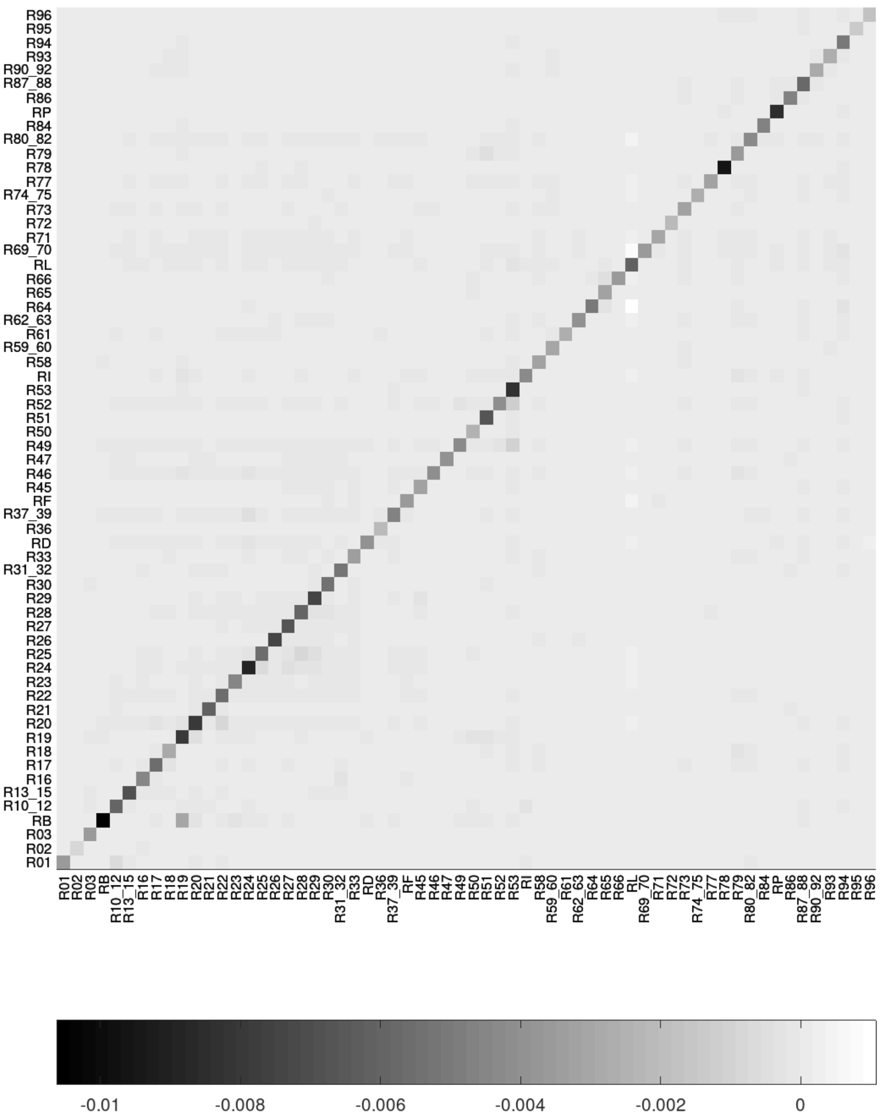

Figure 2: Variation in the net real interest rates of a +1% supply side shock at time 1 obtained computing equilibrium quantities twice. Sectors where the supply side shock occurs are reported on the vertical axis, such that the effects can be read on the corresponding row.

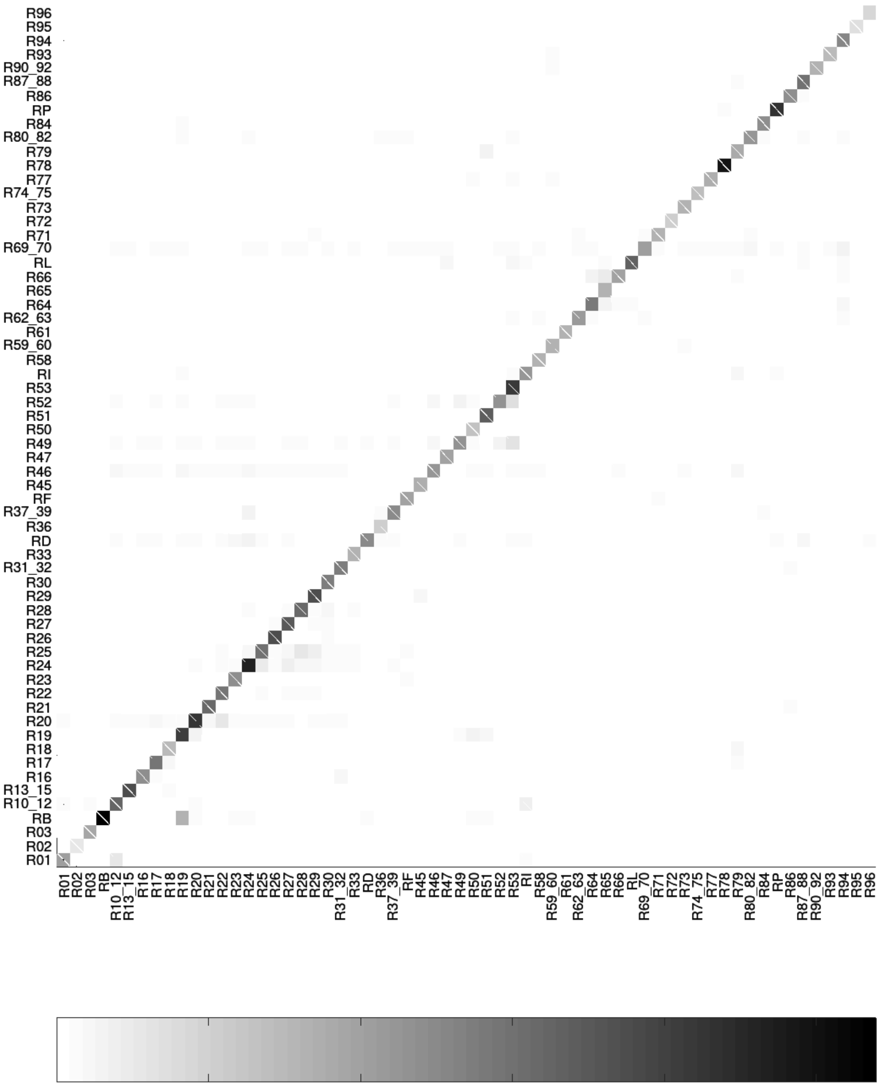

Figure 3: Variation in the net real interest rates of a +1% supply side shock at time 2 obtained computing equilibrium quantities twice. Sectors where the supply side shock occurs are reported on the vertical axis, such that the effects can be read on the corresponding row.

sectoral effects are generally weaker. Noticeable is the negative effect a time 1 shock in the sector “RB - Products from mining and quarrying” has on “R19 - Coke and refined petroleum products” and the positive effect of a time 1 shock in “R64 - Financial services (excluding insurance and pension funding)” exerts on “ RL - Real estate services”. Along the same lines, a time 2 shock in RB has a quite large positive effect on the net real interest rate of R19, a shock in “R25 - Fabricated metal products, except machinery and equipment” has a positive effect on “R28 - Machinery and mechanical appliances n.e.c.”, a shock in “R01 - Products of agriculture and hunting and related services” has a positive effect on “R10 12: Food products, beverages, and tobacco products”, and positive shocks in “R49 - Land transport services and transport via pipelines” and “R52 - Warehousing and support services for transportation” have positive effects on “R53 - Postal and courier services”. Even if these results are somehow expected because of supply chains, they still act as a sort of validation of our analysis.

6.2 Climate Risk Exposure and Skills

Our calibration exercise includes a detailed mapping of economic sectors according to their skill intensity, environmental impact, and exposure to climate shocks. This classification allows for a nuanced understanding of how different segments of the economy may be affected by and contribute to the challenges posed by climate change and the transition to a greener economy.

The distribution of skill intensity across sectors reveals substantial heterogeneity, confirming patterns observed by Goos et al. (2014) and OECD (2021). Sectors with *low skill intensity* are concentrated in traditional industries such as agriculture (R01-R03), mining (RB), food and beverage manufacturing (R10_12), and accommodation and food services (RI) (IPCC, 2022; OECD, 2021). Conversely, *high skill intensity* sectors are predominantly found in knowledge-based industries, including pharmaceuticals (R21), electronics and machinery manufacturing (R26-R29, R30-R33), information and communication services (R61-R62_63), financial and insurance activities (R64, R65), research and development (R72), education (RP), and healthcare (R86) (Goos et al., 2014; OECD, 2021). This suggests that the green transition may pose greater labor market challenges for low-skill sectors, where workers could face higher risks of displacement.

The classification of sectors according to their environmental impact highlights a clear dichotomy, aligned with recent findings by the IPCC (2022) and OECD (2021). *Brown sectors*, characterized by high greenhouse gas emissions and pollution, include agriculture, extractive industries, energy production (RD), construction (RF), and transport activities (R49-R51) (IPCC, 2022; OECD, 2021). These sectors are likely to face greater regulatory pressures and transformation needs under climate mitigation policies. In contrast, *green sectors*—those contributing positively to environmental sustainability—are mainly service-oriented and include pharmaceuticals, information technology, financial and insurance services, professional services (R69_70), research and development, education, healthcare, and cultural activities (OECD, 2021; Goos et al., 2014). *Neutral sectors*, which do not have a strong direct impact on the environment, span a wide range of manufacturing and trade activities (e.g., textiles, metals, retail trade).

Exposure to climate shocks varies markedly across sectors, in line with assessments by the IPCC (2022) and OECD (2021). *High exposure sectors* include agriculture, food production, construction, transportation, and tourism-related activities (RI) (IPCC, 2022; OECD, 2021). These industries are particularly vulnerable to extreme weather events, supply chain disruptions, and resource scarcity. *Low exposure sectors* are typically found in knowledge-intensive services such as IT, finance, professional and scientific services, education, and health (OECD, 2021). Their relatively low dependence on physical assets and natural resources provides a form of inherent resilience to climate-related shocks.

To complement this framework, we incorporate sectoral variation in capital intensity and investment dynamics. As shown by Ding (2022), reductions in trade costs—particularly through liberalization of investment goods markets—lower the relative price of capital services, allowing countries to shift production toward more capital-intensive industries. Sectors that can access and effectively deploy capital—such as advanced manufacturing, ICT, and professional services—experience relative gains in capital income and productivity. These dynamics amplify the resilience and growth potential of green, high-skill, capital-intensive sectors, especially within open economies integrated into global value chains.

However, capital reallocation is not frictionless. Baqaee and Malmberg (2024) emphasize that in the presence of financial frictions and heterogeneity among households and firms, long-run outcomes are governed by a "Golden Rule wedge"—the markup between the rate of return on capital and the cost of investment goods. This wedge reflects the inefficiency of dynamic equilibria and determines the extent to which shocks, including climate-related ones, affect long-run consumption. When capital misallocation is severe, even policy-induced transitions (e.g., from brown to green production) may lead to suboptimal growth if they intensify distortions rather than relax them. Understanding and addressing these capital allocation inefficiencies is thus essential for effective green transition policies.

Cross-referencing all four dimensions—skills, environmental impact, climate exposure, and capital intensity—yields important policy insights. Notably, we can summarize the most relevant results of such an analysis as follows:

1. Several sectors combine *low skill intensity*, *brown environmental impact*, *high exposure to climate shocks*, and *limited capital productivity*—such as agriculture, coal mining, and accommodation and food services. These "quadruply

vulnerable" sectors are at significant risk of structural decline and require integrated policy responses that include worker retraining, investment in adaptation technologies, and easing access to capital (IPCC, 2022; OECD, 2021; Baqaee and Malmberg, 2024).

2. *High skill, green sectors*—including IT services, research and development, education, and healthcare—are capital-intensive, internationally tradable, and less climate-sensitive. These sectors are well positioned to benefit from both environmental and economic transformations, especially in environments with reduced capital goods costs and improved capital allocation (Goos et al., 2014; OECD, 2021; Ding, 2022; Baqaee and Malmberg, 2024).

3. Some *mid-skill, neutral sectors*—such as manufacturing of machinery and metals, wholesale and retail trade—occupy an intermediate position and could be strategically important for ensuring a just transition by absorbing displaced workers from more vulnerable sectors. Their role as transitional buffers is especially relevant in the context of capital market dynamics, as their investment requirements are significant but potentially underserved due to existing distortions (OECD, 2021; Baqaee and Malmberg, 2024).

Overall, the extended mapping underscores the structural complexity of the green transition. In addition to skill and environmental considerations, the interaction between capital services, trade openness, and dynamic inefficiencies plays a crucial role in shaping sectoral trajectories. A successful and equitable transition strategy must therefore integrate environmental, labor market, industrial, and capital allocation policies to address risks while maximizing long-term gains.

6.3 Skill Calibration and Climate-related Shocks

Here we modify the calibration adopted in Section 6.1 to accommodate the previous considerations on households' skills and to investigate the consequence of a supply side shock at time 2 (to be interpreted as a climate-related shock) that hits different sectors in different ways. To do that, we classify each sector along two dimensions (skill intensity and climate exposure) by assigning one among three levels for each dimension: HIGH, NEUTRAL, LOW. Such a classification is shown in Appendix B

For the skill calibration, we proceed as follows. First, we disaggregate the labor factor into two different factors, high skill labor and low skill labor, and consider two different private households, a high skill one and a low skill one, each one completely endowed with the respective labor factor. Then, we split sector wages between high skill and low skill following our classification: a sector classified as HIGH will pay 75% of its wages to the high skill household and 25% to the low skill

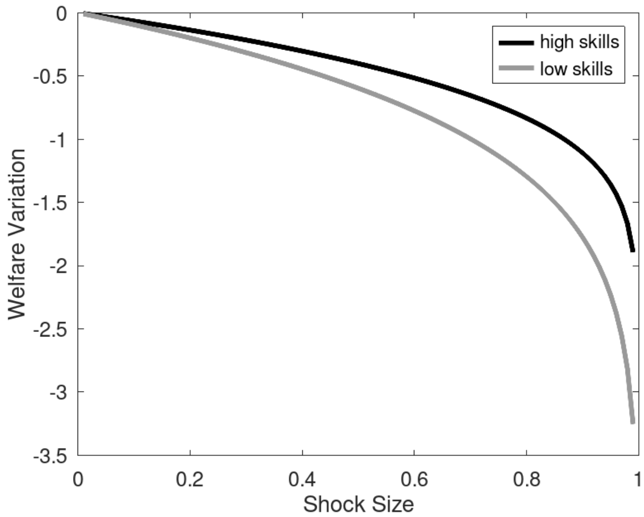

Figure 4: Variation in utility of the high skill and low skill household as a function of shock size in the climate-related supply side shock scenario.

household, a sector classified as LOW will do the opposite, and a NEUTRAL sector splits its total wages equally between the two households. The same procedure is used to split profits and consumption between the two households.

Then, we use the classification of sectors with respect to climate shock exposure to model climate-related supply side shock. More specifically, given a shock size $z \in (0,1)$, we assume that sectors classified as LOW are not hit by the shock (that is, their time 2 total factor productivity z_i^2 remains 1), those classified as NEUTRAL receive a shock equal to half the size of z (i.e., after the shock it is $z_i^2 = 1-z/2$), and those classified as HIGH receive the full shock (that is, $z_i^2 = 1-z$ after the shock). In Figure 4 we show the variation in welfare generated by a shock of size $z \in (0,1)$ for the high- and low-skilled household.[3] As one can notice the drop in utility experienced by the household with low skills is larger than the one observed for the household with high skills. This indicates that climate-related supply side shocks may hit harder low skill households.

In Figure 5, we present the variation in the net real interest rates generated by the same mechanism of climate-related supply shock but focusing on two shock sizes: a relatively small shock (10%) and a relatively large shock (50%).[4] As

[3]Figure 4 has been obtained computing equilibrium quantities twice, in Figure 9 we present the results obtained through the first order approximation. As one can notice, the shape of the curve changes because of the approximation, but the relative positions and trends do not.

[4]We present here the results obtained computing equilibrium quantities twice, in Figure 10 in Appendix C one can find the results obtained by first order approximations. Again, because of the double approximation, the results show larger differences in quantitative terms.

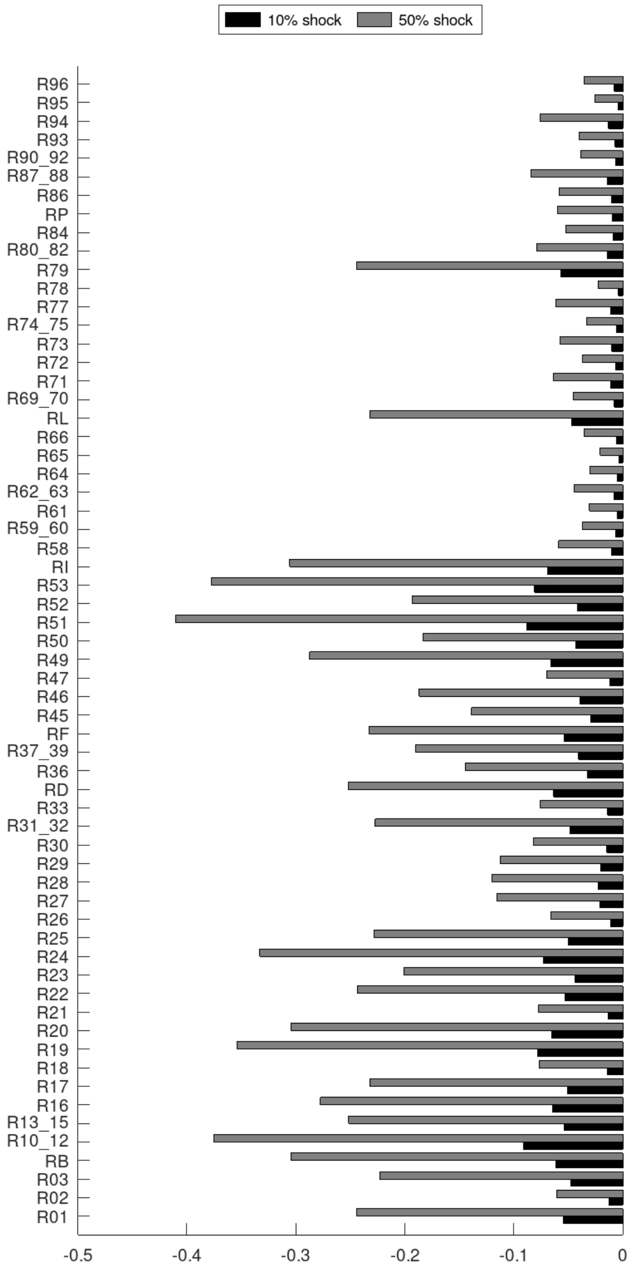

Figure 5: Variation in net real interest rates as a consequence of a shock with relatively small size (10%) and relatively large size (50%) in the climate-related supply side shock scenario.

argued in advance, the effect on net real interest rates of a supply side shock at time 2 shows the same sign of the variation, thus, all the net interest rates decrease. As expected, the rates relative to sectors less exposed to climate-related shocks (e.g., ICT, professional services, the financial sector, health, education, and other services) show smaller variations than those connected to sectors more exposed to climate-related shocks (e.g., agriculture, the food industry, oil refining, metals, chemical products, transportation and postal services). At the same time, production networks play a role in influencing those variation, for instance a sector highly exposed to climate-related shocks such as "R36 – Natural water; water treatment and supply services" experiences a reduction in net real interest rate lower than the one occurring in a less exposed sector such as "RL – Real estate services".

7 Conclusions

In this paper, we develop and analyze a general equilibrium production network model that spans two periods, incorporates a detailed mechanism of capital formation, and features heterogeneous agents. Within this framework, we establish the existence and uniqueness of a competitive equilibrium and show that all relevant economic quantities can be computed explicitly. In particular, we provide an analytical characterization of the effects of supply-side shocks, occurring at different times and in various firms or sectors, on both individual welfare and real interest rates.

To illustrate the model's applicability, we calibrate it using Input-Output data from the Italian economy. Our baseline analysis uncovers distinctive features of the Italian production structure, notably the central roles played by the real estate, food, and tourism-related sectors. We then extend the analysis by distinguishing between high-skilled and low-skilled households to examine the effects of a climate-related negative supply-side shock. This shock simultaneously affects multiple sectors, with its intensity varying according to sectoral exposure. Our results indicate that such climate shocks disproportionately reduce the welfare of low-skilled households and have a non-trivial impact on real interest rates.

CRediT authorship contribution statement

Giulio Bottazzi: Conceptualization, Formal analysis, Investigation, Supervision, Funding acquisition. Daniele Giachini: Conceptualization, Formal analysis, Investigation, Data curation, Software, Writing – original draft, Supervision, Project administration, Funding acquisition. Eleonora Priori: Data curation, Literature review, Investigation, Writing – original draft.

Declaration of competing interest

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Appendix

A Sector codes and names

- **R01**: Products of agriculture and hunting and related services
- **R02**: Products of forestry, logging operations, and related services
- **R03**: Fish and other fishery products; aquaculture products; support services for fishing
- **RB**: Products from mining and quarrying
- **R10_12**: Food products, beverages, and tobacco products
- **R13_15**: Textiles; clothing articles; leather and related products
- **R16**: Wood and products of wood and cork (excluding furniture); articles of straw and plaiting materials

- **R17**: Paper and paper products
- **R18**: Printing and recording services
- **R19**: Coke and refined petroleum products
- **R20**: Chemical products
- **R21**: Basic pharmaceutical products and pharmaceutical preparations
- **R22**: Rubber and plastic products
- **R23**: Other non-metallic mineral products
- **R24**: Metals
- **R25**: Fabricated metal products, except machinery and equipment
- **R26**: Computer, electronic, and optical products
- **R27**: Electrical equipment
- **R28**: Machinery and mechanical appliances n.e.c.
- **R29**: Motor vehicles, trailers, and semi-trailers
- **R30**: Other transport equipment
- **R31_32**: Furniture; other manufactured goods
- **R33**: Repair and installation services of machinery and equipment
- **RD**: Electricity, gas, steam, and air conditioning supply
- **R36**: Natural water; water treatment and supply services
- **R37_39**: Wastewater treatment services; sewage sludge; waste collection, treatment and disposal services; materials recovery services; remediation and other waste management services
- **RF**: Construction work and civil engineering works
- **R45**: Wholesale and retail trade and repair of motor vehicles and motorcycles
- **R46**: Wholesale trade services, excluding motor vehicles and motorcycles
- **R47**: Retail trade services, excluding motor vehicles and motorcycles

- **R49**: Land transport services and transport via pipelines
- **R50**: Water transport services
- **R51**: Air transport services
- **R52**: Warehousing and support services for transportation
- **R53**: Postal and courier services
- **RI**: Accommodation and food service activities
- **R58**: Publishing services
- **R59_60**: Motion picture, video and television program production services; sound recording and music publishing; broadcasting services
- **R61**: Telecommunications services
- **R62_63**: Computer programming, consultancy, and related services; information services
- **R64**: Financial services (excluding insurance and pension funding)
- **R65**: Insurance, reinsurance, and pension funding services, excluding compulsory social insurance
- **R66**: Auxiliary services to financial and insurance services
- **RL**: Real estate services
- **R69_70**: Legal and accounting services; head office services; management consultancy services
- **R71**: Architectural and engineering services; technical testing and analysis services
- **R72**: Scientific research and development services
- **R73**: Advertising and market research services
- **R74_75**: Other professional, scientific, and technical services; veterinary services
- **R77**: Rental and leasing services
- **R78**: Employment services

- **R79**: Travel agency, tour operator and other reservation services and related services

- **R80_82**: Security and investigation services; building and landscape maintenance services; office administrative and other business support services

- **R84**: Public administration and defense services; compulsory social security services

- **RP**: Education services

- **R86**: Human health services

- **R87_88**: Residential care services; social work services without accommodation

- **R90_92**: Creative, arts and entertainment services; library, archive, museum, and other cultural services; gambling and betting services

- **R93**: Sports, amusement, and recreation services

- **R94**: Services provided by membership organizations

- **R95**: Repair services of computers and personal and household goods

- **R96**: Other personal services

B Classification of sectors

Code	Skill Intensity	Shock Exposure	Code	Skill Intensity	Shock Exposure
R01	LOW	HIGH	R50	NEUTRAL	HIGH
R02	LOW	HIGH	R51	HIGH	HIGH
R03	LOW	HIGH	R52	NEUTRAL	NEUTRAL
RB	LOW	NEUTRAL	R53	LOW	NEUTRAL
R10_12	LOW	HIGH	RI	LOW	HIGH
R13_15	LOW	NEUTRAL	R58	HIGH	LOW
R16	NEUTRAL	HIGH	R59_60	HIGH	LOW
R17	NEUTRAL	NEUTRAL	R61	HIGH	LOW
R18	HIGH	LOW	R62_63	HIGH	LOW
R19	LOW	NEUTRAL	R64	HIGH	LOW
R20	NEUTRAL	NEUTRAL	R65	HIGH	LOW
R21	HIGH	LOW	R66	HIGH	LOW
R22	NEUTRAL	NEUTRAL	RL	HIGH	NEUTRAL
R23	NEUTRAL	NEUTRAL	R69_70	NEUTRAL	LOW
R24	NEUTRAL	NEUTRAL	R71	HIGH	LOW
R25	NEUTRAL	NEUTRAL	R72	HIGH	LOW
R26	HIGH	LOW	R73	HIGH	LOW
R27	HIGH	LOW	R74_75	HIGH	LOW
R28	HIGH	LOW	R77	HIGH	LOW
R29	HIGH	LOW	R78	HIGH	LOW
R30	HIGH	LOW	R79	HIGH	HIGH
R31_32	NEUTRAL	NEUTRAL	R80_82	HIGH	LOW
R33	NEUTRAL	LOW	R84	NEUTRAL	LOW
RD	NEUTRAL	HIGH	RP	HIGH	LOW
R36	NEUTRAL	HIGH	R86	HIGH	LOW
R37_39	NEUTRAL	NEUTRAL	R87_88	NEUTRAL	LOW
RF	LOW	HIGH	R90_92	NEUTRAL	LOW
R45	NEUTRAL	NEUTRAL	R93	NEUTRAL	LOW
R46	NEUTRAL	NEUTRAL	R94	NEUTRAL	LOW
R47	NEUTRAL	LOW	R95	HIGH	LOW
R49	NEUTRAL	HIGH	R96	LOW	LOW

C Additional Figures

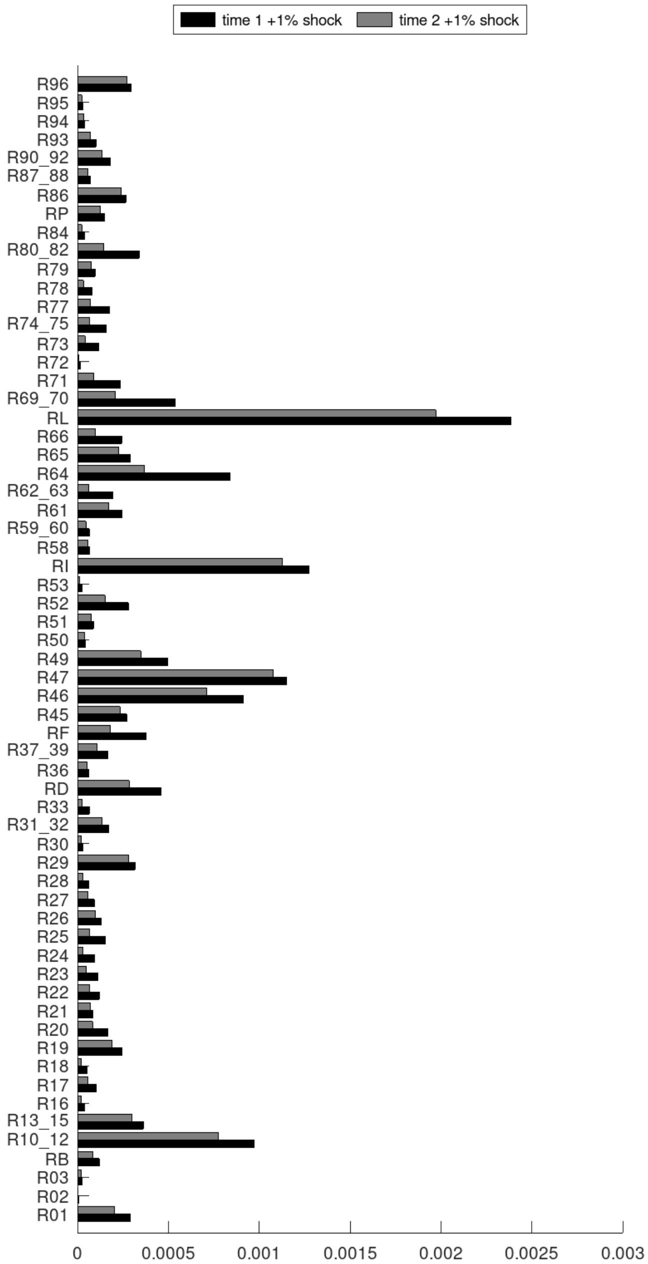

Figure 6: Variation of private household utility as a consequence of a +1% shock in a given sector at a given period obtained by means of first-order approximations.

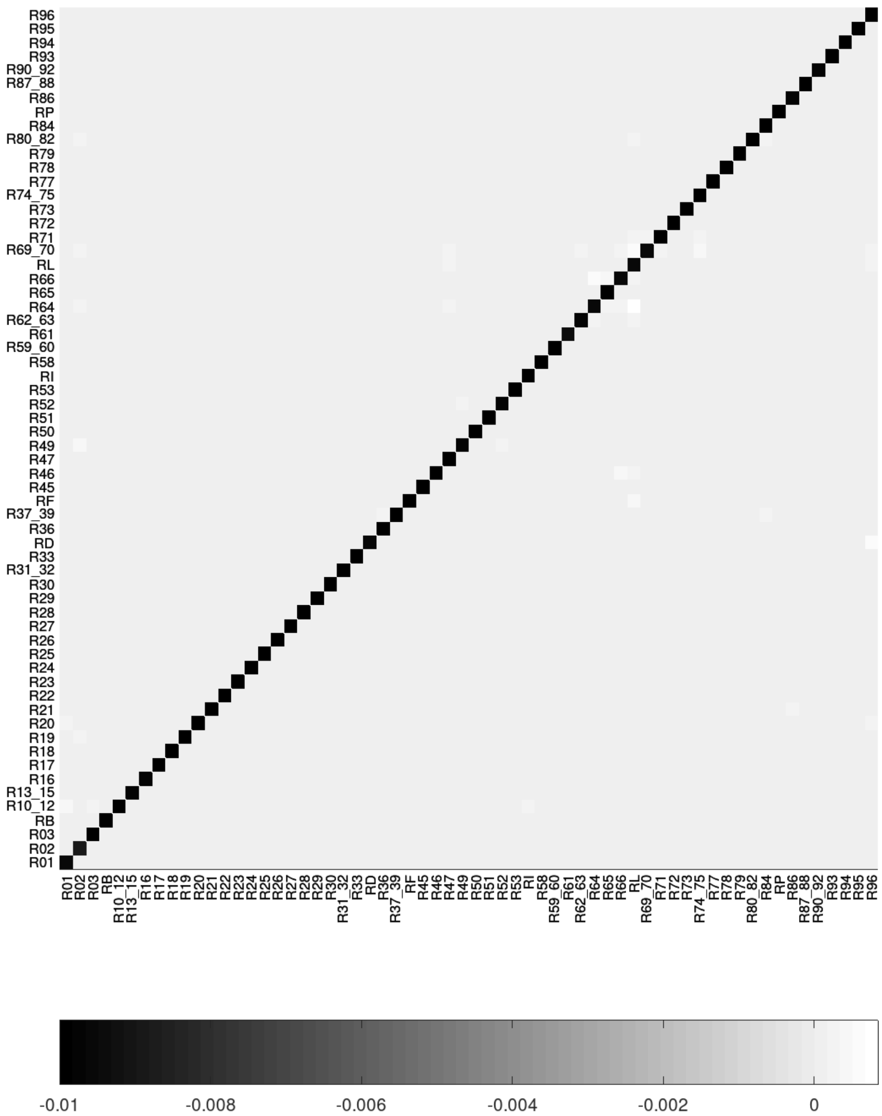

Figure 7: Variation in the net real interest rates of a +1% supply side shock at time 1 obtained from the first order approximations. Sectors where the supply side shock occurs are reported on the vertical axis, such that the effects can be read on the corresponding row.

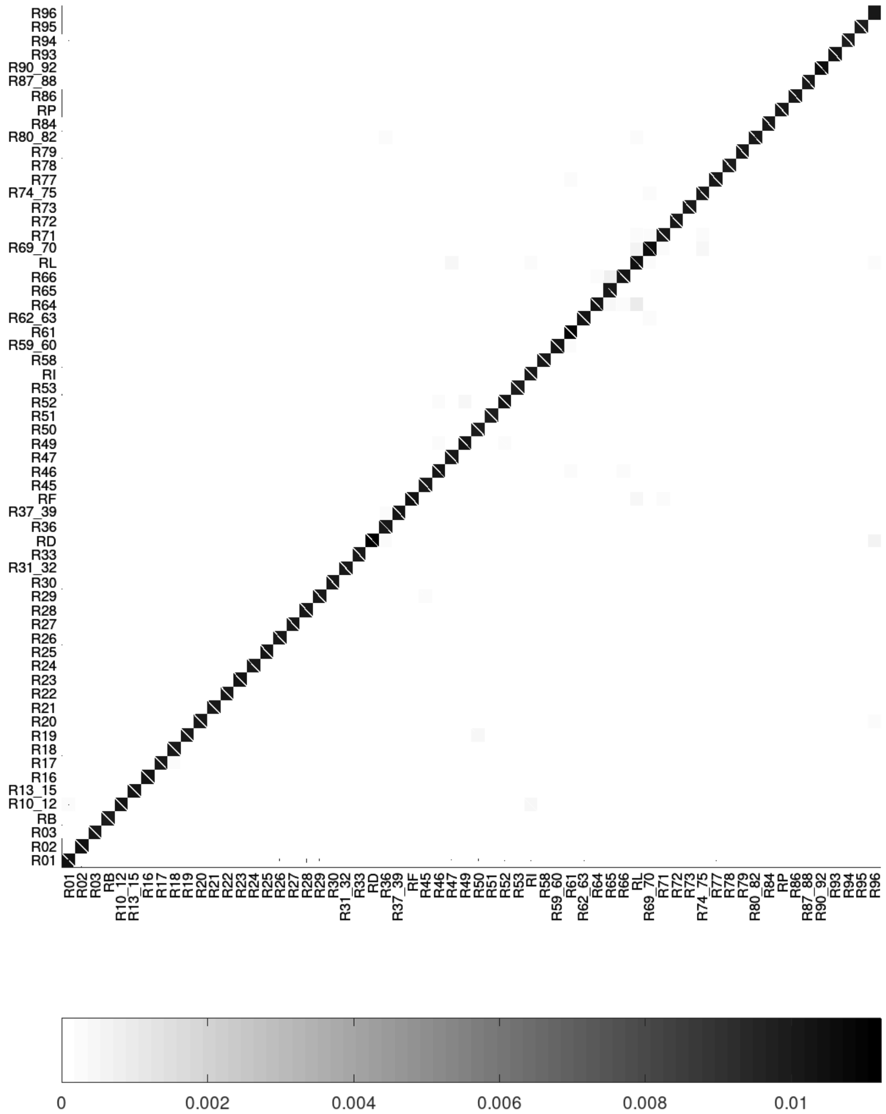

Figure 8: Variation in the net real interest rates of a +1% supply side shock at time 2 obtained from the first order approximations. Sectors where the supply side shock occurs are reported on the vertical axis, such that the effects can be read on the corresponding row.

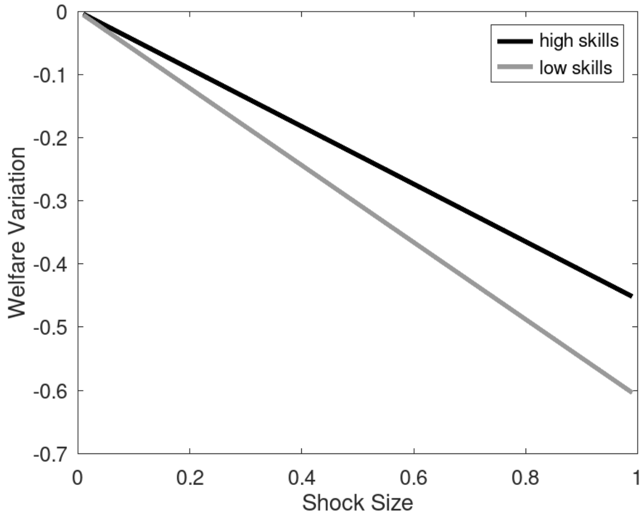

Figure 9: Variation in utility of the high skill and low skill household as a function of shock size in the climate-related supply side shock scenario. Results obtained by means of the first order approximation.

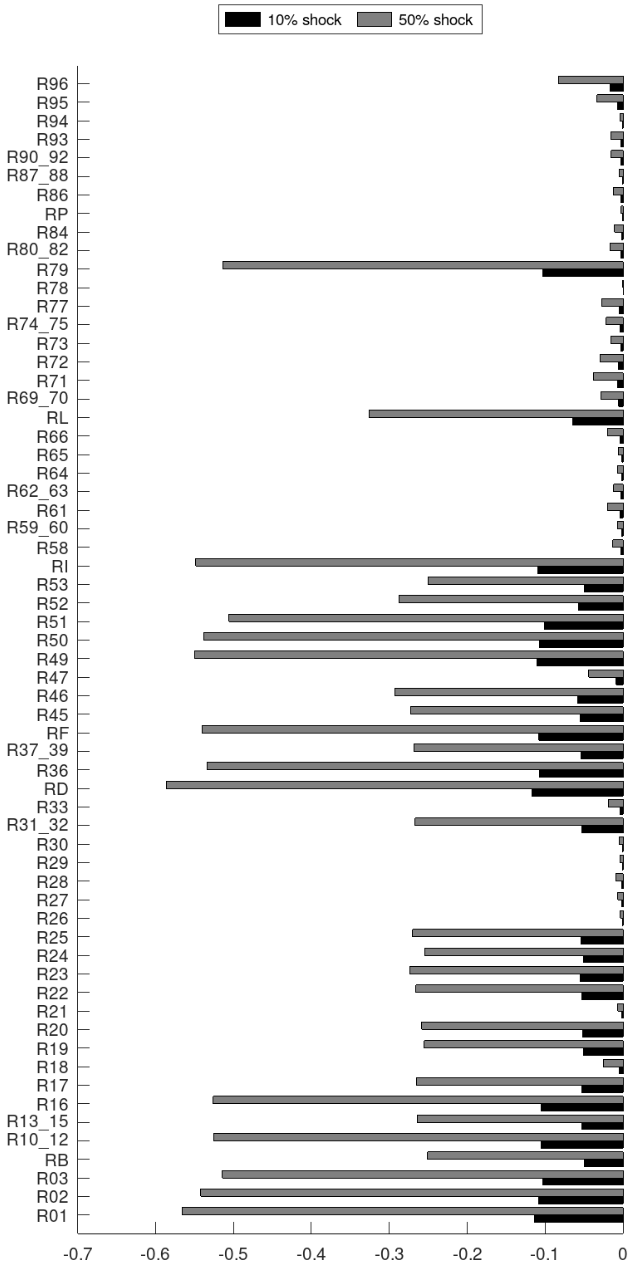

Figure 10: Variation in net real interest rates as a consequence of a shock with relatively small size (10%) and relatively large size (50%) in the climate-related supply side shock scenario. Results obtained by means of the first order approximation.

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