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# Heterogeneity in Macroeconomics: The Compositional Inequality Perspective*

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**Abstract**

This work presents a framework to jointly study individuals' heterogeneity in terms of their capital and labor endowments (*endowment heterogeneity*) and of their saving and consumption behaviors (*behavioral heterogeneity*), from an empirical perspective. By adopting a newly developed synthetic measure of compositional inequality, this work classifies more than 20 economies across over two decades on the basis of their heterogeneity characteristics. Modern economies are far from being characterized by agents with same propensities to save and consume and same endowments (*Representative Agent* systems), or by the existence of rich capital-abundant savers and poor hand-to-mouth consumers (*Kaldorian* systems). Our framework and results are discussed in light of the heterogeneity assumptions underlying several types of macroeconomic models with heterogeneous agents (*Kaldorian*, *TANK* & *HANK*, *OLG*, and *ABM* models). A negative relationship between behavioral heterogeneity and the economy's saving rate is also documented.

**JEL-Classification:** D31, P10, O43

**Keywords:** Heterogeneity in Macroeconomics, Compositional Inequality.

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# 1 Introduction

This paper offers a novel perspective on the role of heterogeneity in macroeconomic modelling. Two types of heterogeneity are considered. The first type is heterogeneity in the distribution of savings and consumption across the income distribution. On average, income-rich individuals save relatively more than income-poor individuals, whose propensity to consume is generally higher (Dynan et al., 2004, Saez and Zucman, 2016, Jappelli and Pistaferri, 2014, Bunn et al., 2018). We call this source of heterogeneity *behavioral heterogeneity*. The second type is heterogeneity in the distribution of capital and labor incomes across the income distribution. Recent studies have shown that, globally, poor individuals derive most of their income from labor, whilst rich individuals derive most of their income from capital (Ranaldi and Milanovic, 2022, Iacono and Palagi, 2022b, Iacono and Ranaldi, 2022). We call this type of heterogeneity *endowment heterogeneity*. By endowment heterogeneity we make reference to the distinction between the endowment of human and physical capital. In our simplified framework, while the former generates labor income, the latter generates capital income.

Macroeconomic models with heterogeneous agents adopt specific behavioural and endowment heterogeneity assumptions. Kaldor was among the first to differentiate between capital-abundant savers and labor-abundant consumers, in a macroeconomic model of growth and distribution (Kaldor, 1955). Recent HANK and Agent Based models are, however, more flexible insofar as they allow for different combinations of heterogeneity characteristics (Kaplan et al., 2018, Dosi et al., 2010). Which set of heterogeneity assumptions best describe modern economic systems? How can we measure behavioral and endowment heterogeneity to inform macroeconomic modelling and attune it to societal characteristics?

This paper presents a framework to jointly study behavioral and endowment heterogeneity, from an empirical perspective. To this end, it combines standard practices in macroeconomic modelling, with recent advances in inequality studies. Specifically, we adopt the concept of *compositional inequality*, as well as an indicator for its measurement (Ranaldi, 2022), to assess the extent of behavioural and endowment heterogeneity across more than 20 economies between 1995 and 2018.

Compositional inequality describes the extent to which the composition of income into two sources, such as capital and labor incomes, or into savings and consumption, is unequally distributed between income-rich and income-poor. The higher the compositional inequality in behaviors and endowments, the more income-rich capitalists save and income-poor laborers consume. The lower the compositional inequality in behaviors and endowment, the more an economy can be represented by a single agent with fixed propensities to save and consume and fixed shares of capital and labor incomes in her total income. We call the former configuration a *Kaldorian System*, and the latter a *Representative Agent System*.

By means of the Income-Factor Concentration (IFC) index, our preferred measure of compositional inequality, and fine-grained microdata on individuals' income components, we document novel empirical evidences on macroeconomic heterogeneity. Three main results stand out. *First*, we document significant cross-country variations along the behavioral and endowment heterogeneity dimensions, as well as little correlation between them. This finding inevitably shows that heterogeneity matters and is country-specific. *Second*, we report a negative relationship between behavioral heterogeneity, on the one hand, and the country's saving rate, on the other: the more equal the propensities to save and consume across individuals (i.e., the lower behavioral heterogeneity), the higher the country's overall saving rate. This relationship is both theoretically and empirically confirmed. *Third*, when a country moves from being a Representative to a Kaldorian System, or when both types of heterogeneity simultaneously increase, its growth rate is firstly positively, and then negatively signed. We therefore find an inverted U-shaped relationship between heterogeneity (in both dimensions) and growth. The cut-off point that determines the change in sign of this relationship is empirically identified. This result sheds light on the extent to which heterogeneity is beneficial, or harmful for growth.

All results from the paper are carefully discussed and brought in conversations with the principal macroeconomic models with heterogeneous agents, as well as their underlying heterogeneity assumptions. Four broad categories of models are considered: Kaldorian, TANK and HANK, OLG, and ABM models. We regard this paper as one of the first attempt at introducing a framework to jointly study behavioral and endowment heterogeneity, from an empirical perspective.

The paper is structured as follows. Section 2 introduces the methodological framework of analysis. Section 3 describes the main empirical results. Section 4 discusses the major implications of our findings for macroeconomic modelling. In this section, we carefully analyse the heterogeneity assumptions underlying several benchmark macroeconomic models with heterogeneous agents. Section 5 concludes the paper.

# 2 Framework

## 2.1 Concept

To empirically analyze behavioral and endowment heterogeneity we rely on the concept of *compositional inequality*, as well as on the *Income-Factor Concentration* (IFC) index, a suitable indicator for its measurement, recently developed in Ranaldi (2022). In this work, we consider two types of compositional inequality: *(i)* compositional inequality in capital and labor incomes, and *(ii)* compositional inequality in saving and consumption.

Compositional inequality describes the extent to which the composition of income into two factors is unequally distributed across the income distribution. Compositional inequality is *maximal* when the two factors are polarized across the income distribution. This is the case when, for instance, the top $p\%$ of the population (such as the top 10%), ranked according to total income, earns income from capital only, and the bottom $(1-p)\%$ (such as the bottom 90%) earns exclusively income from labor. Consider a society composed by two individuals only, $A$ and $B$. Assume the income of $A$, $y_A$, is greater than the income of $B$, $y_B$ ($y_A > y_B$). There is maximal compositional inequality if the income of $A$ is entirely composed by profits, $y_A = \pi$, and the income of $B$ by wages, $y_B = w$.[1] In this stylized society, $A$'s *relative* income coincides with the overall capital income share, and $B$'s *relative* income with the labor income share. Compositional inequality is, instead, minimal when the composition of these two factors is the same across the population. This is the case when, for instance, both individuals, $A$ and $B$, earn the $x\%$ of their income from labor, and $(1-x)\%$ of their income from capital. This implies that the macro functional income distribution is the same as the micro one, for all individuals in society.

In our framework, we consider compositional inequality in capital and labor incomes as a proxy for endowment heterogeneity, and compositional inequality in saving and consumption as a proxy for behavioral heterogeneity. The higher the level of compositional inequality, the greater the degree of heterogeneity.

## 2.2 Methodology

To measure endowment and behavioral heterogeneity we use the IFC index for capital and labor incomes, denoted by $\mathscr{I}_{kl}$, and the IFC index for saving and consumption, denoted by $\mathscr{I}_{sc}$, respectively. To illustrate the mechanics of two indicators of compositional inequality, we start defining the following two equations for individual $i$'s income (share):

$$y_i = \alpha\pi + \beta w \tag{1}$$

$$y_i = \lambda s + \delta c \tag{2}$$

where $y_i = \frac{Y_i}{Y}$ is the individual $i$'s income share, $\pi = \frac{\Pi}{Y}$ and $w = \frac{W}{Y}$ are the aggregate capital and labor shares, $s = \frac{S}{Y}$ and $c = \frac{C}{Y}$ are the saving and consumption rates, $\alpha_i = \frac{\Pi_i}{\Pi}$ and $\beta_i = \frac{W_i}{W}$ are the individual $i$'s relative shares of capital and labor incomes, and $\lambda_i = \frac{S_i}{S}$ and $\delta_i = \frac{C_i}{C}$ the individual $i$'s relative shares of saving and consumption. If we rank individuals according to their total income, $Y_i$ (or $y_i$, since the ranking would be the same), and define the cumulative share of income of the bottom $p\%$ of the distribution as $\mathscr{L}(\boldsymbol{y}, p) = \sum_{j=1}^{p=\frac{i}{n}} y_j$, then the pairs $(p, \mathscr{L}(\boldsymbol{y}, p))$ describe the Lorenz

[1]Compositional inequality would also be maximal if the income sources were exchanged between $A$ and $B$: $A$ earns only income form labor, and $B$ only income from capital.

curve for income. The Gini coefficient of income, a widely used indicator of income inequality, is defined as one minus twice the area under the Lorenz curve. Similarly, by fixing the same ranking according to income, we can define the cumulative shares of capital, labor, saving, and consumption of the bottom $p\%$ of the distribution as $\mathscr{L}(\boldsymbol{\pi}, p) = \sum_{j=1}^{p=\frac{i}{n}} \alpha_j$, $\mathscr{L}(\boldsymbol{w}, p) = \sum_{j=1}^{p=\frac{i}{n}} \beta_j$, $\mathscr{L}(\boldsymbol{s}, p) = \sum_{j=1}^{p=\frac{i}{n}} \lambda_j$, $\mathscr{L}(\boldsymbol{c}, p) = \sum_{j=1}^{p=\frac{i}{n}} \delta_j$. The pairs $(p, \mathscr{L}(\boldsymbol{\pi}, p))$, $(p, \mathscr{L}(\boldsymbol{w}, p))$, $(p, \mathscr{L}(\boldsymbol{s}, p))$ and $(p, \mathscr{L}(\boldsymbol{c}, p))$ describe the concentration curves for capital income, labor income, saving, and consumption.[2] The area under the concentration curve can already be regarded as a rough proxy variable for compositional inequality. Let us take the example of the concentration curve for capital income. When the area is high (low), capital income is concentrated at the bottom (top) of the income distribution, labor income is concentrated at the top (bottom), and endowment heterogeneity is high.

To precisely measure the degree of compositional inequality in a population, we however need to introduce two additional concentration curves: the *zero-*, and the *maximum-concentration curve*. Similarly to the bisector for the Gini coefficient, the zero-concentration curve describes zero inequality in income composition. Differently from the bisector, however, the zero-concentration curve varies across distributions.[3] From an analytical point of view, the zero-concentration curves for capital, labor, saving, and consumption are equivalent to the Lorenz curve, multiplied by the capital, labor, saving, and consumption share of income, respectively. The maximum-concentration curve describes, instead, a hypothetical distribution such that the bottom $p\%$ of the distribution receives one source of income only, while the top $(1 - p)\%$ receives the other source of income.[4]

If we denote by $\mathscr{A}_{kl}$ the area between the concentration curve for capital income, multiplied by the capital share, and the zero-concentration curve for capital income, and by $\mathscr{B}_{kl}$ the the area between the zero- and the maximum-concentration curves for capital income, we can define the IFC index for capital and labor incomes, $\mathscr{I}_{kl}$, as

[2] These curves were first introduced by Kakwani (1977).
[3] In other words, the benchmark of zero compositional inequality is *distribution-specific*.
[4] For an analytical description of the zero- and maximum-concentration curves, see appendix B.

follows

$$\mathscr{I}_{kl} = \frac{\mathscr{A}_{kl}}{\mathscr{B}_{kl}}. \tag{3}$$

Similarly, if we denote by $\mathscr{A}_{sc}$ the area between the concentration curve for saving, multiplied by the saving rate, and the zero-concentration curve for saving, and by $\mathscr{B}_{sc}$ the the area between the zero- and the maximum-concentration curves for saving, we can define the IFC index for savings and consumption, $\mathscr{I}_{sc}$, as follows

$$\mathscr{I}_{sc} = \frac{\mathscr{A}_{sc}}{\mathscr{B}_{sc}}. \tag{4}$$

The $\mathscr{I}_{kl}$ and $\mathscr{I}_{sc}$ range between $-1$ and 1. They are equal to 1 when capital incomes (savings) are concentrated at the top of the income distribution and labor incomes (consumption) at the bottom. They are equal to 0 when every individual has the same composition of capital and labor incomes, or the same propensities to save and consume. The two indicators are, instead, equal to $-1$ when labor income (consumption) is concentrated at the top of the income distribution and capital income (savings) at the bottom. While negative values of these indicators are hard to find in practice, they still represent interesting theoretical possibilities.

In the case of compositional inequality in capital and labor, the sign of the IFC index determines the relationship between the functional and personal distributions of income. A positive IFC implies that an increase in the capital share of income directly translates into an increase in inter-personal income inequality, all else being equal. In the case of compositional inequality in saving and consumption, instead, the sign of the IFC index determines the relationship between the dynamics of the overall saving (consumption) rate and inter-personal income inequality. A positive IFC implies that an increase in the saving rate translates into an increase in inter-personal income inequality, all else being equal. The IFC for saving and consumption can also be considered as a proxy for *saving rate inequality* (Saez and Zucman, 2016).

We define the *Heterogeneity Box* as the set of all possible combinations of the two indicators of compositional inequality, $\mathscr{I}_{kl}$ and $\mathscr{I}_{sc}$ (figure 1). Two representative points of the Box are of particular interest: $\mathbf{R} = (0, 0)$ and $\mathbf{K} = (1, 1)$. $\mathbf{R}$ describes an economy in which every individual has the same shares of capital and labor income in her total income, and the same propensities to save and consume. We call this economy a *Representative Agent* system. **K** describes, instead, an economy in which income-rich individuals save capital income, and income-poor individuals consume labor income. We call this economy a *Kaldorian* system.

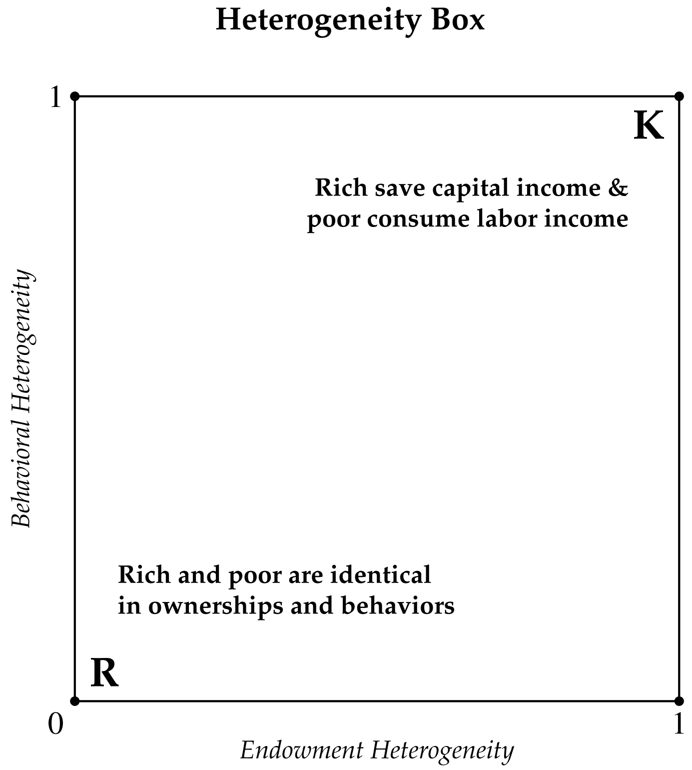



Figure 1: This $1 \times 1$ Box displays all combinations of the positive values of two indices of compositional inequality. Point **K** represents the *Kaldorian* system, in which both behavioral (vertical axis) and endowment (horizontal axis) heterogeneity are maximized. On the contrary side, point **R** represents the *Representative Agent* system, in which both heterogeneity dimensions are minimal. In point **K** the link between income inequality and both the saving rate and the capital share is strong, while it is weak in point **R**.

In the next section, we estimate the distribution of countries in this Box.

# 3 Empirical Findings

## 3.1 Data

To measure the IFC index for capital and labor incomes, we rely on the Ranaldi and Milanovic Database (Ranaldi and Milanovic, 2022). This database includes estimates of the IFC for several definitions of capital and labor incomes, calculated using the Luxembourg Income Study (LIS) Database. In this article, we adopt the benchmark definition present in Ranaldi and Milanovic, whereby capital income is defined as the sum of *rental income*, *interests from deposits*, and *income from dividends*, and labor income as the sum of *wages*, *self-employment income*, and *pension income*, here defined as deferred labor income. The unit of analysis is the individual and all income sources are equally split among household members.[5] Negative income values are excluded from the analysis.

To estimate the IFC for saving and consumption we also rely on the LIS Database. The LIS database provides not only information on individuals' total income, but also on individuals' total consumption expenditure. Savings are defined as the difference between income and consumption[6], both available in LIS. Our estimates of consumption and savings across the distribution are based on a slightly modified definition of total income from the one previously adopted to estimate endowment heterogeneity. Namely, we consider market income *plus* transfers as our benchmark income variable for the estimation of behavioral heterogeneity. This definition allows us to better match the macroeconomic definition of the gross saving rate, which includes net transfer in the denominator. We however recall that our estimates of the saving and consumption rates are household sector's estimate.[7] This is standard practice in

[5] No equivalence scaled are therefore applied.

[6] Our consumption variable includes the following consumption categories: Food and Non-Alcoholic Beverages, Alcohol and Tobacco, Clothing and Footwear, Actual Rent and Utilities, Housing Equipment, Health, Transport, Communication, Recreation and Culture, Education, Restaurants and Hotels, Miscellaneous Goods and Services.

[7] Consumption rates at the macro level instead include non-profit institutions serving households (NPISH) final consumption expenditure and government expenditure on individual consumption goods and services. This explains why, for the majority of the economies, the values of aggregate consumption rates are higher than the household sector's estimates.

empirical studies on the distributions of the propensities to save and consume across the population. Negative saving values are set equal to zero[8] and the unit of analysis is, also in this case, the individual.

Overall, we construct an unbalanced panel composed of 24 countries, observed between 1995 and 2018. Table 1 describes the full list of countries and years available.[9]

| Country | ISO3 | N. Years | Country | ISO3 | N. Years |
|---|---|---|---|---|---|
| Australia | AUS | 2 | Palestine | PSE | 1 |
| China | CHN | 2 | Peru | PER | 4 |
| Estonia | EST | 1 | Poland | POL | 4 |
| France | FRA | 3 | Romania | ROU | 2 |
| Georgia | GEO | 2 | Russia | RUS | 3 |
| Guatemala | GTM | 3 | Serbia | SRB | 1 |
| Hungary | HUN | 6 | Slovenia | SVN | 6 |
| India | IND | 2 | South Africa | ZAF | 5 |
| Israel | ISR | 8 | South Korea | KOR | 4 |
| Italy | ITA | 8 | Switzerland | CHE | 3 |
| Ivory Coast | CIV | 1 | Taiwan | TWN | 8 |
| Mexico | MEX | 11 | Vietnam | VNM | 2 |

Table 1: List of countries in our sample

### 3.2 Descriptive Results

Figure 2 shows the distribution of countries in the Heterogeneity Box. Recall that the horizontal axis represents endowment heterogeneity ($\mathscr{I}_{kl}$), and the vertical axis behavioral heterogeneity ($\mathscr{I}_{sc}$). We record high variations in both heterogeneity dimensions, whether we consider average values per country (panel *a*), or year-specific country estimates (panel *b*). Overall, our estimates of compositional inequality tend to be strictly positive, with the sole exception of South Korea, which displays negative values for compositional inequality in capital and labor income. This finding

[8]The rationale for setting negative saving values equal to zero is to prevent our concentration curves to fall below zero in correspondence to the bottom percentiles of the distribution. This assumption has almost no impact on the main findings of our paper.

[9]For some major economies, such as the United States, information on consumption is not available in LIS database. Nevertheless, in order to ensure the fullest comparability between our measures of compositional inequality across countries, we refrain from complementing our database with alternative sources.

implies that *(i)* income-rich individuals are relatively more capital abundant than poor individuals, and that *(ii)* income-rich individuals have a higher propensity to save than poor individuals. While these general results are not new,[10] important differences emerge between countries. While a bulk of advanced economies, such as Italy and France, are characterized by moderately high levels of both heterogeneity dimensions, some middle income economies, such as Mexico and Vietnam, display extremely high heterogeneity levels in both dimensions. Furthermore, while some countries combine relatively low levels of behavioral heterogeneity with high levels of endowment heterogeneity (i.e., Hungary), others display high behavioral heterogeneity and low endowment heterogeneity (i.e., Russia). In other words, we find examples of countries characterized by individuals with similar propensities to save, but different composition of incomes, and countries where an unequal distribution of the saving rates is associated to a rather equal composition of capital and labor incomes across the income ladder. These first results suggests that macroeconomic models of growth and distribution should be flexible enough to account for various combinations of both types of heterogeneity. Overall, Figure 2 shows a tendency of contemporary economies to display higher variation in behavioral heterogeneity than in endowment heterogeneity, as also confirmed by Figure 5 in Appendix D. Moreover, while behavioral heterogeneity reaches higher values than endowment heterogeneity, the contrary happens for the lower values.

Figure 3 shows the relationship between behavioral heterogeneity (vertical axis) and the aggregate saving rate (horizontal axis) for each country and year present in the sample. We here consider the household sector saving rate, estimated from the household surveys, so as to allow for direct comparability between macro- and micro-level behavioral heterogeneity. A striking negatively-signed association between these two dimensions emerges. High (low) levels of a country's saving rate tend to be associated with low (high) levels of compositional inequality in saving and

[10] As for behavioral heterogeneity, see Dynan et al. (2004), Saez and Zucman (2016), Jappelli and Pistaferri (2014), Bunn et al. (2018) among others, whilst for endowment heterogeneity, see Ranaldi and Milanovic (2022), Iacono and Ranaldi (2022), Iacono and Palagi (2022b), Petrova and Ranaldi (2021), Berman and Milanovic (2020).

consumption. In other words, economies characterized by a relatively equal distribution of the propensities to save across the population display higher saving rates. To theoretically explain this result, let us consider the following relationship derived by Ranaldi and Milanovic (2022) on the link between compositional inequality in capital and labor incomes, on the one hand, and inter-personal income inequality, on the other:

$$\frac{\mathcal{I}_{kl}}{\mathcal{G}} = \frac{\alpha - \pi}{2\mathcal{B}_{kl}}, \tag{5}$$

where $\mathcal{G}$ is the Gini coefficient of total income, $\alpha = \frac{\mathcal{G}_\pi \mathcal{R}_\pi \pi}{\mathcal{G}}$ is the share of capital income inequality to inequality overall, where $\mathcal{G}_\pi$ is the Gini coefficient of capital income, $\mathcal{R}_\pi$ the correlation coefficient between capital and total income, and $\pi$ the capital share of income. Equation 5 states that when the contribution of capital income inequality to overall income inequality, $\alpha$, is larger (lower) than the contribution of capital income to the overall (household sector) national income, $\pi$, compositional inequality is larger (lower) than income inequality. If we consider compositional inequality in saving and consumption instead of compositional inequality in capital and labor incomes and further arrange the terms we obtain:

$$s = \alpha^* - \beta \frac{\mathcal{I}_{sc}}{\mathcal{G}}, \tag{6}$$

where $\alpha^* = \frac{\mathcal{G}_s \mathcal{R}_s s}{\mathcal{G}}$ and $\beta = 2\mathcal{B}_{sc}$. As we can see from equation 6, behavioral heterogeneity, as measured by the IFC index, $\mathcal{I}_{sc}$, and the overall saving rate are negatively associated, whilst the saving rate is positively associated with the Gini coefficient $\mathcal{G}$. The two parameters $\alpha^*$ and $\beta$ determine the intercept and the slope of this relationship, respectively. An increase (decrease) in the contribution of saving inequality to overall income inequality has the effect of shifting the relationship upwards (downwards). On the contrary, a higher (lower) $\beta$, which has the effect of increasing (reducing) the slope of the relationship, is a positive function of $p$, which describes the *size of the capitalist elite* in case of maximum compositional inequality. The higher (lower) the size, the higher (lower) the $\beta$. The next section will also explore this relationship from an econometric perspective.

Figures 6 and 7 display the association between behavioral heterogeneity and income inequality, as measured by the Gini coefficient. A positive relationship is documented between these two measures, confirming the vast empirical evidence on the association between income and the propensity to save (Dynan et al., 2004, among others).

Individuals' ability to save leads to wealth accumulation which, in turn, allows for additional income generation in the form of returns from investments in both physical, and human capital. The more homogeneous the distribution the propensities to save among individuals, the more equal the overall distribution of income in the population.

Section 3.3 studies the relationship between the two dimensions of heterogeneity introduced, saving rate, and growth, from an econometric point of view.

### 3.3 Heterogeneity, saving rate, and growth

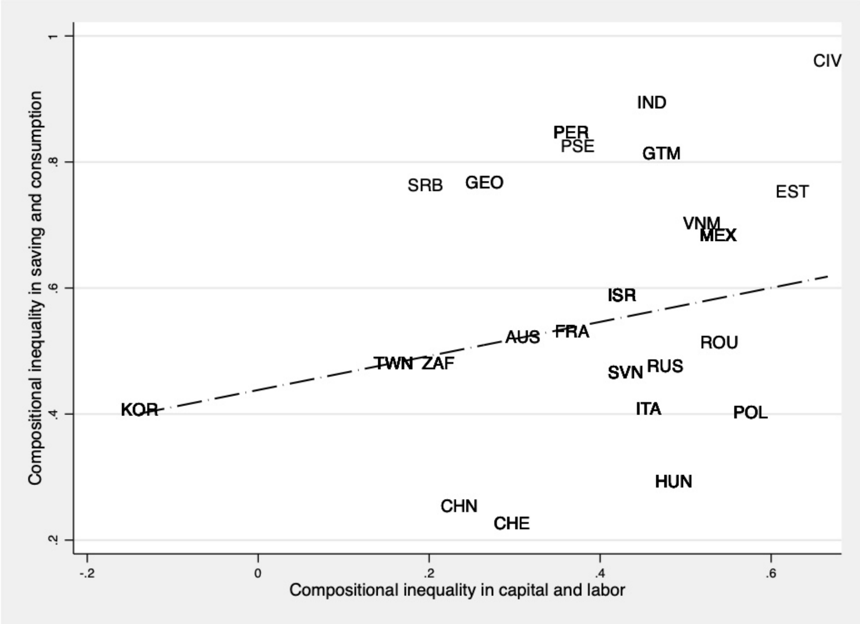
We start with a simple econometric panel fixed-effect framework, defined as follows:

$$Y_{it} = \alpha + \beta X + \mu_i + y_t + \epsilon_{it}, \tag{7}$$

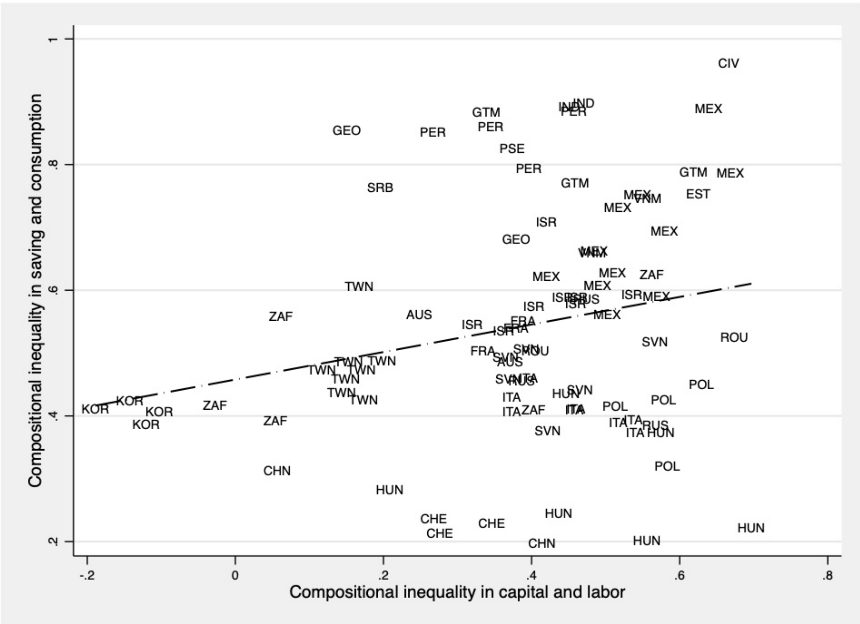
where $Y_{it}$ refers to our dependent variable for country $i$ and in each year $t$, whilst $X$ represents a set of explanatory variables and controls. We include country fixed effects ($\mu_i$) so as to account for time-invariant socio-economic factors, and year fixed effects ($y_t$), which control for macroeconomic shocks. Macroeconomic variables are taken from the IMF Economic Outlook. In all regressions we estimate clustered within-countries and heteroscedasticity-robust standard errors.

Table 2 reports the first results on the association between behavioral and endowment heterogeneity (columns 1 – 2). In line with our descriptive evidence, no correlation emerges between the two IFC indices, even when all controls and fixed effects are included in the model.

We now test whether there is any relationship between heterogeneity at the *macro level* (i.e., variation in the saving and capital income shares), and heterogeneity at the




(a) Average value per country




(b) All countries and years

Figure 2: The two dimensions of compositional inequality are measured by the IFC index. Unweighted averages of both inequality dimensions are considered for each country. Capital income is defined as the sum of rental income, dividends and interests. Labor income is the sum of wages, self-employment income and pensions, here considered as deferred labor income.

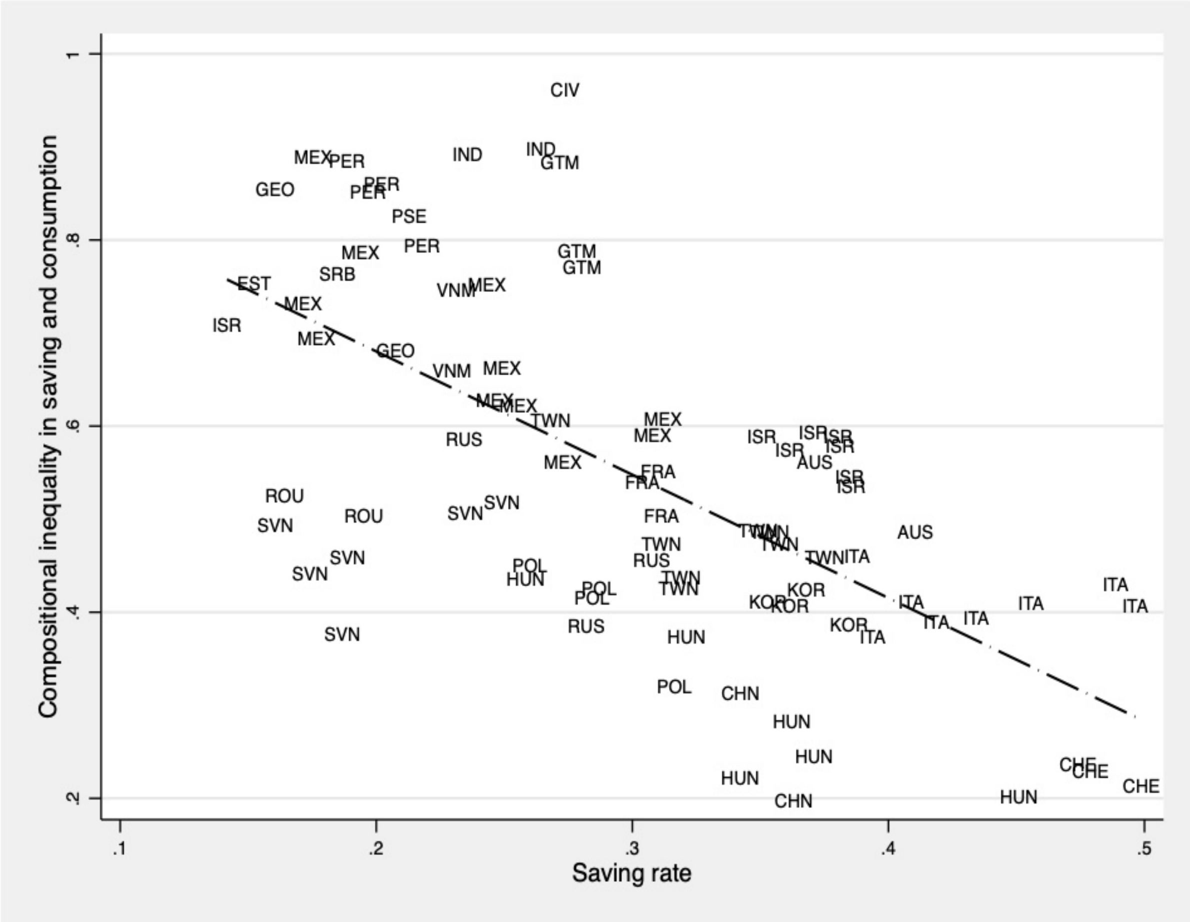



Figure 3: Compositional inequality is measured by the IFC index.

*micro level* (i.e., variation in behavioral and endowment heterogeneity), as measured by the IFCs. Table 2 confirms the presence of a negative and robust association between behavioral heterogeneity ($\mathscr{I}_{sc}$) and the aggregate saving rate (columns 3 – 6). This result is extremely robust to the inclusion of a number of controls, including the level of a country's income inequality, as measured by the total income Gini coefficient, which do not alter the magnitude of the coefficient.[11] The association between income inequality and behavioral heterogeneity is also positive, and significant, confirming both the descriptive and theoretical evidence previously discussed. We do not find any robust correlation between endowment heterogeneity and the macroe-

[11]Recall that these findings are based on estimates of the household sector's saving rate. These results are however qualitatively robust to the inclusion of World Bank macroeconomic saving rates, although with lower statistical significance. Results may be provided upon request.

conomic saving rate, except for when all controls are included in the model. Under this model specification, the correlation between the two is also negatively signed. A reduction in compositional inequality in terms of capital and labor incomes therefore increases the overall saving rate. The magnitude of this effect is, however, smaller than that of compositional inequality in saving and consumption. Finally, we do not find any clear association between endowment heterogeneity, on the one hand, and the capital share of income, on the other, as shown by Table 5 in Appendix A.

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| | $\mathscr{I}_{sc}$ | $\mathscr{I}_{sc}$ | Macro saving | Macro saving | Macro saving | Macro saving |
| $\mathscr{I}_{sc}$ | | | -0.338*** | -0.387*** | -0.395*** | |
| | | | (-3.82) | (-5.32) | (-5.33) | |
| Gini income | | 0.845** | | 0.903*** | 0.908*** | |
| | | (2.48) | | (4.06) | (4.02) | |
| GDP per capita | | 0.000 | | | -0.000000739 | |
| | | (1.10) | | | (-0.38) | |
| Population | | -0.000 | | | -0.000253* | |
| | | (-0.24) | | | (-1.87) | |
| $\mathscr{I}_{kl}$ | 0.0162 | -0.0028 | | | -0.0587** | -0.0493 |
| | (0.22) | (-0.03) | | | (-2.09) | (-1.21) |
| country FE | YES | YES | YES | YES | YES | YES |
| year dummies | YES | YES | YES | YES | YES | YES |
| $N$ | 92 | 88 | 92 | 92 | 88 | 92 |

$t$ statistics in parentheses
$^{*}$ $p < 0.10$, $^{**}$ $p < 0.05$, $^{***}$ $p < 0.01$

Table 2: Econometric regressions on relations between macroeconomic variables

We now discuss the growth implications of different heterogeneity regimes, by estimating Equation 7 where per capita GDP growth is our dependent variable.[12] In addition to the standard controls, we also include an indicator of the position of the country in the bi-dimensional Box (Figure 1). This indicator is defined as the multiplicative interaction term of the two heterogeneity measures, $\mathscr{I}_{sc} \times \mathscr{I}_{kl}$. Table 3 shows the association between behavioral and endowment heterogeneity, on the one hand, and income growth, on the other. We find that an increase in the level of compositional inequality in capital and labor incomes increases growth up to a certain threshold, after which the relationship becomes negative: a marginal increase in the

[12]Data for per capita GDP growth are retrieved from the World Bank. They represent annual percentage growth rates of GDP per capita based on constant local currency.

| p.c. GDP growth | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
|---|---|---|---|---|---|---|---|
| $\mathscr{I}_{sc}$ | -1.874 | 11.79* | -1.523 | 11.74* | 13.10** | 11.23* | 9.609 |
| | (-0.46) | (1.92) | (-0.36) | (1.90) | (2.32) | (1.77) | (1.72) |
| $\mathscr{I}_{kl}$ | 5.682** | 19.96*** | 5.706** | 20.10*** | 22.96*** | 21.02*** | 19.69*** |
| | (2.22) | (4.26) | (2.24) | (4.51) | (4.97) | (4.66) | (4.31) |
| $\mathscr{I}_{sc} \times \mathscr{I}_{kl}$ | | -27.49*** | | -27.77*** | -31.25*** | -27.69*** | -24.54*** |
| | | (-3.09) | | (-3.32) | (-3.50) | (-3.10) | (-2.96) |
| Gini income | | | -3.982 | 2.153 | 8.963 | 3.194 | -3.620 |
| | | | (-0.36) | (0.19) | (0.85) | (0.29) | (-0.41) |
| Current account balance | | | | | -0.113* | | |
| | | | | | (-1.78) | | |
| Population | | | | | | 0.00940 | |
| | | | | | | (0.70) | |
| GDP per capita | | | | | | | -0.000134 |
| | | | | | | | (-1.04) |
| country FE | YES | YES | YES | YES | YES | YES | YES |
| year dummies | YES | YES | YES | YES | YES | YES | YES |
| $N$ | 84 | 84 | 84 | 84 | 81 | 81 | 81 |

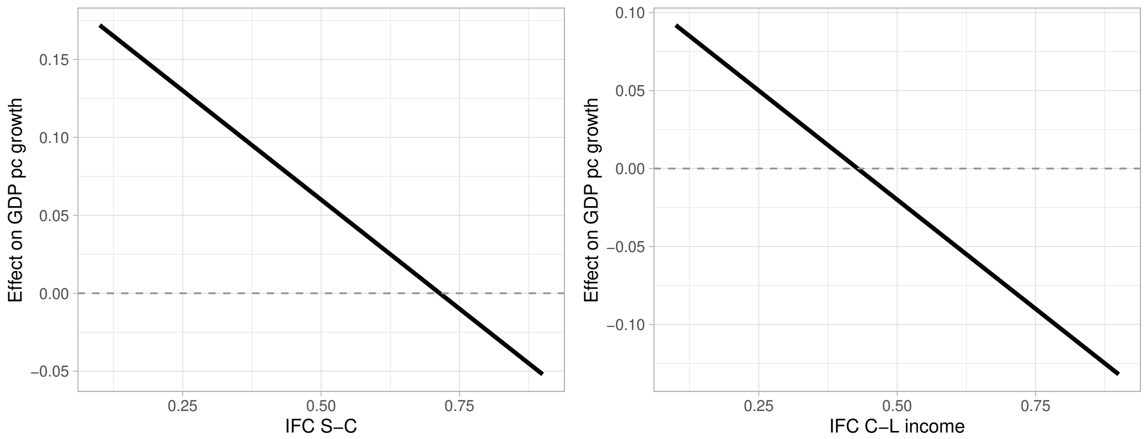
$t$ statistics in parentheses

$^{*}$ $p < 0.10$, $^{**}$ $p < 0.05$, $^{***}$ $p < 0.01$

Table 3: Econometric regressions on per capita GDP growth

level of compositional inequality in capital and labor incomes becomes detrimental for growth. The same dynamics can be found for compositional inequality in saving and consumption. The tipping points are 0.4 and 0.67, respectively (see also Figure 4).[13] The difference between the two tipping points implies that a higher level of behavioral heterogeneity is needed to change the sign of its relationship with growth (from positive, to negative), than for endowment heterogeneity. This result is implicitly the effect of a larger variation in the levels of behavioral heterogeneity with respect to endowment heterogeneity. Notice that no relationship is found between both types of heterogeneity, on the one hand, and per capita GDP, on the other (see Table 6 in Appendix A). The type of economic system considered (be it a Kaldorian, or a Representative Agent model) has, therefore, different implication for growth regimes. If we consider movements along the main diagonal of the Box from point $(0, 0)$, to point $(1, 1)$, growth rates firstly increase, and then decrease after the detected threshold.

[13]The tipping points are calculated considering the baseline model (2) in Table 3. They are obtained starting from the following equation: $g_{it} = \beta_1 \mathscr{I}_{sc} + \beta_2 \mathscr{I}_{kl} + \beta_3 \mathscr{I}_{sc} \mathscr{I}_{kl}$, and by, then, setting both derivatives $\frac{\partial g}{\partial \mathscr{I}_{sc}}$ and $\frac{\partial g}{\partial \mathscr{I}_{kl}}$ equal to 0.




(a) $\mathscr{I}_{sc}$ (b) $\mathscr{I}_{kl}$

Figure 4: Marginal effects of changes in one indicator of compositional inequality on (per capita) GDP growth, keeping the other indicator fixed.

So far, we have focused on the aggregate relationship between heterogeneity, growth, and the saving rate. However, to dig further into the relationship between our two measures of heterogeneity, we can look at endowment and behavioral heterogeneity, from a percentile-level perspective. Specifically, we can study the association between capital income and savings *across* the total income distribution. To this end, we firstly calculate average capital income and savings for each percentile of the total income distribution, here considered as our percentile-level compositional measures (Figures 8 and 9 show the percentile-level associations between these two variables, for all countries in the sample). Secondly, we run a regression model in which the unit of analysis is the percentile, $p$, for country $i$ and time $t$. As done in previous models, we assume each dependent variable be a function of a set of controls, as well as percentile-country fixed effects, and time dummies. Controls also include an interaction term between the income decile and the percentile-specific capital incomes, or savings. The model is the following:

$$Y_{pit} = \alpha + \beta X + \mu_{pi} + y_t + \epsilon_{pit}. \tag{8}$$

Two major results emerge from this analysis. First, there exists a strong, positive association between capital income and saving, both expressed in log terms, at the average level (columns 1 and 2 in Table 4). However, when we break down this relationship across different income percentiles, we observe it remains positively signed for the top deciles (6-10), and negatively signed for the bottom deciles (1-5).[14] Therefore, the presence of an overall positive association between capital incomes and savings across the income distribution is mainly driven by the the top of the distribution. Considering that savings represent one of the main determinants of wealth accumulation (Benhabib et al., 2017, Morelli, 2020), and assuming that saving out of capital income is larger than saving out of labor income, our result supports the evidence whereby there exists increasing returns to scale across the wealth distribution (Fagereng et al., 2020, Iacono and Palagi, 2022a). The absence of a neat, positive relationship between our two measures of compositional inequality, which prevents countries from distributing diagonally in the Box, is therefore the result of a weak positive association between capital income and savings at the bottom, and not the top, of the income ladder. In other words, while the top of the income distribution is characterized, for many countries at different points in time, by the presence of capital-abundant savers, the bottom tend to be more heterogeneous.

Finally, column 3 in Table 4 reinforces our evidence whereby higher behavioral heterogeneity reduces a country's aggregate saving rate, by showing that higher saving levels at the bottom of the distribution are associated with higher saving rates. This is particularly true for deciles 2-6.

[14]Note that the baseline coefficient (Log capital income) refers to the tenth decile.

| | (1) Log savings | (2) Log savings | (3) Macro saving |
|---|---|---|---|
| Log capital income | 0.109*** | 0.101*** | |
| | (4.63) | (3.30) | |
| $D_1\times$ Log capital income | -0.109 | -0.105 | |
| | (-1.58) | (-1.49) | |
| $D_2\times$ Log capital income | -0.194*** | -0.194** | |
| | (-2.64) | (-2.55) | |
| $D_3\times$ Log capital income | -0.0368 | -0.0359 | |
| | (-0.86) | (-0.73) | |
| $D_4\times$ Log capital income | -0.162*** | -0.168*** | |
| | (-3.69) | (-3.27) | |
| $D_5\times$ Log capital income | -0.0752** | -0.0717* | |
| | (-2.12) | (-1.69) | |
| $D_6\times$ Log capital income | -0.0685* | -0.0618 | |
| | (-1.78) | (-1.39) | |
| $D_7\times$ Log capital income | -0.0620* | -0.0624 | |
| | (-1.84) | (-1.55) | |
| $D_8\times$ Log capital income | -0.0373 | -0.0358 | |
| | (-1.35) | (-1.01) | |
| $D_9\times$ Log capital income | -0.0559** | -0.0582 | |
| | (-2.00) | (-1.61) | |
| Income | 0.0000193** | | |
| | (2.13) | | |
| Consumption | -0.000140*** | | |
| | (-4.79) | | |
| Savings | | | 0.00000130*** |
| | | | (5.84) |
| $D_1\times$ Savings | | | 0.00000293 |
| | | | (1.59) |
| $D_2\times$ Savings | | | 0.00000483** |
| | | | (2.08) |
| $D_3\times$ Savings | | | 0.00000701*** |
| | | | (3.03) |
| $D_4\times$ Savings | | | 0.00000594*** |
| | | | (3.31) |
| $D_5\times$ Savings | | | 0.00000661*** |
| | | | (3.38) |
| $D_6\times$ Savings | | | 0.00000507*** |
| | | | (3.43) |
| $D_7\times$ Savings | | | 0.00000356*** |
| | | | (3.06) |
| $D_8\times$ Savings | | | 0.00000246** |
| | | | (2.02) |
| $D_9\times$ Savings | | | 0.00000239*** |
| | | | (2.90) |
| country-percentile FE | YES | YES | YES |
| year dummies | YES | YES | YES |
| $N$ | 5030 | 5030 | 7981 |

$t$ statistics in parentheses

$^{*}$ $p < 0.10$, $^{**}$ $p < 0.05$, $^{***}$ $p < 0.01$

Table 4: Econometric regressions with percentile-specific variables

# 4 Implications for macroeconomic modelling

Our empirical analysis has shown that heterogeneity matters, and is country-specific. The framework developed in this paper allows us to precisely measure the extent of heterogeneity in individuals' behavior and endowment, both between countries and across time. Countries like South Korea and China display, on average, low levels of behavioral and endowment heterogeneity, differently from Mexico and India, which are closer to a Kaldorian system where income-rich individuals save their capital income and income-poor consume their labor income. While not all possible combinations of our compositional inequality indicators reflect the type of countries' heterogeneity in our sample - we find, for instance, no evidence of countries combining low endowment heterogeneity, with high behavioral heterogeneity -, the coverage remains substantial. On the basis of these findings, we argue that a macroeconomic model with heterogeneous agents should be able to account for any possible combination of endowment and behavioral heterogeneity.

How do state-of-the-art macroeconomic models with heterogeneous agents relate to our heterogeneity framework? Do they allow for all possible types of heterogeneity, both statically (*initial* conditions) and dynamically (*asymptotic* conditions)? In this section, we explore these questions by focusing on the following types of macroeconomic models: Kaldorian, TANK and HANK, OLG (à la Stiglitz), and Agent-Based models. While it would be unfeasible, as well as out of the scope of this paper, to generalize the heterogeneity assumptions adopted by every single modelling school, we instead select what we believe are the most representative models (in terms of individual/household behavioral and endowment heterogeneity) of each school. In selecting these benchmark models, we privilege analytical tractability over sophistication.

## 4.1 Kaldorian Model

In his 1955's seminal contribution, Kaldor develops one of the first macroeconomic models of growth and distribution with heterogeneous agents (Kaldor, 1955). This model assumes the existence of two, distinct interest groups: the capitalists, who earn and entirely save all of their income from capital, and the workers, who earn their income from labor and consume most of it.[15] The propensity to save of the capitalists is, therefore, equal to 1 and that of the workers is close to 0. In a subsequent contribution, Pasinetti extends the model developed by Kaldor by also allowing the workers to earn some profits (Pasinetti, 1962).[16] The distributional characteristics of these two groups are exogenously given and, hence, do not change across time. While there is no information regarding the overall level of between-classes income inequality, we assume, as it was implicit in the minds of classical authors, that capitalists' income is larger than workers' income. In compositional inequality terms, this implies that the heterogeneity assumptions implicit in the model developed by Kaldor allow it to cover the segment $(1, \hat{\mathscr{I}}_{sc})$ of the Box,[17] with $\hat{\mathscr{I}}_{sc} > 0$ when the propensity to save of the capitalists is larger than the propensity to save of the workers, and $\hat{\mathscr{I}}_{sc} = 1$ when the propensity to consume of the workers and the propensity to save of the capitalists are, both, equal to 1. The model developed by Pasinetti is, instead, more general, as it covers the inner part of the Box. Extreme compositional inequality scenarios, or simply compositional inequality values above certain thresholds are, in fact, unattainable. Endowment heterogeneity cannot be very high nor very low because of (i) the absence of capitalists' wages, and (ii) the positive value of workers' profits. Behavioral heterogeneity cannot attain its high value because (i) capitalists' propensity to save is always larger than workers' propensity to save, and (ii) workers' propensity to save is larger than zero. A simple extension of the Kaldor-Pasinetti model that allows for all the spectrum of both types of heterogeneity, followed by a discussion of its main long-run macroeconomic implications, can be found in Appendix C.

[15] Kaldor allows in fact workers to save a fraction of their labor income. Moreover, in a later contribution Kaldor emphasizes that the assumption of saving out of profits is not in relations to people, but to firms, which save out of their business profits to finance investments (Kaldor, 1966). Our current reference to Kaldor is, therefore, in relation to his 1955's article (and the subsequent debate with Pasinetti and Samuelson) rather than to his 1966's article.

[16] A revisitation of Pasinetti's model can be found in a the recent contribution by Petach and Tavani (2022).

[17] We here assume a propensity to consume of workers equal to 1, for simplicity.

## 4.2 TANK and HANK Models

Heterogeneous Agent New Keynesian (HANK) models incorporate heterogeneity and uninsurable idiosyncratic income risks into a New Keynesian setting (Aiyagari, 1994).[18] Within the vast literature on the topic,[19] here we consider the analytically tractable TANK and HANK models recently developed by Bilbiie (2020).[20] The tractable nature of Bilbiie's models is desirable for our purposes. Numerous HANK models that allows for high variation in both types of heterogeneity are rather difficult, if not impossible, to analyse in light of our proposed framework.[21] Let us start with the TANK model.

This model is characterized by the existence of a fraction of *hand-to-mouth* individuals, $\lambda$, which are excluded from the asset market. These individuals earn, therefore, income from labor only and have a propensity to consume equal to 1. A fraction $1 - \lambda$ of individuals are, instead, called *savers*, and receive profits from monopolistic competitive firms alongside their labor income. Savers save a strictly positive fraction of their total income.[22] Given that no assumption is made regarding the level of income inequality between these two groups of agents, two inequality scenarios are here considered: the income of the hand-to-mouth consumer is lower (scenario 1), or higher (scenario 2) than that of the savers.[23] Under scenario 1, both $\mathscr{I}_{kl}$ and $\mathscr{I}_{sc}$ are positive, and relatively high, since income-poor individuals exclusively consume labor income. The extent to which endowment heterogeneity is greater (or lower) than

[18]See Achdou et al. (2022) for a recent continuous-time version of the Aiyagari–Bewley–Huggett model.

[19]For a comprehensive, although past, review of the literature on the topic, see Heathcote et al. (2009). For a more recent survey, see Violante (2021).

[20]Similar results would be found if the model developed in Bilbiie et al. (2022) was, instead, considered.

[21]The recent paper by Kaplan et al. (2018), for instance, develops one of the most ambitious heterogeneous model in terms of both households saving and consumption decisions, as well as income composition. However, the joint dynamics of labor productivity (i.e., the distribution of labor income), illiquid and liquid assets (i.e., the distributions of capital incomes), is modelled through a Kolmogorov forward equation, which does not provide any precise information on the association (or copula function) between these variables.

[22]For simplicity, debt-financed consumption is not considered in the analysis.

[23]We exclude the scenario whereby income inequality is null between the two groups, as this would bias our measures of compositional inequality, which need individuals' be ranked according to their total income level.

behavioral heterogeneity is a function of the difference between the capital income share and the saving rate of the savers, $\pi_s$ and $s_s$, respectively. When this difference is positive (negative), endowment heterogeneity is greater (lower) than behavioural heterogeneity.[24] Under scenario 2, however, the two indicators of compositional inequality take negative values and their difference depend on the capital income share and saving rate of the bottom, instead of the top, of the distribution. To sum up, depending on the level of inequality between these two groups, Bilbiie's TANK model would describe economies either in the *north-east*, or in the *south-west* of the Box, where negative values of the two indicators are displayed.

A different situation occurs when the model is extended so as to allow for self-insurance in face of idiosyncratic shocks. As explained in Bilbiie (2020), such extension provides an analytically tractable HANK model. This new version of the model includes, therefore, the presence of two, distinct states: being a hand-to-mouth (H) consumer, or a saver (S). The probability to stay type $H$ is equal to $h$, and the probability to stay type $S$ is equal to $s$. This extension only slightly affects the dynamics of compositional inequalities described under the TANK model, given the non-anonymity property of the IFC index.[25] What affects the IFC index is, however, the change in the relative shares of individuals belonging to the two groups $H$ and $S$. These shares are, in fact, function of the two transition probabilities. It can be shown that the stationary equilibrium of the mass of $H$ is equal to $\frac{1-s}{2-s-h}$ (Bilbiie, 2020). When the transition probabilities are the same ($h = s$), $\lambda = \frac{1}{2}$ and the dynamics of behavioral and endowment heterogeneity in HANK is the same as in TANK. However, when they are different ($h \neq s$), the shape of the Lorenz curve changes and this has, in turn, an impact on the maximum-concentration curve, which is also function of the Lorenz curve for income (see appendix B for details). With that said, we expect this inequality channel to only mildly impact the previously discussed dynamics of the TANK

[24]When $\pi_s > s_s$, capital income at the top relatively to labor income is more concentrated than saving at the top relatively to consumption. To keep the analysis as simple as possible, we are here assuming that the overall shares of capital and of saving to total income are playing no role in the analysis.

[25]The IFC indicator is, in fact, unaffected by any relationship between a specific individual, say individual $j$, on the one hand, and her income composition, or the saving and consumption rates, on the other.

model. We can therefore conclude that the heterogeneity assumptions present in Bilbiie's model range within a central section of the box. This prevents the model from reaching extreme levels of at least one compositional inequality dimension.

### 4.3 OLG Models à la Stiglitz

In a recent paper, Stiglitz assumes the existence of two, distinct groups: the workers, who live for two periods and save for their retirement, and the capitalists, who save a fixed percentage of their income, $s_p$ (Stiglitz, 2015). Workers are also referred to as life-cycle savers. The system's evolution is described according to the capitalists' and workers' accumulations equations, defined as follows:

$$(1+n)k_{t+1}^{c} = (1+s_p f'(k_t))k_t^{c}, \tag{9}$$

and

$$k_{t+1}^{w} = s(k_{t+1})w(k_t), \tag{10}$$

where $w$ denotes workers' wage, $n$ is rate of increase of both workers and capitalists, considered exogenous in the model, whilst $k^w$ and $k^c$ are the workers' and capitalists' wealth, respectively. Without loss of generality, we abstain from Stiglitz' generalization of the model, which allows for transitions from one class to the other, and analyze the heterogeneity assumptions underlying this model.[26] Workers earn income exclusively from labor and their income is equal to $w(k_t)$. On the contrary, capitalists derive their income exclusively from wealth, which is equal to $f'(k_t)k_t^c$. As a consequence, if we assume that capitalists are richer than workers in terms of their total income, the level of endowment heterogeneity is maximal. Let us now focus on behavioral heterogeneity. Workers save a proportion of their income $s(k_{t+1})$ in the first period of their life, $t_i$, and dissave income in the second period, $t_2$. Capitalists save, instead, a fixed proportion of their income, $s_p$, all along their lifetime. This implies that, in both periods, $t_1$ and $t_2$, capitalists save the same proportion of income. Assuming that, once

[26]Similarly to the Bilbiie model described in the previous section, the introduction of transition probabilities have only a mild effect on the overall heterogeneity dynamics.

again, the income of the capitalists is greater than that of the workers, we can conclude that while in period $t_1$ compositional inequality in saving and consumption is relatively small, since both groups save and consume a proportion of their income, in period $t_2$ it further increases. Such an increase reflects the fact that during retirement workers increase their spending, thing that boosts behavioral heterogeneity. We can, therefore, conclude that Stiglitz's model moves from $(1, \mathscr{I}_{cs}^{t_1})$ to $(1, \mathscr{I}_{cs}^{t_2})$ with $\mathscr{I}_{cs}^{t_1} < \mathscr{I}_{cs}^{t_2}$ along the course of time. Similar considerations can also be drawn from the model developed by Mattauch et al. (2022), which can be regarded as a microfoundation of the Stiglitz's model just detailed. While this section has focused on Stiglitz's OLG model, given its straightforward relationship to our multiple-sources-of-income framework, we acknowledge other OLG models could have also been studied in a similar way (Galor and Moav, 2004, Agliari et al., 2020). This paves the way for future research on the matter.

## 4.4 Agent-Based Models

Agent-based models (ABMs) stand out as particularly well-suited for modeling heterogeneity (Haldane and Turrell, 2019). They are micro-founded representations of the economy as a complex system, in which macroeconomic phenomena emerge as a result of repeated interactions among heterogeneous agents (Fagiolo and Roventini, 2016, Dawid and Gatti, 2018, Dosi and Roventini, 2019). This framework is, therefore, particularly suited to model heterogeneity along several dimensions. We now briefly focus on the principal ABMs that, in our view, allows for a high degree of heterogeneity along the two dimensions which are central in our paper. To name a few, the economy in Assenza et al. (2015) is characterized by workers, supplying labor, and capitalists, owning firms. Both workers and capitalists save and accumulate financial wealth and can, in turns, earn capital income in the form of interests on deposits. Assenza et al. (2015)'s modeling of consumption behaviors is, however, rather stylized, insofar as average propensities to consume are homogeneous across the population. Similar assumptions have been made in other benchmark ABMs (see e.g. Dosi et al., 2010). Differently, the model developed by Guerini et al. (2018) allows for a relatively high degree of behavioral heterogeneity, despite capital incomes are neglected, as interest rates on deposits are assumed to be equal to 0. Caiani et al. (2019) differentiate, instead, individuals into four hierarchical classes, ranging from workmen to top managers. All agents earn both wages and interests received on previous period deposits. Propensities to consume are, however, fixed within each social class. Palagi et al. (2021) present a simple agent-based model in which all households earn income from both labor and capital incomes in the form of interests when savings are positive. The model allows for heterogeneous propensities to consume across individuals. Propensities to consume are decreasing functions of income and are shaped by social interactions. A downside of the model is that, given the absence of asset markets (i.e., there are no asset price appreciations, nor depreciations), capital incomes and wealth are rather stylized. Finally, the model in Botta et al. (2021) is here considered as the most flexible in terms of heterogeneity in saving, consumption, labor and capital income across the population. The model includes both financial and non-financial firms paying wages and rents (in the form of dividends on stocks), respectively, to households. Rentier households receive, instead, interests on public bonds and remunerations on collateralized debt obligations. Propensities to consume are heterogeneous and decreasing functions of income. Disregarding taxes, household $i$'s income at each time step in Botta et al. (2021) is described by the following equation:

$$y_{i,t} = w_{i,t} + rsh_{i,t} - i^h_{i,t-1} Lh_{i,t-1}, \tag{11}$$

where $w_{i,t}$ is the exogenous gross wage[27], $rsh_{i,t}$ is the rent received by the household on their shares, $i^h_{i,t-1}$ is the household-specific interest rate, and $Lh_{i,t-1}$ is the household debt stock.

Desired consumption is defined as:

$$c^*_{i,t} = c_y y_{i,t} + c_n \bar{c}_{t-1}, \tag{12}$$

[27]The total wage bill, which is set exogenously as a given proportion $\lambda$ of the capital stock, is distributed over heterogeneous households according to a fixed log-normal distribution with unitary mean and log-standard deviation $\theta$, fixed across time. This implies that differences in wages among households exclusively depend on the total wage bill.

and desired saving as:

$$s^*_{i,t} = y_{i,t} - c^*_{i,t}, \tag{13}$$

where $c_y$ is the propensity to consume out of total income, $\bar{c}_{t-1}$ represents average consumption observed in the previous year, whilst $c_n$ is the strength of the social norm. As discussed by Botta et al. (2021), the evolution of endowment heterogeneity is determined by the dynamics of the total wage bill, which in turns depends on the motion of the capital stock, and that of financial income. This latter is, instead, distributed across households through a recursive process, based on each household's financial commitment out of its gross wage net of taxes (here absent in our framework), and consumption. Given the interconnection between the consumption (saving) and income function via the fixed propensity to save, endowment heterogeneity determines behavioral heterogeneity. As a consequence, we can conclude that, while the model allows for a high degree of endowment heterogeneity, at the same time it excludes extreme compositional inequality scenarios in terms of saving and consumption (see Equation 12). The extent of behavioral and endowment heterogeneity engendered by Botta et al. (2021)'s model is, therefore, limited to a subspace of the box. For instance, the model does not seem to allow for the Kaldorian, nor the Representative Agent configurations to take place. The extent to which it is able to approach the two extreme scenarios therefore depends on its parametrization. Furthermore, to fully grasp the extent of endowment heterogeneity we would need further information on the copula function between the capital and labor income distributions. A simulation exercise would therefore be needed to precisely assess the degree of heterogeneity covered in both dimensions.

Given the flexibility of the ABM approach, a greater richness in terms of both households' behavioral and endowment heterogeneity could be achieved by simply extending existing models and by including assumptions on the joint distribution of capital and labor incomes.

# 5 Conclusion

This paper brought in conversation standard practices in theoretical macroeconomic modelling with recent methodological, as well as empirical advances in inequality studies. Specifically, it developed a framework to empirically test the extent of household heterogeneity in modern economies. Two types of heterogeneity are considered: *endowment* and *behavioral heterogeneity*. The first type refers to heterogeneity in individuals' income composition in terms of capital and labor incomes. The higher (lower) endowment heterogeneity, the more (less) the economy is characterized by rich capital income receivers and poor laborers. The second type refers to heterogeneity in individuals' income composition in terms of savings and consumption. The higher (lower) behavioral heterogeneity, the more (less) the economy is characterized by rich savers and poor hand-to-mouth consumers. We measure these two types of heterogeneity using an indicator of compositional inequality recently developed by Ranaldi (2022), the IFC index, in more than 20 economies between 1995 and 2018. Three main empirical results stand out from our analysis.

*First*, we documented significant cross-country variations along the behavioral and endowment heterogeneity dimensions, as well as little correlation between them. Countries like South Korea and China display, on average, low levels of behavioral and endowment heterogeneity, differently from Mexico and India which are closer to a Kaldorian, than a Representative Agent, system, in which income-rich individuals save their capital income and income-poor individuals consume their labor income. *Second*, we reported a negative relationship between behavioral heterogeneity, on the one hand, and the country's saving rate, on the other. In other words, the more equal the propensities to save and consume across individuals (i.e., the lower behavioral heterogeneity), the higher the country's overall saving rate. To the best of our knowledge, this paper is the first to report this empirical and theoretical regularity across several countries and years. *Third*, following a simultaneous increase in both types of heterogeneity, a country's growth rate is firstly positively, and then negatively signed. Therefore, it exists an inverted U-shaped relationship between growth and household heterogeneity.

Our framework was discussed in light of the heterogeneity assumptions underlying four types of macroeconomic models with heterogeneous agents: Kaldorian, TANK and HANK, OLG, and ABM models. Our analysis showed that no model allows for the full range of behavioral and endowment heterogeneity, as defined in our framework.

On the basis of these results, we strongly encourage future generations of macroeconomic models with heterogeneous agents to account for the full spectrum of behavioral and endowment heterogeneity. We believe heterogeneity assumptions should always be brought in conversation with the actual distributions of capital and labor incomes, and of saving and consumption, emerging from the data. Our framework helps organize and synthesize these information, in a rather stylized, and intuitive manner.

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# A Additional Tables

| | (1) Macro capital share | (2) Macro capital share | (3) Macro capital share | (4) Macro capital share |
|---|---|---|---|---|
| $\mathscr{I}_{sc}$ | 0.00157 | 0.0108 | 0.00361 | |
| | (0.04) | (0.28) | (0.08) | |
| Gini income | | -0.173 | -0.131$^{*}$ | |
| | | (-1.54) | (-1.78) | |
| GDP per capita | | | 0.000000291 | |
| | | | (0.51) | |
| Population | | | 0.000386 | |
| | | | (1.23) | |
| $\mathscr{I}_{kl}$ | | | 0.00839 | 0.00784 |
| | | | (0.57) | (0.46) |
| country FE | YES | YES | YES | YES |
| year dummies | YES | YES | YES | YES |
| $N$ | 92 | 92 | 88 | 92 |

$t$ statistics in parentheses

$^{*}$ $p < 0.10$, $^{**}$ $p < 0.05$, $^{***}$ $p < 0.01$

Table 5: Regressions on macroeconomic capital share

| | (1) Per capita GDP | (2) Per capita GDP | (3) Per capita GDP | (4) Per capita GDP |
|---|---|---|---|---|
| $\mathscr{I}_{kl}$ | 1318.1 | 1009.3 | 4229.2 | 130.9 |
| | (0.20) | (0.13) | (0.98) | (0.02) |
| $\mathscr{I}_{sc}$ | 5153.1 | 4812.5 | 5649.4 | 1002.0 |
| | (1.65) | (0.68) | (1.38) | (0.21) |
| $\mathscr{I}_{sc} \times \mathscr{I}_{kl}$ | | 655.5 | | 8988.4 |
| | | (0.07) | | (1.22) |
| Gini income | | | -49583.6$^{**}$ | -51877.6$^{**}$ |
| | | | (-2.11) | (-2.11) |
| country FE | YES | YES | YES | YES |
| year dummies | YES | YES | YES | YES |
| $N$ | 88 | 88 | 88 | 88 |

$t$ statistics in parentheses

$^{*}$ $p < 0.10$, $^{**}$ $p < 0.05$, $^{***}$ $p < 0.01$

Table 6: Regressions on per capita GDP (constant prices).

# B Zero- and Maximum-Concentration Curves

In this section, we analytically define the zero- and maximum-concentration curves. For simplicity, we define them for compositional inequality in capital and labor incomes. It is straightforward to derive these curves for compositional inequality in saving and consumption.

The zero-concentration curve for capital income, $\mathscr{L}^e(\pi, p)$, is defined as follows:

$$\mathscr{L}^e(\pi, p) = \pi \sum_{j=1}^{i} y_j \quad \forall i = 1, \ldots, n. \tag{14}$$

This curve can be see as the Lorenz curve for total income, multiplied by the capital share, $\pi$. It describes a distribution of income sources where the composition of capital and labor income is the same for all individuals. Notice that this curve is a function of both the Lorenz curve for income, and the capital share. Therefore, differently from the egalitarian line used to construct the Gini coefficient, which is the same for all distortions, this curve is distribution-specific.

The maximum-concentration curve for capital income, $\mathscr{L}^{max}(\pi, p)$, can take two forms, depending on whether the concentration curve for capital income lies below, or above the zero-concentration curve. In the former case, it can be defined as:

$$\mathscr{L}^{max}(\pi, p) = \mathscr{L}^{m}(\pi, p) = \begin{cases} 0 & \text{for } p \leq p^{''} \\ \mathscr{L}(\mathbf{y}, p) - z_{-} & \text{for } p > p^{''}, \end{cases} \tag{15}$$

whilst in the latter case, as follows:

$$\mathscr{L}^{max}(\pi, p) = \mathscr{L}^{M}(\pi, p) = \begin{cases} \mathscr{L}(\mathbf{y}, p) & \text{for } p \leq p^{'} \\ z & \text{for } p > p^{'}, \end{cases} \tag{16}$$

with $p^{'}$ s.t. $\mathscr{L}(\mathbf{y}, p^{'}) = \pi$, $p^{''}$ s.t. $\mathscr{L}(\mathbf{y}, p^{''}) = 1 - \pi$. Under this scenario, the maximum-concentration curve equals zero up to a given income percentile $p^{''}$, and then takes the shape of the Lorenz curve. In the second case, the maximum-concentration curve takes the shape of the Lorenz curve up to a given income percentile $p^{'}$, and then it is constant. The choice of the percentiles $p^{'}$ and $p^{''}$ depends on the shape of the Lorenz curve and on the capital share.[28]

# C Kaldorian Model: An Extension

In this Section, we show how the two compositional inequality dimensions can be embedded in a simple Kaldorian macroeconomic model of growth and distribution, and how they shape its dynamics. For simplicity, we assume the economy is composed by two groups only, with incomes $Y_1$ and $Y_2$. Let us consider the following accounting identity:

$$Y = \Pi + W, \tag{17}$$

where $\Pi$ and $W$ represent total capital and labor incomes in the economy, respectively. As done in the main body of the paper, we can express total income of each group as follows:

$$Y_i = \alpha_i \Pi + \beta_i W \ \ \forall i = 1, 2, \tag{18}$$

where $\alpha_i = \frac{\Pi_i}{\Pi}$ and $\beta_i = \frac{W_i}{W}$ are the relative shares of capital and labor income of group $i$, with $i = 1, 2$. We define total saving of group $i$ as $S_i$, hence $s_i = \frac{S_i}{Y_i}$ is the saving rate of group $i$. As total saving is the sum of $S_1$ and $S_2$, $S = S_1 + S_2$, after some rearrangements, we can write:

$$S = s_1 Y_1 + s_2 Y_2 = \Pi\left(\alpha_1 - \beta_1\right)\left(s_1 - s_2\right) + Y\left(s_1\beta_1 + s_2\beta_2\right). \tag{19}$$

Let us now consider the condition in which the system is in a (long-run) dynamic equilibrium, that is $S = I$, where $I$ is the amount of investment necessary to withstand either population growth and technological progress. Furthermore, if we supposed to be in a closed economy without government spending, from equation 19 we can define both the capital share of income $\frac{\Pi}{Y}$ and the rate of profit $\frac{\Pi}{K}$ in the economy as:[29]

$$\frac{\Pi}{Y} = \pi = \frac{1}{\alpha_1 - \beta_1}\frac{1}{s_1 - s_2}\frac{I}{Y} - \frac{s_1\beta_1 + s_2\beta_2}{(\alpha_1 - \beta_1)(s_1 - s_2)}, \tag{20}$$

[28] For further details on the choice of the two percentiles, see Ranaldi (2022)

[29] Note that the term $\frac{s_1\beta_1 + s_2\beta_2}{(\alpha_1-\beta_1)(s_1-s_2)}$ can also be written as $\frac{\beta_1}{\alpha_1-\beta_1} + \frac{s_2}{(\alpha_1-\beta_1)(s_1-s_2)}$.

and:

$$\frac{\Pi}{K} = \frac{1}{\alpha_1 - \beta_1} \frac{1}{s_1 - s_2} \frac{I}{K} - \frac{s_1\beta_1 + s_2\beta_2}{(\alpha_1 - \beta_1)(s_1 - s_2)} \frac{Y}{K}. \tag{21}$$

Equations 20 and 21 generalize the equations of the capital share and of the profit rate introduced by Kaldor (1955).[30] The choice of $\frac{\Pi}{Y}$ and $\frac{\Pi}{K}$ as dependent variables follows directly from the works of Kaldor (1955), Pasinetti (1962), and Samuelson and Modigliani (1966). Particularly, to use Kaldor's words: "The interpretative value of the model [...] depends on the "Keynesian" hypothesis that investment, or rather, the ratio of investment to output, can be treated as an independent variable, invariant with respect to changes in the two savings propensities" (p. 95). However, our major objective simply remains to show that an economic system's position in the Box features in shaping its long-run macroeconomic outcomes. At this point of the analysis, if we further assume that $Y_1 > Y_2$, we can observe that the term $\alpha_1 - \beta_1$ is one of the three main components of the IFC index for two individuals ($n = 2$) (see Ranaldi (2022)). In other words, this term is equivalent to the difference between the areas of the concentration curves for labor and capital income ($\tilde{\mu}_w - \tilde{\mu}_\pi$) in the framework with a population of size $n > 2$.[31] Therefore, we can consider the term $\alpha_1 - \beta_1$ as a proxy for income composition inequality in capital and labor: $\alpha_1 - \beta_1 \approx \mathscr{I}_f(\boldsymbol{\pi})$.[32] Additionally, in order for the equations 20 and 21 to be economically meaningful, we shall assume that $\alpha_1 - \beta_1 > 0$. Such an assumption is reasonable as the sign of the IFC index (which solely depends on its numerator, $\tilde{\mu}_w - \tilde{\mu}_\pi$), is positive for the majority of countries considered.

[30]Kaldor develops the Post-Keynesian theory of income distribution and of the rate of profit. In his article, the author assumes maximal compositional inequality in capital and labor among the two groups, as $Y_1 = \Pi$ and $Y_2 = W$. This assumption is equivalent to say that $\alpha_1 = 1$ and $\beta_2 = 1$ (and, therefore, that $\alpha_2 = 0$ an $\beta_1 = 0$).

[31]We remind that the numerator of the IFC index equals the difference between these two curves, multiplied by 2.

[32]Recall that the term $\alpha_1 - \beta_1$ is the determinant of the matrix of relative coefficients $\mathbf{A}$, which is $\mathbf{A} = \begin{pmatrix} \alpha_1 & \beta_1 \\ \alpha_2 & \beta_2 \end{pmatrix}$. Indeed, the determinant of $A$ is $|A| = \alpha_1\beta_2 - \alpha_2\beta_1 = \alpha_1 - \beta_2$. Interestingly, when $\mathbf{A}$ is an identity, hence $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then the two equations for $\frac{\Pi}{Y}$ and $\frac{\Pi}{K}$ are as in Kaldor (1955). The latter can be shown by simply noticing that when the matrix is an identity, one group earns all the capital income, and the other group all the labor income in the economy. Differently, when $\mathbf{A}$ is of the following form $\begin{pmatrix} a & 0 \\ b & c \end{pmatrix}$, with $a, b, c \neq 0$, then the two equations for $\frac{\Pi}{Y}$ and $\frac{\Pi}{K}$ are as in Pasinetti (1962).

Likewise, the term $s_1 - s_2$ can be regarded as an increasing function of saving and consumption composition inequality: when the rich save relatively more than the poor with respect to their total income, then savings are relatively more concentrated at top rather than at the bottom of the income ranking. Therefore, we can write: $s_1 - s_2 = f(\mathscr{I}_{s/c}(\mathbf{s}))$, with $\frac{\partial f}{\partial \mathscr{I}_{s/c}(\mathbf{s})} > 0$.

In light of the previous considerations, and noticing that $\frac{Y}{K} = \frac{1}{\beta}$ and $\frac{I}{K} = \dot{K}$, where $\boldsymbol{\beta}$ is the capital-output ratio, and $\dot{K}$ is the rate of capital accumulation, equations 20 and 21 can be further expressed as:

$$\frac{\Pi}{Y} = \pi = \frac{1}{\mathscr{I}_f(\boldsymbol{\pi})}\frac{1}{f(\mathscr{I}_{s/c}(\mathbf{s}))}\frac{I}{Y} - \frac{s_1\beta_1 + s_2\beta_2}{\mathscr{I}_f(\boldsymbol{\pi})f(\mathscr{I}_{s/c}(\mathbf{s}))}, \tag{22}$$

and:

$$\frac{\Pi}{K} = \frac{1}{\mathscr{I}_f(\boldsymbol{\pi})}\frac{1}{f(\mathscr{I}_{s/c}(\mathbf{s}))}\dot{K} - \frac{s_1\beta_1 + s_2\beta_2}{\mathscr{I}_f(\boldsymbol{\pi})f(\mathscr{I}_{s/c}(\mathbf{s}))}\frac{1}{\boldsymbol{\beta}}. \tag{23}$$

Equations 22 and 23 suggest that endowment and behavioral heterogeneity are central to understand the long-run dynamics of the capital share of income and of the rate of profit. Particularly, if we define $\mathscr{S}$ as follows: $\mathscr{S} = \frac{1}{\mathscr{I}_f(\boldsymbol{\pi})}\frac{1}{f(\mathscr{I}_{s/c}(\mathbf{s}))}$, then we can say that a country's position in the Box, which is a function of $\mathscr{S}$, determines the "'sensitivity of distribution', since it indicates the change in the share of profits in income which follows upon a change in the share of investment in output" (Kaldor, 1955). Considering that $\mathscr{S}$ is high in a Kaldorian System, and low in a Representative Agent System, three observations can be put forward:

1. A rise in the investment-to-income ratio $\frac{I}{Y}$ increases the capital share of income $\pi$ more in the Representative Agent System than in the Kaldorian System.[33]

2. A rise in the capital accumulation $\dot{K}$ increases the rate of profit $\frac{\Pi}{K}$ more in the Representative Agent System than in the Kaldorian System.

3. A rise in the capital-to-income ratio $\boldsymbol{\beta}$ increases the rate of profit $\frac{\Pi}{K}$ more in the Representative Agent System than in the Kaldorian System.

[33]Post-Keynesian models generally treat the investment rate $\frac{Y}{Y}$ as an endogenous variable, whilst the capital share of income $\frac{P}{Y}$ as an exogenous variable. This leads to an inversion of the causality order between these two variables: from the capital share to the investment rate (and not vice versa). However, for reasons of consistency with the original model adopted, we keep here the same causality order as in Kaldor and Pasinetti's models.

Let us now further analyze the observations above. The first tells us that in a Representative Agent System a sudden boost in the investment rate increases the capital income share relatively more than in a Kaldorian System. While such rise in the investment rate does not hamper income inequality in a Representative Agent System (as the composition of the two factors is equally distributed across the population), it does boost income inequality in a Kaldorian System, in which the capital income is mainly concentrated at the top end of the income ranking. Let us now focus on the third and the fourth observations. The third observation states that when we increase the stock of capital in the economy, then the capital tends to be more profitable in a Representative Agent System than in a Kaldorian System. In other words, under an increase in the stock of capital, the capital tends to be more profitable when better distributed across the population. Instead, the fourth observation states that under a rise in the capital-to-income ratio, or the Piketty's $\beta$, the capital tends to be more profitable when it is better distributed across the population. These results simply mean to illustrate that the two concepts of income composition inequality and saving and consumption composition inequality feature prominently in shaping the long-term relations between key macroeconomics variables, such as the capital-to-output ratio, the rate of profit, the capital share of income and the investment rate. Although the construction of a sound macroeconomic model of the Box goes well beyond the scope of our work, we believe equations 22 and 23 bring interesting insights for future research on the matter.

# D Additional Figures

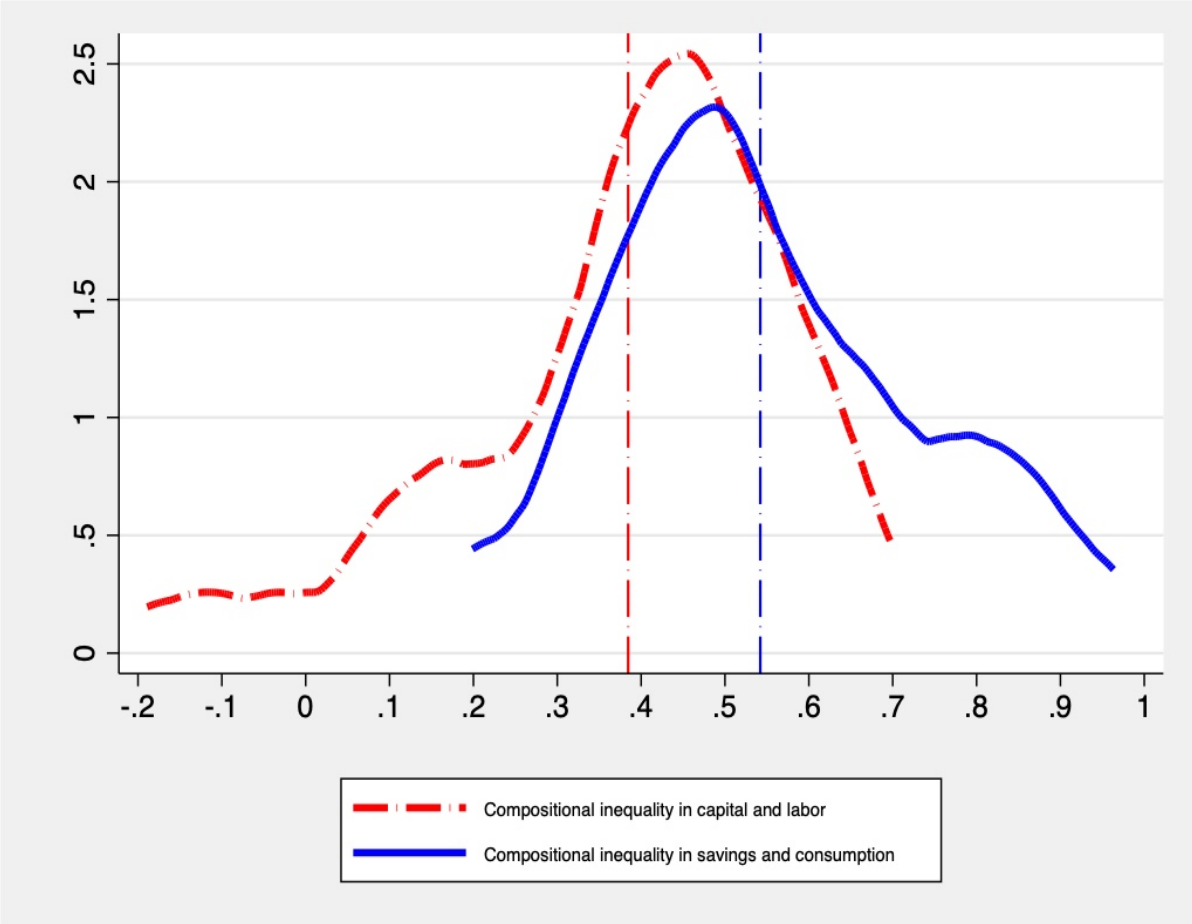


Figure 5: Kernel densities of compositional inequality in capital and labor (red line) and in saving and consumption (blue line) across all countries and years, as measured by the IFC index. The two vertical lines are the unweighted averages of the two inequality dimensions across countries and years.

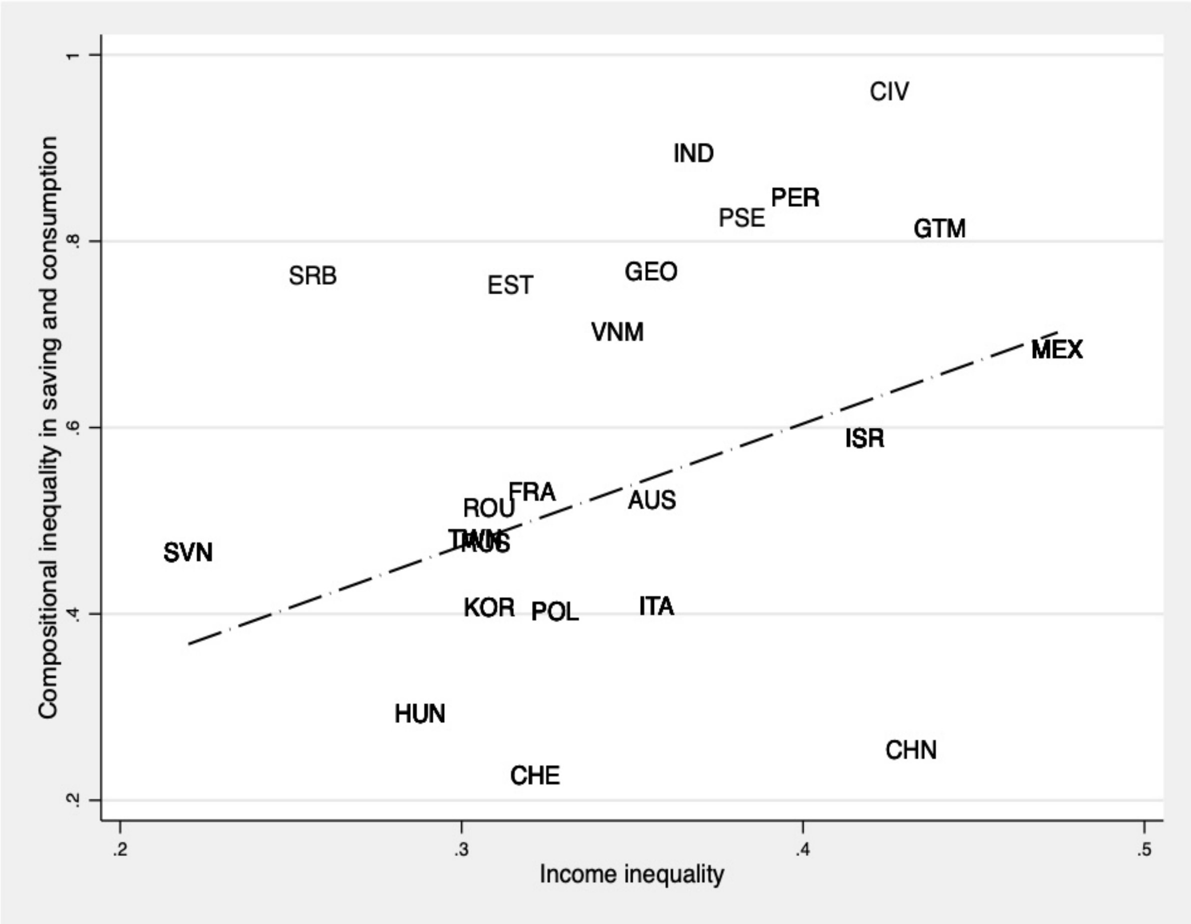


Figure 6: Compositional inequality is measured by the IFC index, whilst income inequality by the Gini coefficient. Unweighted averages of both inequality dimensions are considered for each country.

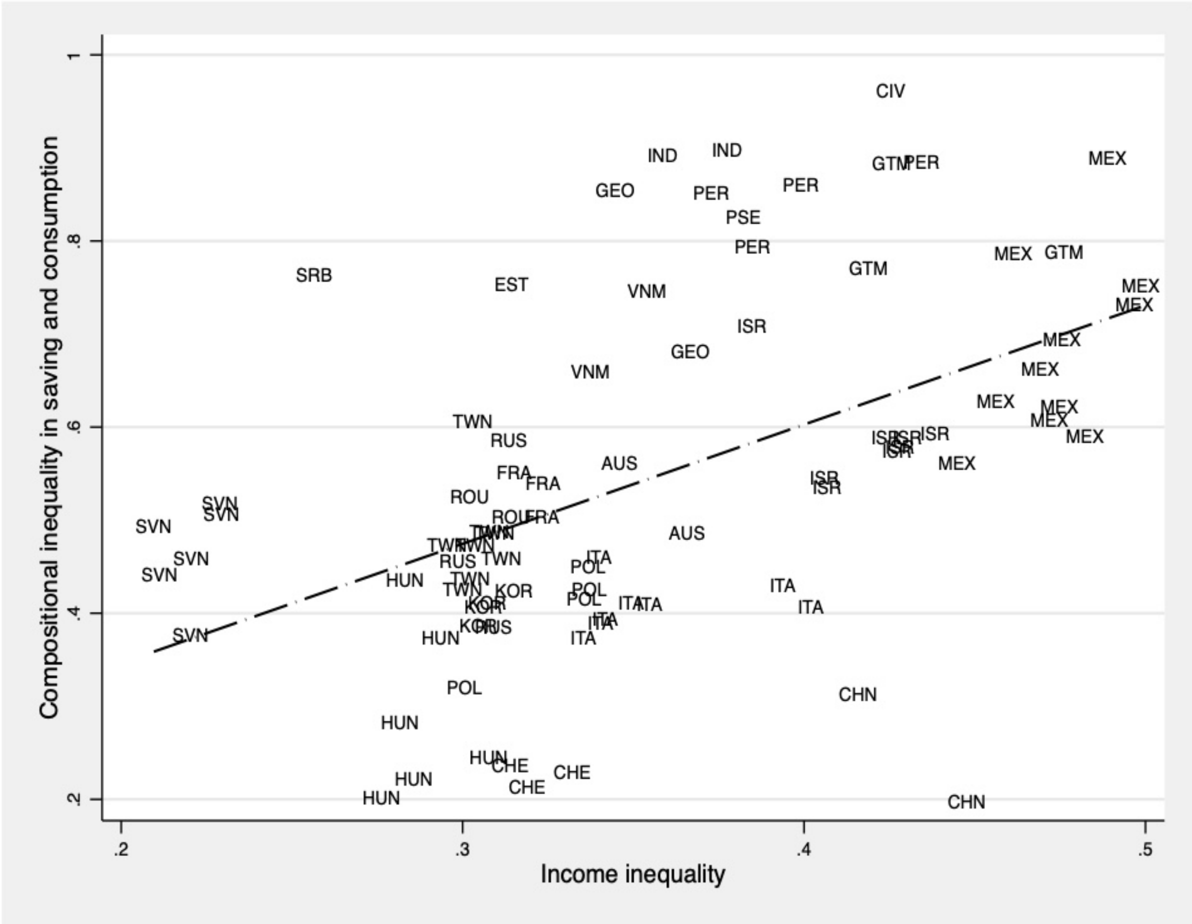


Figure 7: Compositional inequality is measured by the IFC index, whilst income inequality by the Gini coefficient.

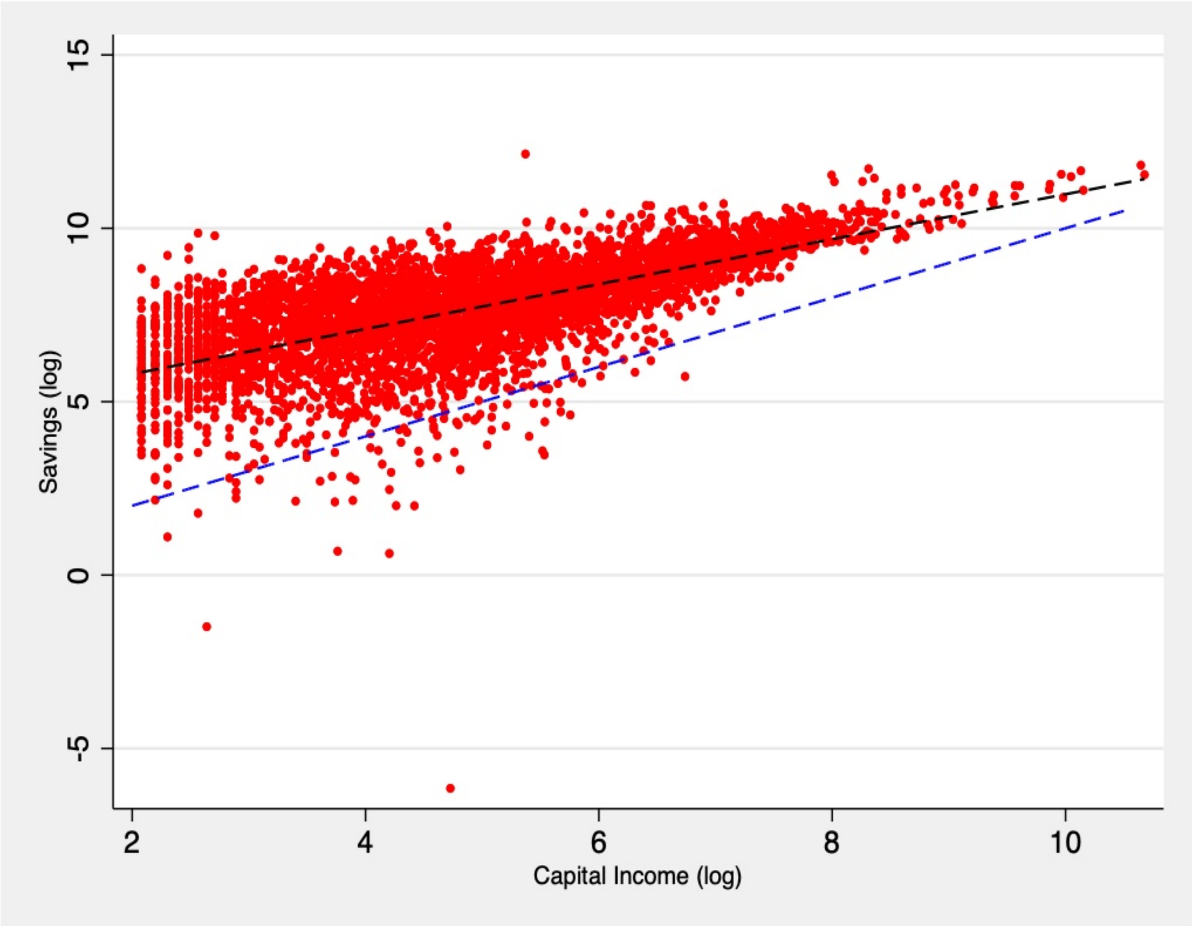


Figure 8: Each dot corresponds to the average values of capital income and savings for a specific percentile of the total income distribution. Capital income is defined as the sum of rental income, dividends and interests.

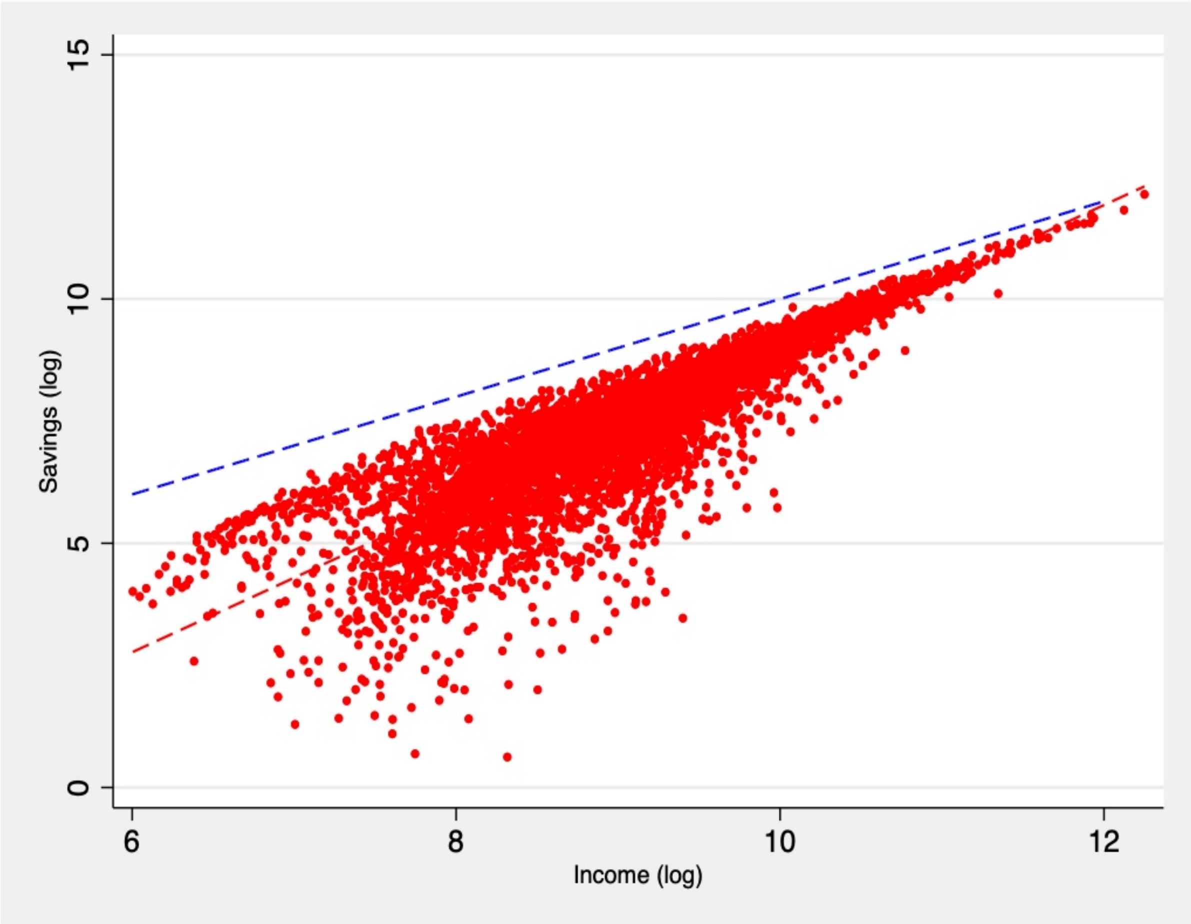


Figure 9: Each dot corresponds to the average values of total income and savings for a specific percentile of the total income distribution.