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Persistence in firm growth: inference from conditional quantile transition matrices

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Persistence in firm growth: inference from conditional quantile transition matrices

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Abstract

We propose a new methodology to assess the degree of persistence in firm growth, based on Conditional Quantile Transition Probability Matrices (CQTPMs) and well-known indexes of intra-distributional mobility. Improving upon previous studies, the method allows for exact statistical inference about TPMs properties, at the same time controlling for spurious sources of persistence due to confounding factors such as firm size, and sector-, country- and time-effects. We apply our methodology to study manufacturing firms in the UK and four major European economies over the period 2010-2017. The findings reveal that, despite we reject the null of fully independent firm growth process, growth patterns display considerable turbulence and large bouncing effects. We also document that productivity, openness to trade, and business dynamism are the primary sources of firm growth persistence across sectors. Our approach is flexible and suitable to wide applicability in firm empirics, beyond firm growth studies, as a tool to examine persistence in other dimensions of firm performance.

JEL codes: C14, D22, L25

Keywords: Firm growth persistence; Transition probability matrices; Mobility indexes; Non-parametric statistics.

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1 Introduction

To what degree are firm growth rates persistent? Does *success breeds success*, in the sense that currently expanding firms show a higher probability of further expanding their market share, while those that are shrinking are destined to continue shrinking over time? Or, conversely, do industry dynamics unfold through mean-reverting or even random growth patterns, ultimately leading to instability in firm growth rates over time?

The answers to these questions have relevant implications for both theory and policy. From a policy perspective, studying firm growth persistence is important for understanding the extent to which financial support schemes or regulatory changes targeting firm growth, can actually promote durable economic growth, employment and value creation in sectors and countries. Growth policies have more likelihood of producing long-lasting effects –provided the right firms are targeted– if firm growth rates show a high persistence, implying that firms that are already growing tend to repeat their positive performance over time. Conversely, policies to sustain growth are likely to have only temporary effects, even when the correct firms are targeted, if firm growth exhibits low or even negative persistence. In this case, targeted firms that initially grow, are very likely to stop growing relatively quickly. At the same time, the empirical assessment of firm growth persistence is also important for anti-trust policy. In fact, although growth may not continue forever, for instance due to a saturation of demand, strong persistence suggests a tendency toward the rapid emergence of firms with strong market power. That would call for timely policy interventions in those sectors in which market concentration is undesirable. Evidence of low or negative persistence instead suggests that industry dynamics follow a process that involves substantial market shares reshuffling and instability, thereby reducing the likelihood that strongly dominant firms will emerge and persist in a market.

From a theoretical viewpoint, studying firm growth persistence helps in comparing the empirical merit of alternative models of firm growth and firm-industry dynamics. Firstly, any sign of persistence, even low, would be at odds with models that describe the evolution of the firm size over time as the outcome of purely random, serially independent growth shocks (Gibrat, 1931; Geroski, 2000). Secondly, the degree of persistence observed in the data could help discriminate between models that assume convex vs. non-convex adjustment costs (Rothschild, 1971). Convexities entail some degree of persistence in firm growth due to a smooth convergence toward optimal size, while non-convexities predict low or negative persistence due to lumpiness in investment and (S,s)-practices characterizing firm dynamics (see the review in Caballero, 1999). In addition, considerable persistence in growth patterns is implied by models such as those in the evolutionary tradition, which rationalize industry dynamics as stemming from the interaction between persistently superior vs. persistently inferior firms (Nelson and Winter, 1982; Silverberg et al., 1988; Dosi et al., 1995; Metcalfe, 1998; Dosi et al., 2000). In this family of models, in fact, persistent differences in relative firm-specific capabilities (e.g. in efficiency or innovation, due to cumulativeness of knowledge, increasing returns, stability of routines and organizational structures, or path-dependence), lead to persistent differences in market outcomes, particularly in terms of profitability and growth. Notably, neoclassical equilibrium models of industry dynamics with firm heterogeneity, deliver similar predictions, although the core mechanisms differ across models, according to either passive learning (Jovanovic, 1982; Hopenhayn, 1992) or active learning (Ericson and Pakes, 1995).

The considerable information content that firm growth persistence carries for both theory and policy, has generated a large empirical literature, which we briefly review in Section 2. In summary, there are three types of empirical analyses. The vast majority of works study the average autocorrelation or autoregressive (AR) structure of firm growth rates, with inconclusive results, ranging from positive to negative to insignificant autocorrelations. More recent studies have extended the AR analysis by applying quantile regression (QREG) techniques. These report negative autocorrelations at both the bottom and the top quantiles of the growth rates distribution, but disagree as to whether the relative abundance of small-micro firms in these quantiles represents a convincing explanation for their results. Thirdly, a limited number of papers exploit transition probability matrices (TPMs) defined on growth rates quantiles (Quantile TPMs, QTPMs) to examine intra-distributional dynamics (Dopke and Weber, 2010; Capasso et al., 2013; Daunfeldt and Halvarsson, 2015; Mathew, 2017; Dosi et al., 2020). These QTPM studies tend to show that persistence is overall relatively low, as the majority of firms frequently move across the growth rates quantiles over time. Also, they find peculiar patterns in the extreme quantiles. The firms in the top quantiles (i.e., the top-performers) and in the bottom quantiles (the under-performers), both show a higher probability of retaining their relative positioning over time than other firms, but they also undergo significant anti-persistent, bouncing effects.

This paper relates to this third stream of research that exploits TPMs and QTPMs in search of a more general characterization of growth persistence than the AR model. We improve upon previous works in two substantial ways, which we detail in Section 4. First, we introduce the Conditional Quantile TPMs (CQTPMs) to correctly assess the frequencies in QTPM cells, accounting for the dependence on additional confounding variables. We use this technique to remove any spurious persistence possibly arising from the well-documented relation between properties of firm growth QTPMs and firm size (Daunfeldt and Halvarsson, 2015; Capasso et al., 2013). Through the CQTPM approach, we can replace the ex-ante defined firm size classes used in previous studies, with a conditional definition that adapts to the evolution of the size distribution inside each industrial sector. We also control for time-, country- and sector-specific effects, thus obtaining CQTPMs that are not affected by the spurious dependence possibly induced across firm growth quantiles by factors such as business cycle phases, demand dynamics, or technology patterns in individual sectors or countries.

Our second improvement, is the development of a framework to draw formal inferences regarding the overall degree of persistence in the intra-distributional dynamics described by transition matrices. Starting from the CQTPMs, we consider two mobility indexes −the Prais/Shorrocks index (Prais, 1955; Shorrocks, 1978) and the Bartholomew index (Bartholomew, 1982)− on which we can build a formal test for the null hypothesis of independent growth rates. Without a precise inferential analysis, the qualitative discussion of these indexes attempted in previous studies (Dopke and Weber, 2010; Dosi et al., 2020) is inconclusive. Drawing a precise inference is difficult, since the asymptotic properties of the elements of the CQTPMs and the associated mobility indexes depend on the joint density of the variables under study. We show that, under the null of independence, inferential analysis is in fact feasible through a relatively easy Monte Carlo exercise. The asymptotic standard errors derived via the Monte Carlo analysis enable us to define a standardized, asymptotically normally distributed version of the mobility indexes. This forms the basis for a formal test of the null of independence.

In the empirical part of the paper, we apply CQTPMs and mobility indexes to examine persistence in sales growth dynamics at the aggregate economy and the sectoral level, exploiting a cross-country firm-level dataset. As we describe in Section 3, our data include a large sample of manufacturing firms active over the period 2010-2017 in the four major European economies (France, Germany, Italy, Spain) and the UK. In Section 5, we show that country-level CQTPMs are more persistent than under the null of fully independent growth rates. However, they also reveal a good deal of turbulence in the distributional dynamics. In particular, top-performers and under-performers are more persistent than other firms, and are also more likely to experience the bouncing effects reported in previous studies. On the other hand, our analysis of sectoral CQTPMs (by 2-digit sectors) reveals considerable variation in the extent of persistence across sectors, countries and time. Since we find that this variation is primarily driven by sector-specific effects, we then explore, in Section 6, the relation between sectoral standardised indexes and a set of sectoral characteristics. Dosi et al. (2020) made a similar attempt, correlating (unstandardized) mobility indexes with industry growth. In our case, we explore a wider set of industry-level variables, including sectoral characteristics that are commonly considered to be tightly linked to patterns of firm growth and firm-industry dynamics. We find that productivity, business dynamism and openness to international markets inversely relate to persistence. We discuss the implications of our study in Section 7, together with suggestions for future research.

2 Empirical literature on firm growth persistence

The empirical assessment of firm growth persistence has traditionally relied upon estimation of autoregressive (AR) firm growth models, in panels of firms active in a given sector or country over time, possibly controlling for additional covariates (typically initial firm size). This literature on the AR structure of firm growth rates is vast and the results not easy to compare, as the studies differ by firm growth proxies, country, sector and time period considered. A fair summary is that these works are far from delivering a consistent picture. Early studies, based on relatively small samples of large firms (see e.g., Ijiri and Simon, 1967; Kumar, 1985; Dunne and Hughes, 1994), find positive autocorrelation. The subsequent works do not confirm this finding in larger and more detailed samples. Positive autocorrelation is reported in Chesher (1979) for the UK listed companies, in Wagner (1992) for the German manufacturing sector, in Bottazzi and Secchi (2003) for US manufacturing, up to the evidence of long-lasting autocorrelation (till the 7^{th} lag) found in Bottazzi et al. (2001) for the case of the international drug industry. Conversely, a number of works report negative serial correlation, e.g. in Boeri and Cramer (1992) for Germany, in Goddard et al. (2002) for Japanese listed firms, and in Bottazzi et al. (2007) and Bottazzi et al. (2011) across Italian and French manufacturing firms, respectively. Together, there are also studies that do not find any serial correlation at all, such as Geroski and Mazzucato (2002) for the US automobile sector and Bottazzi et al. (2002) for Italian manufacturing industries, as well as studies documenting that the sign of the autoregressive coefficients varies depending on the lag considered, as in Coad (2007).

Most studies following the "AR approach" to firm growth persistence, estimate AR firm growth models either via standard estimators (OLS and panel) or LAD regression. They thus focus on the central tendency of the sample. Some more recent papers extend the AR analysis by applying quantile regression (QREG) techniques, allowing to examine serial correlation along the quantiles of the growth rates distribution (see, e.g. Lotti et al., 2003; Coad, 2007; Coad and Hölzl, 2009; Capasso et al., 2013; Daunfeldt and Halvarsson, 2015). The results of these studies tend to agree that growth rates autocorrelation (or anti-correlation) is generally weak, no matter the quantile considered. There is however some interesting variation. Growth rates serial correlation is very low or practically zero in the central quantiles, while negative autocorrelation is found across both low-performing firms in the bottom quantiles and high-growth firms in the top quantiles. Negative autocorrelation in the top quantiles is particularly relevant, as the result speaks to the debate on the role of high-growth firms for long-run growth. Indeed, this finding casts doubts that high-growth firms persist in their growth performance, complementing previous studies evidence that high-growth firms are most often one-hit wonders rarely repeating high-growth performance over time (Daunfeldt and Halvarsson, 2015), while the few persistent high-growth firms do not seem to bring clear advantages in terms of productivity and other key dimension of performance (Bianchini et al., 2017; Moschella et al., 2019).[1] In relation to that, there is mixed about whether negative autocorrelation in extreme quantiles can

[1] See also Erhardt (2021), showing that the extent of high-growth persistence observed empirically, is highly sensitive to alternative measures of high-growth persistence employed in the literature.

be explained by firm size. The findings in Coad (2007) on a sample of French manufacturing firms, suggest that negative autocorrelation in the tails of the growth rates distribution is mostly due to small firms, especially in the case of small high-growth firms in top quantiles. Coad and Hölzl (2009) corroborate this conclusion, in a comprehensive sample of Austrian service firms, showing in particular that negative autocorrelation is peculiar of very small, micro firms. In contrast, the estimated QREG-AR coefficients are significantly negative even for larger firms, in the near-population sample of Dutch firms examined in Capasso et al. (2013).

A major limitation of studies that focus on the AR structure of firm growth rates, no matter whether they examine central tendency or QREG estimators, lies in the implicit assumption that growth rates of all firms follow the same parametric and linear process over time. This is a restrictive untested hypothesis, which does not necessarily hold true. Further, the AR model describes the growth dynamics of each single firm independently from the dynamics of the other firms in the reference population, disregarding intra-distributional dynamics over time.

A more general characterization of persistence, addressing these limitations, is offered by the very few papers that examine persistence in the intra-distributional dynamics of growth rates through the analysis of TPMs, in particular of Quantile TPMs (QTPMs), allowing to assess mobility/stability across growth rates quantiles (Dopke and Weber, 2010; Capasso et al., 2013; Daunfeldt and Halvarsson, 2015; Mathew, 2017; Dosi et al., 2020). A comparison of results across these studies is complicated by differences in samples, level of analysis (country vs. sector), definition of QTPM states (quartiles, deciles or percentiles of growth rates) and length of transition (usually one year, but in some cases longer, 3-to-5 year transitions). A few common findings emerge, however. First, the main diagonal elements of the estimated QTPMs are typically far below 1, meaning that the vast majority of firms do not keep their relative positioning over time. This is qualitatively interpreted as evidence of strong deviation from a fully persistent process. Second, there are persistent out-performers and persistent under-performers in top and bottom quantiles. Third, anti-persistent dynamics are often found in the off-diagonal cells, particularly in extreme quantiles, as indeed firms in extreme quantiles in the initial period have a relatively high probability to end-up in the opposite extreme quantiles. Bouncing effects of this type are more apparent in studies taking a comparatively more fine-grained definition of quantile-states (deciles or percentiles of growth), such as in Dopke and Weber (2010) for German non-financial firms, in Capasso et al. (2013) for the Dutch manufacturing, and in Daunfeldt and Halvarsson (2015) for the population of Swedish firms. The evidence is more nuanced in Mathew (2017) and Dosi et al. (2020), examining movements across growth rates quartiles, respectively for Indian and US-COMPUSTAT firms. Apart from these few common findings, the QTPMs reported in the studies display considerable variability according to a number of factors, in particular by firm size (Capasso et al., 2013; Daunfeldt and Halvarsson, 2015), sector of activity (Mathew, 2017; Dosi et al., 2020) and also with respect to time and business cycle (Dopke and Weber, 2010).

A key weakness of all these studies is that the discussion of QTPMs remains largely qualitative. There is no attempt to provide formal inferential analysis about the properties of the matrices, not even in the papers (as in Dopke and Weber, 2010; Dosi et al., 2020) introducing the mobility indexes we use in the present study. The authors in general compare values in QTPMs cells and mobility indexes obtained in different sub-samples, trying to interpret their relative values as an indication of persistence. However, without a proper measure of the confidence interval that can be assigned to the computed statistics, these qualitative comparisons are seldom informative.

Table 1: # of firms available to compute growth rates

Year	France	Germany	Italy	Spain	UK
[2010/2011]	50,478	15,090	80,559	59,669	10,077
[2011/2012]	52,671	19,009	83,505	59,817	10,121
[2012/2013]	53,400	33,964	86,676	59,604	10,360
[2013/2014]	44,794	36,272	90,796	60,959	10,530
[2014/2015]	39,021	32,687	102,892	62,284	10,551
[2015/2016]	30,882	32,506	112,056	63,709	10,478
[2016/2017]	25,888	28,936	127,413	62,605	10,736

3 Data and descriptive evidence on firm growth rates

The empirical analysis of this paper exploits the ORBIS database maintained by Bureau Van Dijk. ORBIS is a widely used source of information on financial statements and other firm characteristics, covering over 200 million firms across the globe. Although limitations are well-known, especially in terms of under-representation of micro firms (below 10 employees), it constitutes the best available source for cross-country analysis (Kalemli-Ozcan et al., 2015; Bajgar et al., 2020). We have access to data for France, Germany, Italy, Spain and the UK, over the period 2010-2017. We focus on manufacturing firms, classified according to their sector of primary activity at 2-digit level (NACE Rev.2 classification).[2] We define the growth of firm i in year t as the log-difference

$$g_{i,t} = s_{i,t} - s_{i,t-1} \tag{1}$$

where

$$s_{i,t} = log(S_{i,t}) - \frac{1}{N} \sum_{i}^{N} log(S_{i,t}) \tag{2}$$

is the logarithm of firm annual revenues $S_{i,t}$ normalised by removing the average annual revenues computed over all the firms active in the same (2-digit) sector of firm i. This normalization is often used in firm growth empirics to account for common factors affecting the size of all firms in the same sector. It implies that g measures the growth of relative size, thus capturing market shares dynamics.

We take annual sales as the empirical proxy of firm size but other proxies are possible and have been used in firm growth empirics, as for instance employment or tangible assets. The latter proxies describe the input side of the growth processes and are suited to capture growth of production capacity, relating to labour and investment dynamics. Taking sales as a proxy of size, instead, implies that the measured growth rates more closely adhere to the notion that theories of firm-industry dynamics typically consider, that is the ability to succeed in the output market.

Table 1 shows the number of firms for which we can compute 1-year growth rates, that is firms reporting non-missing values of sales over two consecutive years. ORBIS being an unbalanced panel, the observations vary by year.

[2]ORBIS reports 4-digit industries. However, as we show in Appendix A, if we take finer sectoral disaggregation than the 2-digit level (at 3 or 4-digit), the number of firms is too small to run sensible statistical analysis. In fact, as it will become clear in the following, computation of CQTPMs is heavily data demanding. Notice also that we do not consider NACE 12 (Manufacture of tobacco products), NACE 15 (Manufacture of leather and related products), and NACE 19 (Manufacture of coke and refined petroleum products) since the number of firms in these sectors is already too small (less than 20 firms) at the 2-digit level, in all countries.

Table 2: AR persistence

Year	France	Germany	Italy	Spain	UK
[2011/2012]	0.003	0.003	0.019***	0.027***	-0.026**
	(0.003)	(0.005)	(0.004)	(0.006)	(0.013)
[2012/2013]	0.002	0.000	0.007**	0.011**	-0.052***
	(0.003)	(0.001)	(0.003)	(0.006)	(0.016)
[2013/2014]	0.002	0.006**	0.013***	-0.020***	0.025**
	(0.004)	(0.003)	(0.003)	(0.005)	(0.011)
[2014/2015]	0.018***	0.011***	0.018***	0.008*	-0.018
	(0.007)	(0.004)	(0.003)	(0.004)	(0.011)
[2015/2016]	0.000	0.004	0.001	0.008*	-0.030*
	(0.006)	(0.003)	(0.002)	(0.004)	(0.016)
[2016/2017]	0.006	0.010*	0.001	-0.001	0.007
	(0.006)	(0.005)	(0.002)	(0.005)	(0.010)
All years	0.004**	0.005***	0.009***	0.006***	-0.010**
	(0.002)	(0.002)	(0.001)	(0.002)	(0.004)

Notes: LAD estimates of the AR coefficient β in Equation (3). Bootstrap standard errors in parentheses. Asterisks denote significance levels: $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$.

3.1 Autoregressive analysis of growth persistence

An AR model of firm growth of the form

$$g_{i,t} = \beta\, g_{i,t-1} + \epsilon_{i,t} \tag{3}$$

or variation thereof (e.g., including further lags on the right hand side) represents the empirical approach most commonly followed in the literature to assess persistence in growth rates. It therefore represents a useful benchmark to start with. An AR(1) coefficient β statistically equal to zero indicates no persistence, while a significantly positive (negative) estimated β provides evidence of serially autocorrelated (anti-correlated) growth episodes over time. Ideally, one would like to estimate β separately for each firm, but the time series dimension of standard firm-level datasets is usually too short to allow for reliable firm-by-firm estimation.[3] Having 7 years available in our data for computing firm growth rates, we follow the common practice to pool firm-year observations, implying that the estimated β captures the average growth rates autocorrelation in the sample.

Table 2 reports country estimates of the AR specification in Equation (3), obtained through the LAD estimator, which is robust to non-normalities in the distribution of the considered variable. This estimator is appropriate since our data replicate the stylised fact that firm growth rates exhibit a fat-tailed, tent-shape behaviour (see Figure 1, left panel), robustly documented in the literature across countries, levels of sectoral aggregation and time period considered (see e.g., Stanley et al., 1996; Amaral et al., 1997; Bottazzi and Secchi, 2003, 2006a; Coad, 2009; Bottazzi et al., 2011, 2014).

When pooling data over the available years (last line in Table 2), coefficient estimates are close to zero, implying very low persistence, positive in all countries but the UK. Separate estimates by year confirm that AR persistence is generally low. Despite some variability over time and across countries, coefficient estimates are not statistically different from zero in most cases, or otherwise range in between -0.052 and 0.027 when statistically significant.

[3]See Dosi et al. (2020) for a notable exception, exploiting long firm-level time series from US-COMPUSTAT data.

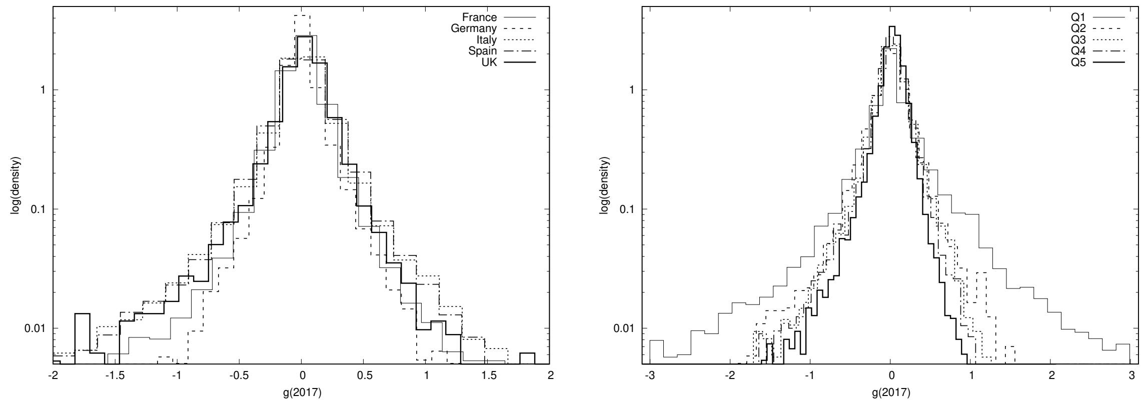

Figure 1: *Left:* Empirical density of firm (sales) growth rates, estimates for aggregate manufacturing in the year 2017, by country. *Right:* Empirical density of firm growth rates conditional on firm size, estimates for aggregate manufacturing in Italy, reporting sales growth rates in 2017 by quintiles of firm size (sales) in 2016. Comparable results are obtained in other years, in all countries.

Table 3: Growth rates distribution conditional on size - AEP parameters

	Size quintiles				
	Q1	**Q2**	**Q3**	**Q4**	**Q5**
a_l	0.210	0.180	0.155	0.147	0.124
	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)
a_r	0.239	0.161	0.147	0.140	0.125
	(0.004)	(0.002)	(0.002)	(0.001)	(0.001)
b_l	0.410	0.611	0.601	0.603	0.588
	(0.005)	(0.008)	(0.008)	(0.007)	(0.007)
b_r	0.375	0.692	0.774	0.914	1.084
	(0.004)	(0.009)	(0.011)	(0.013)	(0.016)

Notes: Scale (a_l and a_r) and shape (b_l and b_r) parameters of AEP estimates of growth rates of Italian manufacturing firms in 2017, by quintiles of firm size distribution in 2016. Standard errors in parenthesis. Comparable results are obtained in other years in the sample period, in all countries.

3.2 Relation between growth rates and firm size

A general problem affecting the estimation of AR models like in Equation (3) is the heteroskedastic nature of growth rates. More specifically, the often reported existence of dependence between firm size and the distribution of firm growth rates (Stanley et al., 1996; Amaral et al., 1997; Bottazzi et al., 2001; Bottazzi and Secchi, 2003, 2006b; Calvino et al., 2018). In fact, size-growth dependencies are present and strong in our sample. As we show in Figure 1 (right panel) taking data for Italian firms across the years 2017-2016, the empirical density of sales growth rates changes across classes of initial firm size, identified here by quintiles of the initial sales distribution. In fact, Maximum Likelihood estimates of shape and scale parameters of the asymmetric exponential power (AEP) distribution, in Table 3, reveal that smaller firms show higher average growth and larger growth variance.[4] We found consistent results also in the other years and countries covered in our data, in line with previous findings in the literature.

A possible strategy to addressing size-growth dependence within the AR approach, could be by extending Equation (3) to include an explicit scaling function accounting for heteroskedastic shocks (Bottazzi et al., 2007). In this paper we propose a different solution, accounting for size-growth dependence within the overall goal to study firm growth persistence through transition matrices, allowing for a more general characterization of persistence than in a parametric AR structure.

4 Methodology

This paper improves upon previous uses of the Quantile TPMs (QTPMs) and related mobility indexes for the empirical analysis of persistence in firm growth rates. Compared to standard TPMs, QTPMs have a clear advantage. Standard TPMs rely upon pre-defined partitions of the support of the variables used to define rows and columns of the matrix. This can be reasonable for certain kind of data for which a natural or formal partition is commonly accepted, but it is hardly justified when dealing with over time changes in the growth rates distribution. QTPMs are more robust in this respect, since they do not depend on the specific shape of the marginal distributions of the variables considered. The partitioning of the support of the variables is based on quantile functions, and therefore it is insensitive to any invertible monotonic transformation applied to the variables themselves. In firm growth analysis, this means that the matrices obtained for –say– different sectors or countries, can be directly compared even if the growth rates distributions are, in those cases, different.

Despite the characterization of probabilistic dependence provided by QTPMs is far more general than the restrictive parametric assumption implicit in AR models, there are two inherent difficulties in the application of QTPMs to studying firm dynamics. First, the direct application of the tool to a bivariate distribution might be affected by the dependence of the two considered variables on other variables that are not specifically considered. To overcome this difficulty, in Section 4.1 we introduce the Conditional Quantile TPM (CQTPM). Second, inference with QTPMs is more difficult than with standard TPMs. Analogously to the elements of a standard TPM, the elements of the empirical QTPM are consistent, efficient and asymptotically normally distributed estimators of the corresponding "true" elements, which could be obtained under complete knowledge of the underlying distributions. However, the asymptotic variance-covariance structure of matrix elements of a QTPM is more complicated than in the case of a standard TPM. In Section 4.2 we show how the choice of an appropriate null, that is the null of independence, helps solving this difficulty and makes the inferential analysis based on mobility indexes relatively simple.

4.1 Conditional Quantile TPMs

To see how the CQTPMs work, let us start with the definition of its unconditional version, the QTPM. Assume to have a sample of N paired observations $S = \{(x_n, y_n)\}$ with $n = 1, \ldots, N$. The first element x_n is often associated with the *initial state* and the second element y_n with the *final state* of some observed variable. Let $\hat{F}_x$ and $\hat{F}_y$ be the empirical distribution of the values $\{x_n\}$ and $\{y_n\}$ respectively. In other terms, $\hat{F}_x$ and $\hat{F}_y$ are the marginals of the joint empirical distribution $\hat{F}(x, y)$, obtained from the sample S. The respective empirical

[4]The AEP distribution introduced in Bottazzi and Secchi (2011) is largely used in firm growth literature to assess asymmetries and fat-tails in growth rates. Left and right shape parameters (b_l and b_r) smaller than two correspond to tails fatter than a Gaussian. The left and right scale parameter (a_l and a_r) capture width of the support below and above the modal value.

quantile functions are defined as $\hat{Q}_x(u) = \inf\{x \mid \hat{F}_x(x) \geq u\}$ and $\hat{Q}_y(u) = \inf\{y \mid \hat{F}_y(x) \geq u\}$ for $u \in [0, 1]$.[5] Given a partition of the interval $[0, 1]$ in Q equispaced intervals of size $1/Q$, consider the quantities $\hat{x}_i = \hat{Q}_x(i/Q)$ and $\hat{y}_i = \hat{Q}_y(i/Q)$ for $i = 0, 1, \dots, Q$. In particular, $\hat{x}_0 = \hat{y}_0 = -\infty$. Then, the QTPM matrix $\hat{P}(S)$ associated to the sample S is a $Q \times Q$ matrix defined as

$$\hat{P}_{i,j}(S) = \frac{\sum_{n=1}^{N} I\{\hat{x}_{i-1} < x_n \leq \hat{x}_i, \hat{y}_{j-1} < y_n \leq \hat{y}_j\}}{\sum_{n=1}^{N} I\{\hat{x}_{i-1} < x_n \leq \hat{x}_i\}}\,, \quad i, j = 1, \dots, Q\,, \tag{4}$$

where $I\{\cdot\}$ is the indicator function, taking value 1 if its argument is true, and 0 otherwise. The (i, j) element of the matrix $\hat{P}$ contains the number of paired observations whose first component is between $\hat{x}_{i-1}$ and $\hat{x}_i$ (included) and whose second component is between $\hat{y}_{i-1}$ and $\hat{y}_i$ (included), divided by the number of observations whose first component is between $\hat{x}_{i-1}$ and $\hat{x}_i$ (included), irrespective of the value of the second component. Since the partition of the interval $[0, 1]$ is equispaced, the denominator in (4) is approximately equal to N/Q.[6] Because the sample QTPM is a consistent and asymptotically efficient estimator of the true QTPM (see for instance, Section 3.2.2 in Formby et al., 2004), if the observations are drawn from a joint distribution $F(x, y)$ with marginals F_x and F_y, when $N \to \infty$ one has

$$\begin{aligned}\hat{P}_{i,j}(S)/Q \to F(F_x^{-1}(i/Q), F_y^{-1}(j/Q)) + F(F_x^{-1}((i-1)/Q), F_y^{-1}((j-1)/Q)) \\ - F(F_x^{-1}(i/Q), F_y^{-1}((j-1)/Q)) - F(F_x^{-1}((i-1)/Q), F_y^{-1}(j/Q))\,.\end{aligned} \tag{5}$$

The matrix $\hat{P}(S)$ is bi-stochastic, that is the sum of the elements of each row and each column is equal to 1: $\sum_{j=1}^{Q} \hat{P}_{i,j} = \sum_{j=1}^{Q} \hat{P}_{i,j} = 1$ for any i, j.

The QTPM does not contain more information than the joint empirical distribution $\hat{F}(x, y)$, but it is convenient in highlighting the existence of dependence between the two components x and y of the paired observations. If larger values of x are more often paired with larger values of y, then the entries of the matrix P laying along or near the main diagonal will show relatively larger values. If the opposite holds, that is if larger values of x are more likely paired with smaller values of y, then the elements away from the diagonal will have the larger values.[7] Differently from standard regression models trying to establish a specific functional relation between the variables x and y (e.g., in our context, the linear AR model relating g_t with g_{t-1}), the way QTPMs capture probabilistic dependence is not constrained by a specific parametrization, and can easily account for the presence of non-linear effects.

There is however an important issue that may prevent the interpretation of the patterns observed in the QTPM as conveying direct and precise information about the probabilistic dependence between the two considered variables. If there existed a third variable z on which the realizations of both x and y depend, then spurious patterns would appear in the QTPM due to the clustering of observations of x and y around the realizations of the third variable. To keep the parallel with regression analysis, this is similar to the bias that would arise due to omitting z in a regression between x and y.[8]

[5] The quantile function is theoretically defined as the inverse of the distribution function. A more elaborate definition is required when the empirical distribution function is considered because the latter is in general not invertible.

[6] Deviations are due to the sample size not being perfectly divisible by Q.

[7] Larger and smaller must not to be intended in absolute terms, but rather relative to the values observed in the sample itself.

[8] This problem could also spoil the interpretation of the QTPM as the transition matrix of a Markov process between states defined by values of x and y, and its use to obtain an invariant measure of the process. In fact, in a Markov process the initial state contains all information necessary to determine the distribution of the

In the context of this paper, where the focus is on dependence in firm growth rates over time (that is, x is g_{t-1} and y is g_t), just computing the QTPM as done in previous firm growth studies, would assume that the probabilities in the QTPM cells correctly reflect the joint distribution of the two states, disregarding that firm growth may depend on other variables. That would ignore, for instance, the size-growth dependencies discussed in Section 3.2. But, in fact, any other omitted variable that correlates with firm growth –not only firm size– might create a spurious tendency toward overpopulating main diagonal entries of the matrix, resulting into an overestimation of persistence.

The conditional version of the QTPM, the CQTPM, exactly allows to account for "variable dependence" in QTPM analysis. Assume to augment the sample of ordered couples under investigation with the observations on a third variable z potentially related to the first two. The sample S is now made of N triplets $S = \{(x_n, y_n, z_n)\}$ with $n = 1, \dots, N$. Then, in order to condition upon z, one can simply split the sample according to the values of z itself and examine the QTPMs between x and y within each sub-sample defined according to the values of z. If z is a discrete variable, the split procedure is obvious as one simply builds different sub-samples, each collecting observations on x and y for each different discrete value of z. The QTPMs computed according to Equation (4) within each sub-sample, are the CQTPMs in this case, as they are conditional upon the realization of z by construction. If the z variable is continuous, instead, one can resort to the quantile definition. Labeling as $\hat{Q}_z$ the marginal empirical quantile function associated to z, the sample S can be split into L equipopulated sub-samples S_l defined as

$$S_l = \{(x_n, y_n) \mid \hat{z}_{l-1} < z_n \leq \hat{z}_l\} \ , \tag{6}$$

by setting $\hat{z}_l = \hat{Q}_z(l/L)$, with $l = 0, \dots, L$. Then, a CQTPM $\hat{P}(S_l|z)$ is build for each sub-sample S_l applying Equation (4) above. The rationale is that when the third component z_n is constrained in a limited range of values, its effect on the first two variables x_n and y_n does not change significantly and thus can be neglected when building the transition matrix. The support of observations (x_n, y_n) is in general different for different sub-samples S_l, so that the different matrices $\hat{P}(S_l|z)$ are built using different empirical quantile functions $\hat{F}_x$ and $\hat{F}_y$.

Once the CQTPMs relative to the different sub-samples are computed (one CQTPM for each discrete value of z or one for each quantile-based sub-sample S_l), they can be examined separately to assess properties of the relation between x and y which are now free of bias due to dependence on z. For instance, to identify properties that are robust vis-a-vis properties that vary across different values of z. Alternatively, it might be convenient to combine the CQTPMs computed over the different sub-samples, to obtain an "aggregate" CQTPM that describes the (now unbiased) relation between x and y. Upon checking for homogeneity, an aggregate CQTPM can be computed by averaging, as $\bar{P}(S|z) = \sum_{l=1}^{L} \hat{P}(S_l)/L$. One reason for averaging is increasing the size of the sample. This will be relevant in the inference analysis of the following sections.

The entire procedure to compute CQTPMs, can be generalized to the case where there are several variable $\{z_1, \dots, z_K\}$ to be controlled for. One simply needs to: (i) split the original sample on x and y in equipopulated bins with respect to the values of the K (discrete or continuous) omitted variables; (ii) compute the CQTPM relative to x and y separately on each sub-sample; and (iii), if useful, average across sub-samples to obtain the "aggregate" CQTPM. However, one should be parsimonious about the number of variables to condition upon, as the size of the sub-samples may decrease rapidly with the number of conditioning variables. This

final states. This would hold in the case of QTPMs only if the first observation were a sufficient statistic for the realizations of the second, which is in general not guaranteed to be true. See, for instance, the exercise performed over productivity levels in Bartelsman and Dhrymes (1998) –that the authors rightly acknowledge as inconclusive– or the estimate of the Markov process driving the dynamics of firms between "growth regimes" in Dopke and Weber (2010).

is the price one is paying to enjoy a more flexible characterization than in a standard regression model.

4.2 Inference through mobility indexes

While transition matrices (in general, not only QTPMs or CQTPMs) provide a rich non-parametric description of the dependence between two variables, just looking at the numbers in specific cells or comparing cell values across matrices, might not be particularly informative. The identification of interesting patterns by visual inspection is not trivial when the matrices are large or there are many matrices to be compared. In addition, there are no guarantees that the supposedly identified patterns are statistically significant.

A number of so-called mobility indexes has been proposed in the literature to summarize the properties of or to extract specific information from QTPMs. Starting from the observed couples (x, y), these indexes capture the tendency that the relative position of the realized value y in the empirical distribution of values $\{y_n\}$, is similar to the relative position occupied by the realization of initial variable x in the empirical distribution of values $\{x_n\}$.

We consider two indexes originating from studies of income distribution and recently "imported" in studies of firm growth persistence. The first is the Prais/Shorrocks index (Prais, 1955; Shorrocks, 1978), defined as

$$I_s(P) = \frac{Q - tr(P)}{Q - 1} \quad , \tag{7}$$

where P is the QTPM (or the CQTPM), Q is the number of quantiles and tr denotes the trace of the matrix. The second index we consider is the Bartholomew index (Bartholomew, 1982)

$$I_b(P) = \frac{1}{Q-1} \sum_{i=1}^{Q} \sum_{j=1}^{Q} \frac{n_i}{n} P_{ij} |i - j| \quad , \tag{8}$$

where i and j indicate, respectively, initial and final quantiles identifying an entry of the QTPM (or the CQTPM), and n_i/n is the number of observations in the initial quantile i over the total number of observations, thus approximately equal to $1/Q$.

The two measures offer complementary yet different characterization of the degree of mobility, and thus persistence, in a matrix. The Shorrocks index I_s conveys information about the probability to remain in the initial quantile. It just considers movements in or out from the diagonal of the matrix. If there is no mobility, that is when all observations y_n are in the same quantile of their respective x_n, the value of the index is zero. Then, the index increases with mobility: the more observations are characterized by a change in the relative order of the variables, the larger the index. It reaches its maximum value equal to $Q/(Q - 1)$ when all the y_n occupy different quantiles than the respective x_n.

The Bartholomew index provides a different account of off-diagonal movements. It also equals zero under no mobility, when all observations y_n remain in the same quantile of their respective x_n. However, off-diagonal entries contribute to the value of the Bartholomew index in the case there is some mobility, with a weight $|i - j|$ that increases with the distance between the initial and final quantile. In this respect, the Bartholomew index is more apt than the Shorrocks' index to account for anti-persistent dynamics and bouncing effects across distant quantiles that have been observed in previous qualitative studies of firm growth QTPMs. A jump in relative growth rates from –say– the first to the tent decile, contributes to the Shorrocks index in the same way a jump from the first to the third quantile does. The Bartholomew index gives different weights to these two jumps.

In order to statistically compare the indexes computed over different matrices, or to draw inference about whether the index values obtained for a given matrix differ statistically from a given benchmark, one needs to assign a standard error to them. Theoretically, the indexes are statistics defined over the entries of the matrix. Thus, their asymptotic properties derive from the asymptotic properties of the latter. In case the matrix is a QTPM, one faces the problem that the asymptotic variance-covariance structure of the matrix entries is more complicated than in the case of a standard TPM. There are two reasons for this. First, while in a standard TPM the boundaries of the cells in which the samples are split are fixed, in a QTPM they are themselves estimated from the empirical quantile function and, as such, subject to noise (see the previous Section and Formby et al., 2004, p. 189). Second, in a QTPM the number of observations in each row and column is constrained to be exactly N/Q. The increased complexity of the variance-covariance structure of the matrix elements induced by these two effects, implies that simple approaches as the one presented in Schluter (1998) are not suited. In particular, the delta method analysis in Formby et al. (2004) shows that the asymptotic variance-covariance structure of the elements of the QTPM depends on the joint probability density of the underlying variables. Thus, in order to derive the asymptotic behaviour of the elements of the QTPM and of the associated mobility indexes, one needs an estimate of the underlying joint density. In many situations, this requirement makes the direct use of the asymptotic expression excessively cumbersome. A viable alternative could be to apply bootstrap techniques, along the lines suggested in Biewen (2002) and Richey and Rosburg (2018) for the case of mobility indexes derived from standard TPMs. In the case of QTPMs the bootstrap approach would be even more recommendable than in the case of standard TPMs. However, when *Conditional* QTPMs are considered –as in this paper– the bootstrap procedure is complicated by the need to find an appropriate stratification of the sample with respect to the omitted/control variable.

In our case, a simpler approach is possible. The key observation is that our inferential analysis involves comparing the empirical indexes with the (asymptotic) distribution they have under the null of independent growth rates. In fact, inspired by the classical Gibrat's model of size-growth dynamics, independence of firm growth rates over time is the natural benchmark in our context. It has been already used to gauge qualitative considerations about firm growth QTPM entries in Dosi et al. (2020) and Capasso et al. (2013).

In general terms, the null of independence implies that, starting from any initial quantile, there is the same probability to end up in any one of the final quantiles. If Q quantiles are considered, this corresponds to a theoretical matrix with $1/Q$ in all the entries. Under this null, the derivation of the properties of the indexes greatly simplifies. First, one can easily derive that, under the null, the expected values of the observed indexes $\hat{I}_s$ and $\hat{I}_b$ read

$$\mathbb{E}[\hat{I}_s] = 1 \text{ and } \mathbb{E}[\hat{I}_b] = \frac{Q+1}{3\,Q} \; . \tag{9}$$

Second, the invariance of the QTPM under invertible monotonic transformation of the underlying random variables implies that, *when the variables are independent*, the asymptotic behavior of the variance-covariance structure of the elements of the QTPM becomes distribution independent and one can use a Monte Carlo approach to derive it.

Specifically, since the elements of the QTPM are consistent, efficient and asymptotically normal, then the considered indexes, being smooth functions of these quantities, are themselves consistent, efficient and asymptotically normal. Therefore, under the null of independence, with sample size N going to infinity, the indexes $\hat{I}_s$ and $\hat{I}_b$ are normally distributed with mean given in (9) and variance

$$\mathbb{V}_N[\hat{I}_x] \sim \frac{C_x(Q)}{N} \, , \quad x = s, b \; . \tag{10}$$

The asymptotic coefficients $C_s(Q)$ and $C_b(Q)$ only depend on the number Q of quantiles considered and can be therefore computed via Monte Carlo simulations with any distribution. Since

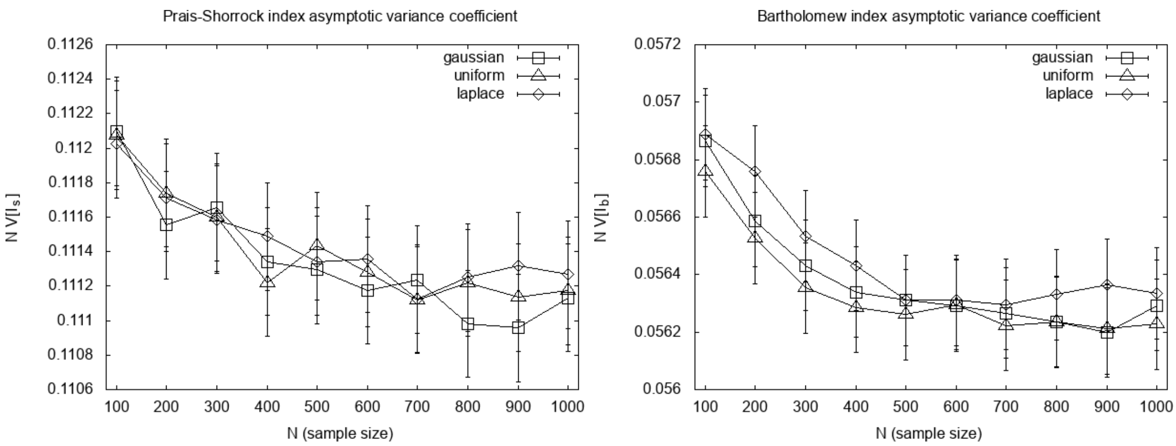

Figure 2: Monte Carlo analysis of rescaled variance ($N\,\mathbb{V}_N[\hat{I}]$, from Eq. 10), for the Shorrocks (left) and the Bartholomew (right) index. Figures obtained using R=10^6 replications of the respective index, each replication computed over a sample of N independently drawn couples and using QTPMs with Q=10. For the generation of the underlying data we test three different distributions: Standard Normal, Uniform and Laplace.

in the paper we will perform the empirical analysis using deciles, we are interested in the case Q=10. Accordingly, we perform the following exercise. For a given sample size N, we randomly generate R independent samples of N couples of independent observations drawn from a given distribution. On each sample, we compute the QTPM with Q=10, and the Prais-Shorrocks and Bartholomew indexes associated to this matrix. We end up with R independent realizations for each index, $\hat{I}_x(r;N)$, with $r = 1,\ldots,R$ and $x = s,b$. Then, we compute their mean and variance

$$\mathbb{E}_N[\hat{I}_x] = \frac{1}{R}\sum_{r=1}^{R}\hat{I}_x(r;N)\,, \quad \mathbb{V}_N[\hat{I}_x] = \frac{1}{R-1}\sum_{r=1}^{R}\left(\hat{I}_x(r;N) - \mathbb{E}_N[\hat{I}_x]\right)^2, \quad x = s,b\,.$$

When N goes to infinity, $\mathbb{E}_N[\hat{I}_x]$ goes to the values $\mathbb{E}[\hat{I}_x]$ reported in Equation (9). Concerning the behavior of the variance, we report in Figure 2 the quantities $N\,\mathbb{V}_N[\hat{I}_s]$ (left panel) and $N\,\mathbb{V}_N[\hat{I}_b]$ (right panel) obtained over R=10^6 independent replications, three different distribution of the underlying variables (Standard Normal, Uniform in $[0,1]$ and Laplace centered in 0 with tail coefficient a=1), and sample size ranging from N=100 to N=1000. The confidence intervals in the plots are derived from the Chi-Square quantile function with R−1 degrees of freedom, $Q_{\chi^2}(q, R-1)$, under the assumption that the indexes are normally distributed. The bands represent a 95% confidence level and are obtained multiplying the computed value times $(R-1)\,Q_{\chi^2}(0.025, R-1)$ or times $(R-1)\,Q_{\chi^2}(0.975, R-1)$, respectively for the upper and lower bound.[9]

The numbers reported in the plots confirm that the behavior does not depend on the underlying distribution. While a clearly visible downward slope signals the presence of sub-asymptotic corrections, their effect is so small that they are already negligible, for any practical purpose, when the size of the sample is as small as N=100. This is below the size of the samples we will consider to build CQTPMs in this paper. In the inferential analysis of the following sections, when an expression for the variance of the indexes is required, we will use $C_s(10)$=0.1120 and $C_b(10)$=0.0563, corresponding to the average of the two coefficients over the three considered distribution for N=1000 (taken to the last significant digit).

[9]In the case of R=10^6 replications, the bounds are 0.9972 and 1.0028.

5 Distributional analysis of firm growth persistence

We exploit the CQTPM framework developed above to condition out possible dependence of growth rates on time, country, sector, and firm size. We consider the observations on firm growth rates separately for each transition between year t and $t+1$ allowed by the sample time period, split them further by country and, within each country, by (2-digit) industries. This implicitly controls for spurious persistence in growth rates due to country, sector or time factors, exploiting the simple fact that these potentially omitted sources of dependence are naturally coded as discrete variables (they are discrete z variables, in the wording of Section 4.1).

Then, within each of the transition-country-sector sub-samples obtained this way, we control for growth-size dependencies as follows. We start with the triplet $\{g_{n,t}, g_{n,t+1}, s_{n,t}\}$, where $s_{n,t}$ is initial firm size (in terms of sales, in line with the definition of g), and build 5 equipopulated sub-samples according to the quintiles of the distribution of initial firm sizes (i.e., L=5 in Equation 6 above). Then, within each size-quintile sub-sample, we build a CQTPM by taking the deciles of the marginal empirical distribution of growth rates in t and $t+1$ as the initial and final states of the transition (i.e., Q=10). Lastly, we "aggregate back" the 5 QTPMs computed over the size-quintiles via averaging them, thus obtaining a CQTPM for each of the transition-country-sector sub-samples. In total, we are left with 600 CQTPMs to work with (5 countries, 20 2-digit sectors, 6 transitions).[10]

Conditioning on firm size is particularly important. As discussed in Section 3.2, growth rates are known to depend on firm size. QTPMs computed by sector or country ignoring growth-size dependencies, might show more persistence in some cells than in others just because firms of similar size are likely to exhibit similar growth rates in both initial and final period. In fact, Daunfeldt and Halvarsson (2015) and Capasso et al. (2013) report apparent differences between the QTPMs computed separately across firms belonging to different size classes. Instead of using ex-ante defined size classes, as these previous studies do, our procedure to build CQTPMs adapts to the specific characteristics and evolution of the size distribution inside each sector-country-transition sample.

5.1 Country-level analysis

We start by examining CQTPMs properties at the country level. These are obtained by averaging the 20 sectoral CQTPMs obtained for each country in a given transition (controlling for firm size as described above), leaving us with 6 separate matrices to study for each country.

Figure 3 shows the CQTPMs corresponding to the 2016/2017 transition. Numbers by row represent shares of firms staying in the same decile or moving to a different decile of yearly growth rates over the two years. Given the size of the samples we are working with, the statistical estimation error of matrix elements is of the order of one thousand. Thus, we only include significant digits in all our figures, while the gray scale helps identifying the main patterns.

Two main patterns emerge, common to all countries. First, we observe some tendency to remain in the same decile, as shown by the darker stripe along the main diagonal. This tendency is stronger in the top and bottom deciles, revealing the presence of persistent over- and under-performing firms. Together, and second, there is a relatively high probability of moving to the opposite decile, as suggested by the darker stripe along the secondary diagonal. In particular, there is a relatively higher probability of switching from Q1 to Q10, or vice-versa.

[10] We tested the robustness of our results by experimenting with two different definitions of the CQTPMs. First, we conditioned upon initial size deciles instead of initial size quintiles. Second, we kept conditioning on quintiles of initial size as in the baseline analysis, but considered 20 equipopulated growth bins instead of growth deciles (computing Monte Carlo values of C_s and C_b for $Q = 20$). These alternative specifications did not affect significantly our main findings.

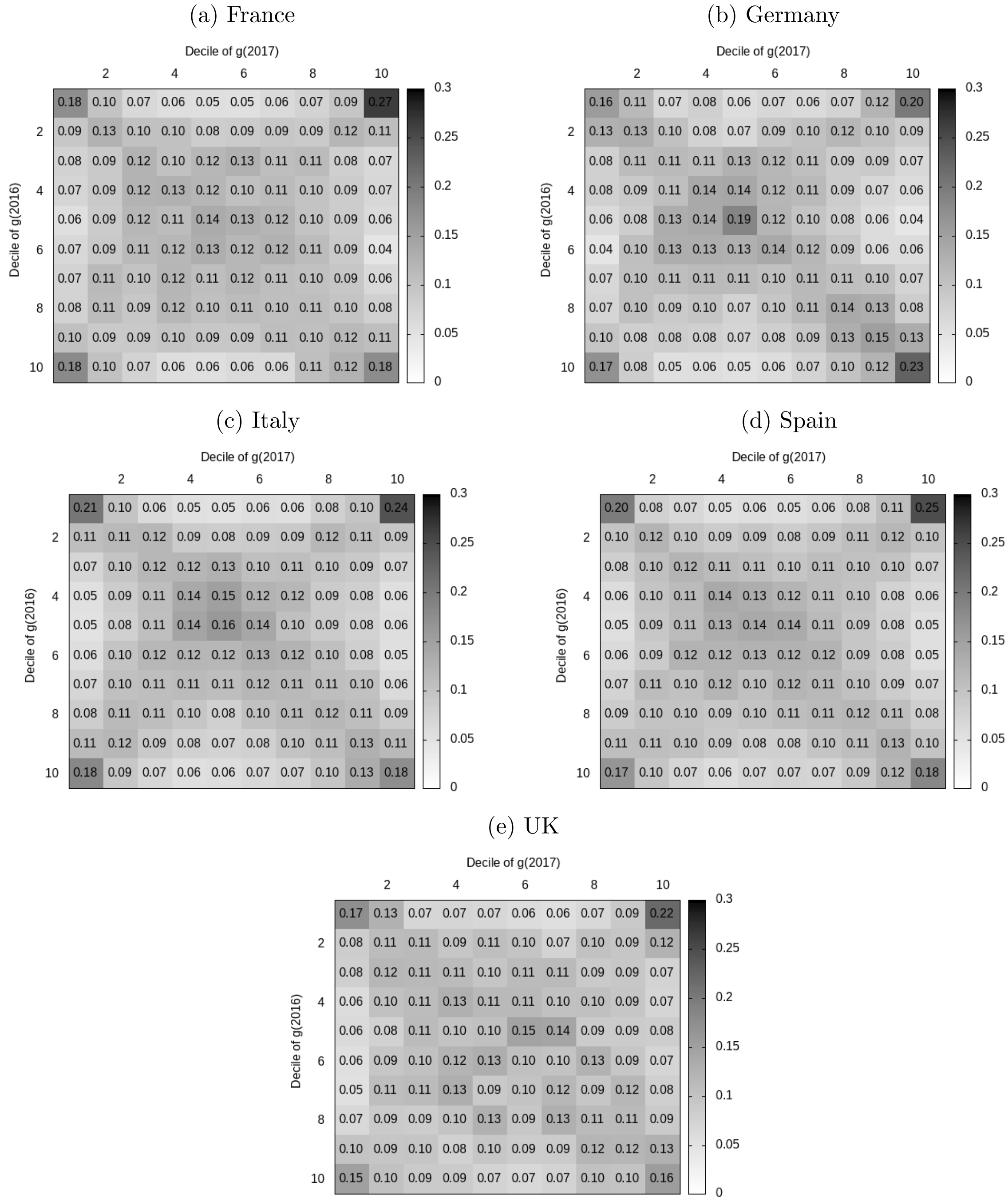

Figure 3: Conditional Quantile TPMs, defined on growth rates deciles – by country, 2016/2017 transition.

Table 4: Mobility indexes and test of the null of independence

	Transition	France	Germany	Italy	Spain	UK
Shorrocks	[2011/2012]	0.967***	0.955***	0.959***	0.960***	0.985***
		(-21.876)	(-14.382)	(-34.195)	(-28.237)	(-4.453)
	[2012/2013]	0.970***	0.934***	0.962***	0.964***	0.968***
		(-19.809)	(-23.955)	(-34.441)	(-25.388)	(-9.244)
	[2013/2014]	0.970***	0.934***	0.960***	0.964***	0.960***
		(-18.486)	(-30.512)	(-34.513)	(-25.939)	(-11.659)
	[2014/2015]	0.969***	0.941***	0.958***	0.956***	0.969***
		(-17.389)	(-30.312)	(-37.836)	(-31.715)	(-9.297)
	[2015/2016]	0.970***	0.936***	0.953***	0.958***	0.973***
		(-15.145)	(-31.845)	(-44.355)	(-30.365)	(-7.897)
	[2016/2017]	0.966***	0.945***	0.956***	0.957***	0.974***
		(-15.769)	(-25.576)	(-43.359)	(-30.762)	(-7.680)
Bartholomew	[2011/2012]	0.352***	0.339***	0.342***	0.338***	0.364
		(-13.462)	(-12.649)	(-28.561)	(-29.113)	(-1.043)
	[2012/2013]	0.357***	0.335***	0.346***	0.347***	0.367
		(-8.769)	(-16.054)	(-24.635)	(-20.003)	(0.286)
	[2013/2014]	0.358***	0.331***	0.345***	0.352***	0.332***
		(-7.581)	(-22.805)	(-27.217)	(-15.056)	(-14.491)
	[2014/2015]	0.352***	0.336***	0.341***	0.344***	0.354***
		(-11.497)	(-22.394)	(-31.943)	(-22.880)	(-5.420)
	[2015/2016]	0.354***	0.333***	0.345***	0.344***	0.359***
		(-9.260)	(-23.659)	(-29.145)	(-23.097)	(-3.350)
	[2016/2017]	0.352***	0.332***	0.345***	0.350***	0.349***
		(-9.177)	(-22.534)	(-29.687)	(-16.809)	(-7.135)

Notes: Shorrocks and Bartholomew mobility indexes (standardized values in parenthesis) computed on CQTPMs, by country and transition years. Asterisks refer to an F-test of the null hypothesis that the CQTPMs exhibit independence, implemented as an assessment of the distance between observed standardized indexes and their expected value under the null (1 for Shorrocks; $0.3\bar{6}$ for Bartholomew). Significance level: $^{***}p < 0.01$.

This is evidence of sizable, anti-persistent bouncing effects particularly affecting extreme growth events. Consistent patterns replicate also in all the other transition years, in all countries (see Appendix B). The behavior observed in extreme quantiles is in accordance with previous studies which examined unconditional TPMs. Our analysis confirms that they survive also after controlling for firm size and country-, sector- and time-effects.

These peculiar properties of growth dynamics would be totally invisible within the standard AR regression approach. Moreover, they show that the linear AR model discussed in Section 3.1 is a poor description of the underlying process. In fact, were the AR model able to provide a satisfactory characterization of the underlying dynamics, we should observe a similar behavior across all initial states, that is across all the 10 rows of a matrix. Apparently, this is never the case in any of the countries considered.

Notice also that –here as well as in the rest of the paper– we do not present a pooled analysis aggregating CQTPMs over time. That would require averaging the CQTPMs obtained across the different transitions. This is not allowed in our data, since we verified that the CQTPMs computed for the different transitions do not pass an homogeneity test (details in Appendix C). This may be a further source of bias in previous studies that apply TPMs or QTPMs to firm growth persistence, where the reported matrices are often obtained after pooling data over time, without previously checking for homogeneity.

The qualitative analysis of the matrices is already revealing of interesting patterns. However, a central question remains: to what extent the numbers observed in the CQTPMs cells deviate, in a statistical sense, from the "0.1 benchmark" that one expects under the null of independent growth rates? Are the intra-distributional movements observed in the empirical CQTPMs large or small, compared to the benchmark of independence? The theoretical and Monte Carlo results developed above about mobility indexes properties help addressing these questions.

As a preliminary step, notice that, since the different CQTPMs involve a different number of firms, the values of the indexes directly computed on the observed CQTPMs cannot be compared across the different matrices. However, since we derived their properties under the null for the case Q=10, we can compute a standardized version of the indexes, as a function of the sample size

$$\tilde{I}_s = \frac{\hat{I}_s - \mathbb{E}[\hat{I}_s]}{\sqrt{\mathbb{V}_N[\hat{I}_s]}} = 3\sqrt{N}(\hat{I}_s - 1)\,, \quad \tilde{I}_b = \frac{\hat{I}_b - \mathbb{E}[\hat{I}_b]}{\sqrt{\mathbb{V}_N[\hat{I}_b]}} = 4.22\sqrt{N}(\hat{I}_b - 11/30)\,. \tag{11}$$

The nice feature of these quantities is that they are asymptotically distributed as a $\mathcal{N}(0,1)$ and, thus, they are comparable across different samples and different matrices. By definition, their value indicate how many standard deviations the empirical data are away from their expected value under the null of independence. Negative values indicate that there is more persistence in the observed CQTPMs than under the null, while the opposite holds for positive values.

Table 4 reports un-standardized mobility indexes and their standardized version, computed for the country-level CQTPMs discussed above, by country and transition. Previous studies provide qualitative comparisons between unstandardized values and some theoretically meaningful value (e.g, corresponding to "no firms remain in the initial quantile"). Figures in Table 4 show that such comparisons might be deceiving, while the standardization procedure is essential in drawing definite conclusions about the nature of the process. To see this, consider the Shorrocks' index. If all firms remained in the same quantile over time (the case of "no mobility", i.e. main diagonal elements all equal to 1), we would expect to observe un-standardized values close to 0. Conversely, if no firms preserved their initial quantile over time (i.e., all zeroes in the main diagonal), the expected value of the un-standardized index would be $10/9 = 1.1$. The numbers observed in Table 4, all in the range 0.94-0.98, are not that far from this case, while they seem quite far from zero. However, they are also not too far from the benchmark value of 1 that would emerge under the null of independence (recall Equation 9). The standardized values allow to decisively discriminate between the two alternatives. In fact, they show that the un-standardized figures are several standard deviations smaller than under the null, implying that there is more persistence in the data than an independent growth process would imply.

The emergence of consistently negative standardised values suggest that this is in fact a general pattern. To corroborate this, Table 4 also reports an F-test for the statistical significance of the distance between the observed standardized values and the expected value under the null of independence. The F-tests confirm that the observed negative deviations from the benchmark are statistically significant.

The analysis of the Bartholomew indexes yields consistent patterns and support similar conclusions. The un-standardized indexes are well above 0, i.e. the value expected under no mobility, and they are also well below the value of $7/9 = 0.\bar{7}$ which would be expected under "max mobility", if all firms made the longest possible jump compared to their initial quantile.[11] In fact, the un-standardized values are very close to the value expected under the null of independence for Q=10, which is $0.3\bar{6}$ (see Equation 9). Again, the standardized values and the associated F-tests reveal negative and statistically significant deviations from the null. This confirms that firm growth is more persistent than an independent process, in all countries considered and across all transitions.[12]

[11] Note that the theoretical maximum value of the Bartholomew index is a function of the number of quantiles Q. It equals $(3\,Q-2)/(4\,(Q-1))$ if Q is even, and equals $(3\,Q+1)/(4\,Q)$ if Q is odd.

[12] As shown in Appendix D, the conclusion remains the same when the CQTPMs is computed over 3-year growth rates.

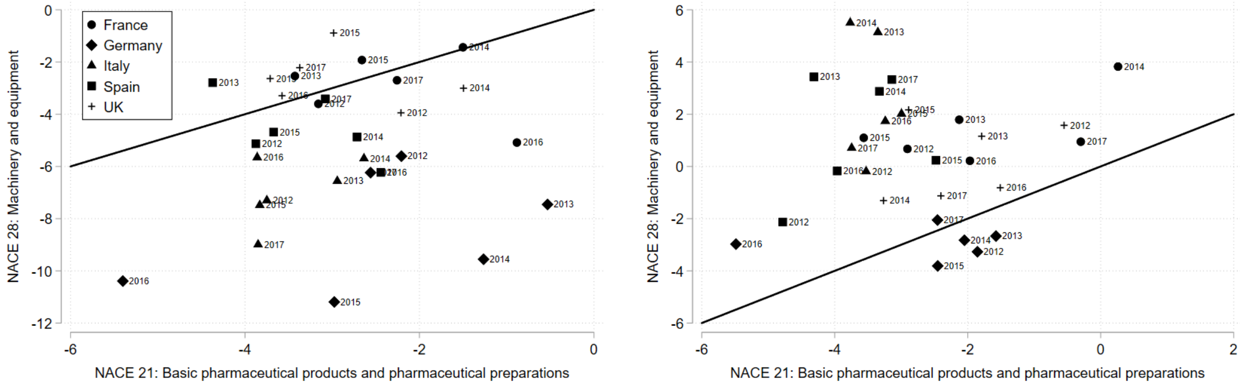

Figure 4: Comparison of Standardized Shorrocks (left) and Standardized Bartholomew (right) in two selected sectors. Points refer to different countries and transitions. The line is the 45° sloping bisector.

Within this general result, the Shorrocks and the Bartholomew indexes also reveal differences across countries. The UK and, to a lesser extent, France appear as the countries with less persistent growth rates, as the value of the indexes are less negative than in other countries. This ranking in the degree of persistence across countries, essentially replicates in all years, although there is some variability within each country over time.

A notable difference between the two measures, is that the standardized Bartholomew indexes are generally less negative compared to the corresponding standardized Shorrocks' indexes. That is, despite the general rejection of the null of independence, the Bartholomew indexes suggest more mobility (lower persistence) than the Shorrocks indexes do. This is compatible with the Bartholomew statistic giving more weight to off-diagonal movements and, thus, being more suited to account for the bouncing effects we observed in the top and bottom deciles of the CQTPMs.

Overall, the analysis of country-level CQTPMs supports the idea that firm growth intra-distributional dynamics are more persistent than an independent process would imply, even after controlling for biases that could arise due to dependence of growth rates on firm size and country-, sector- or time-specific factors. This emerges out of relatively low persistence in most CQTPMs cells, coupled with the peculiar dynamics in the corners of the matrices, characterizing firms experiencing extreme relative growth episodes.

5.2 Sector-level analysis

The CQTPM framework we developed can also be exploited to examining similarities and differences in firm growth persistence across sectors. As an example of the variety of sectoral patterns across the 600 2-digit sectoral CQTPMs we can work with, the scatter plots in Figure 4 correlate the standardized values of the Shorrocks and the Bartholomew indexes computed by transition and country for two sectors, "Basic pharmaceutical and pharmaceutical preparations" (NACE 21) and "Machinery and equipment" (NACE 28).

In general, no matter the index considered, the plots reveal a good deal of sectoral heterogeneity. The supports spanned by both indexes are wide and the points are scattered away from the 45° degree line, implying that the two sectors show different extent of persistence. We also observe apparent differences across the two indexes, much likely due to the ability of the Bartholomew index to account for off-diagonal movements. According to the Shorrocks indexes, although both sectors display higher persistence than under the null of independence (negative standardized values), NACE 28 shows more persistent growth dynamics (less negative standardized value). The Bartholomew indexes, instead, feature various points where negative deviations from the null observed for NACE 21 associate with positive deviations for NACE 28, in the same transition and country. This supports that firms in NACE 21 display more persistent dynamics than firms in NACE 28. Similarly heterogeneous patterns emerge also across other 2-digit sectors covered in our data. This evidence confirms the choice to compute sector-wide CQTPMs in the first place. More generally, it also suggests that sector-specific factors may indeed be a relevant source of variation in firm growth persistence.

Notice, however, that in the plots of Figure 4, the points referring to the same country exhibit some tendency to cluster relatively close to each other. This would hint that country-specific heterogeneity, already observed in country-level CQTPMs, survive at the level of single sectors. Conversely, we do not observe significant clustering by transition year, suggesting that the variation due to time-effects is relatively modest, once the sector and the country dimensions are fixed.

To disentangle statistically the relative explanatory power of country, sector and time factors, we estimate the following variance decomposition regression model

$$\tilde{I}_{j,c,t} = \alpha + \delta_j + \delta_c + \delta_t + \epsilon_{j,c,t} \tag{12}$$

where the dependent $\tilde{I}$ is either the standardized Shorrocks or the standardized Bartholomew index associated to the CQTPM of sector j in country c over the transition between t-1 and t, while the δ covariates represent full sets of sector, country and transition fixed-effects.

Estimation results are reported in Table 5 for different specifications. When we include country dummies only (in model 1 and 5, France is the baseline), the relative ordering of the coefficients is broadly in line with the results in Table 4. Italian and Spanish firms display more persistence than firms in other countries, while France and particularly the UK data show lower persistence. Recall that in the exercise of Table 4, we were averaging the sector-year CQTPMs of a given country and then computing the indexes. In the regression model in Equation (12), we are doing somewhat the opposite: the coefficients associated to the country dummies are proportional to the averages of the sectoral indexes across all sectors within a given country. All values need to be interpreted as a deviation with respect to the constant and,for both indexes, the net value is negative for all countries.

The estimates of models where we only include sector fixed-effects (model 2 and model 6, NACE 32 is the baseline) reveal that the coefficients on the sectoral dummies are more heterogeneous than the country dummy coefficients. They span a support of around 10, while the support of country dummies is about 5.6. In the regression on the Shorrocks indexes, all sectoral dummies have a negative net value (considering the constant). This confirms that there is generally more persistence than under the null of independence, with firms in NACE 10 ("Manufacture of food products") having particularly persistent growth rates. On the other hand, in the regression taking the Bartholomew index as the dependent variable, NACE 28 ("Machinery and equipment") is the only sector with positive net contribution.

The dummies relative to the different transition years (model 3 and model 5) are not significant, in line with the intuition from Figure 4 that time provides a negligible contribution to total variation.[13]

The three sets of dummies explain, together, 63.5% of total variation in Shorrocks indexes and 62.1% of variation of Bartholomew indexes (see the "complete" models 4 and 8). Because of the symmetric nature of the sample (same number of observations for each sector and same sectors for each country), one can estimate the contribution of each group of dummy variables to the models' R^2 by just dropping the other dummies. It turns out that, for both the indexes, country and sector dummies together account for essentially the whole explained variance.

[13]This finding suggests that growth dynamics is consistent over time, despite the rejection of homogeneity across QTPMs, examined in Appendix C.

Table 5: Decomposition of sector, country and time contribution to firm growth persistence

		Standardized Shorrocks				Standardized Bartholomew			
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Country	Germany	-1.478**			-1.478**	-1.448**			-1.448**
		(0.465)			(0.344)	(0.520)			(0.338)
	Italy	-3.757**			-3.757**	-2.927**			-2.927**
		(0.515)			(0.324)	(0.610)			(0.373)
	Spain	-1.915**			-1.915**	-1.678**			-1.678**
		(0.441)			(0.238)	(0.602)			(0.424)
	UK	1.908**			1.908**	1.216*			1.216***
		(0.405)			(0.309)	(0.530)			(0.412)
Sector	10		-6.223**		-6.223**		-8.995**		-8.995**
			(1.416)		(1.070)		(1.439)		(1.285)
	11		2.821**		2.821**		2.742**		2.742**
			(0.526)		(0.498)		(0.699)		(0.574)
	13		2.367**		2.367**		1.801*		1.801**
			(0.687)		(0.500)		(0.722)		(0.490)
	14		1.202		1.202		0.059		0.059
			(0.933)		(0.642)		(0.771)		(0.577)
	16		2.124**		2.124**		3.529**		3.529**
			(0.664)		(0.493)		(0.612)		(0.528)
	17		2.666**		2.666**		1.424*		1.424**
			(0.635)		(0.464)		(0.622)		(0.440)
	18		0.247		0.247		0.273		0.273
			(0.831)		(0.453)		(0.615)		(0.443)
	20		1.361*		1.361**		-0.057		-0.057
			(0.683)		(0.486)		(0.684)		(0.510)
	21		3.284**		3.284**		2.136**		2.136**
			(0.536)		(0.536)		(0.548)		(0.432)
	22		1.555*		1.555**		0.625		0.625
			(0.629)		(0.387)		(0.678)		(0.471)
	23		1.846*		1.846**		3.222**		3.222**
			(0.765)		(0.472)		(0.705)		(0.514)
	24		3.442**		3.442**		2.940**		2.940**
			(0.710)		(0.517)		(0.580)		(0.458)
	25		-1.183		-1.183		3.411**		3.411**
			(1.178)		(0.632)		(0.865)		(0.597)
	26		1.713*		1.713**		2.207**		2.207**
			(0.736)		(0.593)		(0.583)		(0.465)
	27		2.094**		2.094**		2.404**		2.404**
			(0.703)		(0.523)		(0.516)		(0.438)
	28		1.086		1.086*		5.330**		5.330**
			(0.741)		(0.525)		(0.616)		(0.659)
	29		2.965**		2.965**		2.105**		2.105**
			(0.615)		(0.477)		(0.554)		(0.417)
	30		3.931**		3.931**		3.558**		3.558**
			(0.606)		(0.463)		(0.452)		(0.433)
	31		2.695**		2.695**		3.712**		3.712**
			(0.669)		(0.481)		(0.628)		(0.544)
Transition	[2011/2012]			0.736	0.736*			0.050	0.050
				(0.477)	(0.290)			(0.538)	(0.368)
	[2012/2013]			0.351	0.351			0.612	0.612
				(0.500)	(0.332)			(0.569)	(0.412)
	[2013/2014]			-0.078	-0.078			-0.144	-0.144
				(0.525)	(0.327)			(0.525)	(0.391)
	[2014/2015]			-0.248	-0.248			-0.403	-0.403
				(0.547)	(0.318)			(0.598)	(0.381)
	[2015/2016]			-0.405	-0.405			-0.074	-0.074
				(0.551)	(0.388)			(0.559)	(0.369)
	Cons	-3.700**	-6.248**	-4.808**	-5.259**	-2.264**	-4.852**	-3.238**	-3.892**
		(0.357)	(0.512)	(0.377)	(0.458)	(0.477)	(0.450)	(0.405)	(0.441)
	Obs	600	600	600	600	600	600	600	600
	R^2	0.275	0.350	0.011	0.636	0.126	0.488	0.006	0.620

Notes: OLS estimates of Equation (12). Baseline categories are: NACE 32 (Other manufacturing) for sector dummies; transition 2016/2017 for the transition dummies; France for country dummies. Bootstrap standard errors in parenthesis. Asterisks denote significance levels: $^{*}p < 0.05$, $^{**}p < 0.01$.

Country dummies account for 27.5% of the variance of the Shorrocks indexes and for 12.6% of the variance of the Bartholomew index. Sector dummies capture more: 35% of the variance of the Shorrocks indexes and 48.8% of the variance of the Bartholomew indexes.

Overall, the analysis of sector-level CQTPMs supports that sectoral specificities stand out as the main source of deviations of intra-distributional persistence from the null of independent firm growth rates.

6 Sectoral determinants of growth persistence

The primary role of sector specific factors emerged from the analysis of sectoral CQTPMs, suggests the presence of a relevant relation between firm growth persistence and the economic characteristics of sectors. In this Section we further explore this relation.

We relate sectoral persistence, as measured by standardised indexes, to a set of sectoral variables which are plausibly linked to patterns of firm growth and industry dynamics. Specifically, we consider profitability, productivity, market concentration, business dynamism and openness to international markets. Profitability is defined as gross operating margins over total sales. Since we study manufacturing sectors, focusing on profits from operating performance excluding the effect of financial assets and liabilities seems more appropriate. Data are taken from the Structural Business Statistics (SBS) database maintained by EUROSTAT. We define productivity as labour productivity (LP), measured in terms of real value added per hours worked, available at sectoral level from the EU-KLEMS database. As a measure of concentration, we take the standard Hirschman-Herfindahl index, computed on sales of the firms active in the same 2-digit sector in ORBIS (by year and country). Business dynamism is proxied via the churning rate, defined as the sum of firm birth and death rates per year in each sector, that we collect from EUROSTAT-SBS. Finally, we define openness as the ratio between the number of exporting firms and the total number of firms in a sector. Figures to compute this ratio are taken from EUROSTAT-Trade by Enterprise Characteristics (TEC) and EUROSTAT-SBS, respectively for the numerator and the denominator. Table 6 summarizes the definition of variables, their sources and coverage. Coverage varies according to whether it was possible to find complete information for all the 600 sector-country-transition combinations spanned by the definition of CQTPMs. The only problematic variable is Openness, for which we have only 280 observations, due to the limited number of 2-digit sectors included in the TEC database.

The signs to be expected in the relationships between firm growth persistence and the sectoral characteristics considered, are not all completely clear a-priori, in particular for profits, productivity and openness. If high profitability levels in a sector are interpreted as a signal of market power, then high barriers to entry or the incumbents' ability to hamper competition should stabilize relative firm growth rankings and keep persistence higher, compared to low profitability sectors. On the other hand, high profitability in a given sector may indicate attractive investment/profit opportunities, which may be accompanied by substantial entry attempts and, as a consequence, increased turbulence and reduced persistence in growth rates. The relation with productivity is equally difficult to predict. In fact, high productivity sectors are environments where competitiveness is on average high, but this may lead to opposite predictions. On the one hand, since performing better than the average is difficult in such environments, one could expect relative growth and market shares to be more stable compared to low-productivity sectors. On the other hand, firms in highly competitive environments are arguably subject to stronger selective pressures, which is likely to increase turbulence. Concerning openness, one has to consider that involvement in international markets is both an opportunity and a threat to firms. Accessing export markets may help to sustain sales growth, especially when the domestic market is stagnant, and induce more stability in growth rates. At the same time, however, firms operating in more open sectors are also increasingly subject to

Table 6: Sectoral characteristics

Variable	Source	Description	#Obs
Profitability	EUROSTAT-SBS	Gross operating surplus over sales turnover (%)	597
Labour Productivity (LP)	EU KLEMS	(log of) Value added per hours worked by persons engaged	600
Concentration	ORBIS	Herfindahl-Hirschman Index x 100	600
Business dynamism	EUROSTAT-SBS	Sum of birth and death rates of firms (%)	585
Openness	EUROSTAT-SBS and TEC	Exporting firms over total firms (%)	280

Notes: Sectoral variables are defined at 2-digit NACE level and by country, over the period 2011-2017.

adverse external shocks, and typically face fiercer competition. This may create turbulence in growth dynamics, resulting in a nuanced relation between openness and firm growth persistence.

Sharper predictions seem possible about the role of concentration and business dynamism. We expect persistence in growth rates to be relatively higher in more concentrated sectors, since concentration is a signal of market power, either due to the structural characteristics of the sector or to the anti-competitive behavior of incumbents, which counters changes in relative market shares over time. Lastly, persistence is naturally expected to decrease with business dynamism, since the sectoral turbulence due to entry and exit, by definition, involves instability in relative market shares.

To investigate how these characteristics affect growth rates persistence, we consider a series of regression models where the standardized mobility indexes associated to sectoral CQTPMs are regressed on the sectoral variables. To avoid simultaneity issues, we take lagged the variables. Specifically, the standardized indexes computed over the transition between $t-1$ and t, are regressed against sectoral variables measured in $t-2$

$$\tilde{I}_{c,j,t} = \alpha + \beta\,\mathbf{X}_{c,j,t-2} + \epsilon_{c,j,t}\,, \tag{13}$$

where $\mathbf{X}$ is the vector of sectoral characteristics. Recall that negative values of standardized indexes indicate more persistence than under the null of independent growth rates. Therefore, negative estimates of the β coefficients imply a positive association between sectoral characteristics and persistence.

Preliminary estimates where each variable is included alone in the model (not reported for brevity), reveal that profitability, productivity and business dynamism show a statistically significant and negative association (positive β) with persistence, while openness does not display statistically significant association. These findings emerge irrespective of whether we consider the Shorrocks or the Bartholomew index as the dependent variable. Concentration also negatively associates with persistence, but the correlation is statistically significant only against the Shorrocks indexes.

In Table 7 we examine multivariate specifications, including all the sectoral variables together. The estimates in column 1 report baseline results without country, sector and time dummies. Looking at the Shorrocks index, we see that persistence decreases (positive coefficients) with productivity, business dynamism and openness. The regression with the Bartholomew index confirms a significant and inverse relation (positive β) between persistence and productivity.

We then extend the model by adding fixed-effects, to control for unobserved heterogeneity along the sources of variation in the data. Given our interest in sectoral characteristics, inclusion of sector fixed-effects needs to be carefully considered. Indeed, sector fixed-effects may absorb the statistical significance of the relations, if sectoral characteristics vary mostly across sectors, rather than within sector. In column 2 we only include country and time fixed-effects. Business dynamism and openness display a statistically significant coefficient vis-a-vis the Shorrocks indexes, while openness is the only variable showing a statistically significant association with

Table 7: Firm growth persistence and sectoral characteristics

	All variables			Excl. Openness		
	(1)	(2)	(3)	(4)	(5)	(6)
Dependent: Standardized Shorrocks						
Profitability	-0.023	-0.045	-0.038	0.052	-0.082	-0.141*
	(0.040)	(0.038)	(0.061)	(0.023)	(0.032)	(0.036)
LP	0.258*	0.560	1.996**	0.019	1.854**	1.415**
	(0.085)	(0.443)	(0.602)	(0.066)	(0.327)	(0.382)
Concentration	0.046	0.015	0.012	0.036	0.022	-0.005
	(0.029)	(0.017)	(0.023)	(0.021)	(0.011)	(0.012)
Business dynamism	0.318**	0.144*	0.093	0.277**	0.097	0.078
	(0.055)	(0.063)	(0.079)	(0.036)	(0.039)	(0.061)
Openness	0.055**	0.064**	0.065**			
	(0.015)	(0.016)	(0.025)			
Cons	-11.044**	-8.291**	-13.886**	-8.990**	-11.850**	-10.060**
	(1.240)	(2.054)	(3.104)	(0.539)	(1.651)	(1.893)
Country FE	no	yes	yes	no	yes	yes
Time FE	no	yes	yes	no	yes	yes
Sector FE	no	no	yes	no	no	yes
R^2	0.176	0.436	0.688	0.125	0.324	0.653
Obs	275	275	275	582	582	582
Dependent: Standardized Bartholomew						
Profitability	0.004	0.023	0.010	0.066	-0.018	-0.073
	(0.037)	(0.055)	(0.070)	(0.027)	(0.032)	(0.042)
LP	0.267*	-0.378	2.423**	0.062	0.796	2.173**
	(0.087)	(0.451)	(0.653)	(0.060)	(0.361)	(0.450)
Concentration	0.019	0.001	0.018	0.014	0.003	0.012
	(0.029)	(0.024)	(0.019)	(0.017)	(0.013)	(0.012)
Business dynamism	0.147	0.042	0.207	0.150**	-0.005	0.167*
	(0.065)	(0.086)	(0.097)	(0.038)	(0.062)	(0.054)
Openness	0.035	0.049*	0.059			
	(0.022)	(0.020)	(0.031)			
Cons	-6.873**	-2.170	-16.215**	-5.975**	-5.152	-13.510**
	(1.659)	(2.683)	(3.557)	(0.623)	(2.301)	(2.170)
Country FE	no	yes	yes	no	yes	yes
Time FE	no	yes	yes	no	yes	yes
Sector FE	no	no	yes	no	no	yes
R^2	0.049	0.138	0.662	0.039	0.137	0.645
Obs	275	275	275	582	582	582

Notes: OLS estimates. All models include country, sector and transition fixed-effects. Bootstrap standard errors in parenthesis. Asterisks denote significance levels: * $p < 0.05$, ** $p < 0.01$.

the Bartholomew indexes. The positive coefficients of these variables confirm that they associate with reduced persistence. Since identification works across sectors, the findings are informative on the association between standardized indexes and sectoral variables *in deviation* from their average values computed across industries, within country and transition year. So for instance, the positive coefficient on business dynamism means that the sectors where churning is above the average sectoral churning observed in a country in a given transition, display lower persistence than the average sectoral persistence observed in a country in a given transition.

In column 3 we also add sector fixed-effects. The identification of parameters exploits the deviation of indexes and regressors from their within-sector specific average, computed within country and transition year. In the specification with the standardized Shorrocks indexes, productivity and openness are the only statistically significant variables. They both associate with reduced persistence. The same conclusion holds for productivity, but not for openness, when considering the regression on the Bartholomew indexes.

In columns 4-6, we perform a robustness check excluding openness from the models' specification. As mentioned, data on openness are missing for about one half of the 600 sector-country-year combinations for which we can compute CQTPMs and associated mobility indexes. The estimates on the other sectoral characteristics might be biased, if the sector-country-time combinations where we can observe openness, are systematically different. The estimation results show that this is not generally the case. We broadly confirm the conclusion from the baseline estimates that productivity and business dynamism display statistically significant association with persistence, while concentration does not.

Overall, productivity, business dynamism and openness to trade stand out as the variables with more stable patterns of statistical significance. They all tend to display an inverse relation with firm growth persistence.

7 Discussion and conclusion

We have applied CQTPMs and related mobility indexes to draw precise inference on persistence in intra-distributional dynamics of firm growth rates, exploring its determinants across sectors and countries. The analysis is based on a sample of manufacturing firms active in four major European economies and the UK over the period 2010-2017.

Our first and main finding is that, although there is more persistence than under an independent growth process, a good deal of intra-distributional mobility characterizes firm growth dynamics. This result contributes to the long-standing debate about the validity of Gibrat's classical model of firm growth and the "illusion of randomness" (Henderson et al., 2012; Derbyshire and Garnsey, 2014; Coad et al., 2015). In fact, strictly speaking, our analysis supports a rejection of any model of firm growth based on independent growth shocks. At the same time, however, our evidence conflicts with theories predicting high stability in growth rates rankings, induced by fitter firms experiencing sequences of positive growth events and less fit firms continuously shrinking over time. In this respect, our results resonate with previous evidence that growth rates are, if not totally erratic, at least quite difficult to predict (even with machine learning algorithms, see Coad and Srhoj, 2020).

As a qualification of the general finding of considerable turbulence in intra-distributional dynamics, CQTPMs display more persistence for relatively fast-shrinking and fast-growing firms. Persistence in the top of the growth rates distribution is supportive of the attention that high-growth firms receive in the literature. However, it is precisely in the extreme deciles that we also find evidence of anti-persistent, bouncing effects, entailing that firms in the extreme deciles are likely to experience large jumps to the opposite extreme deciles of the distribution. These episodes of extreme volatility and reversal document a good deal of lumpiness in growth processes, whereby large (positive or negative) adjustments tend to be followed by periods of relative inaction. Previous studies on firm growth TPMs relate this particular dynamics in extreme quantiles to the relative abundance, in those quantiles, of small and hence more volatile firms. Our analysis of CQTPMs show that such dynamics represent a pervasive property of the growth process, which is still present even after conditioning on firm size (and also on country, time and sector). These patterns also relate to the stylized fact that growth rates distributions exhibit thick tails. Frequently occurring large growth shocks (positive or negative) are not just due to the presence of a fixed set of top and bottom performing firms. They also result from significant intra-distributional mobility, volatility and bouncing effects.

From a policy perspective, the considerable turbulence in firm growth patterns that we document, is hardly good news for policies targeting the growth of specific groups of firms and aimed to achieve long-lasting effects. As there are generally low chances that firms growing in a given period will steadily grow over time, growth policies are likely to have a volatile and transient effect. In particular, the bouncing effects observed in the top deciles, lend additional support to previous studies showing that high-growth firms are often "one-hit wonders". Conversely, instability of growth rates rankings over time is good news for anti-trust policies. In the sectors and countries analyzed, there seems to be no serious concern for a tendency toward strong cumulative growth, potentially leading to excess dominant position in markets. At the same time, however, the growth process does not appear to naturally contribute to a gradual reduction of market concentration.

Our findings also highlight the role of sectoral factors as drivers of firm growth persistence. Previous studies have provided qualitative evidence on variation of TPMs properties at the aggregate country level or at the level of specific sectors, typically without conditioning on size. Our multi-country, multi-sector analysis of CQTPMs, simultaneously conditioning on firm size and time-, country- and sector-specific factors, reveals that sectoral specificities explain considerably more variation than country-specific and time-specific factors do. That is, growth rate persistence does not primarily depend on country context and institutions, nor does it vary significantly with specific year-by-year contingencies. Rather, it correlates significantly with some structural characteristics of sectors, such as productivity, business dynamism, and openness to trade. This provides an initial basis to inform about how industry performance and dynamics may interplay with policies targeting firm growth persistence, in case the latter is seen as a target for policy.

There are several extensions of the analysis which we did not consider in this study, mostly due to the characteristics of the available data, yet which seem particularly promising for future research.

Firstly, our methodology could be extended to consider further potentially confounding factors that may create spurious persistence in intra-distributional dynamics. Conditioning on the sector, country, and time effects, and on firm size –as we have done here– seemed a natural starting point. Growth-size dependencies play a prominent role in firm growth theory and empirics, while sector, country and time effects are obvious sources of dependence in growth patterns. Future research could include additional conditioning. The analysis in Coad et al. (2018), although the authors do not examine CQTPMs, suggests that firm age could be a particularly interesting candidate. But any other firm-level characteristics potentially related to firm growth –such as efficiency, innovativeness, or access to finance– could be easily integrated into the procedure to obtain conditional matrices. Also, with larger data (e.g. covering the entire population of firms in a country) one could take a more fine-grained disaggregation of sectors to compute the CQTPMs. This would enable to focus on the intra-distributional dynamics occurring across firms that can be more reliably assumed to compete in the same market, compared to the 2-digit sectors we have used here. The CQTPMs framework we developed is flexible enough to be adapted to all these cases. The only caveat is that more and more data points are required whenever an additional conditioning is applied.

Another interesting avenue for further research relates to the time scale over which intra-distributional dynamics are studied. Given the relatively short sample period available, we mainly focused on yearly growth rates and one-year transitions and just examined a robustness check confirming our main results for 3-years growth rates. However, with longer-in-time data, our methodology could be exploited to explore persistence over longer transitions (e.g., 5-year or 10-year growth) and/or over longer time horizons (e.g., over several decades). Theories of firm-industry dynamics that reject independence, predict stronger persistence in relative growth over the short-run, due to capabilities and firm-specific attributes being relatively slow-changing variables. Yet, those models abstract from a clear definition of the time scale over which their predictions unfold. For instance, do firms' structural advantages apply over yearly growth rates or over longer time scales, such as 3-5 years of growth? How long is the short-run? 5 years or perhaps a decade? By gathering empirical evidence on growth persistence over different time scales, one could verify over which time scales the theoretical predictions are more consistent with the data. That would help better judge those theories. It would also help to connect the empirics of firm growth persistence with genuinely long-run theories, which predict successive phases of stability and instability in industry dynamics, according to Schumpeterian creative destruction vs. creative accumulation, and over the development of the product life-cycle.

With its limitations and opportunities for further development, this work provides a benchmark framework for future research on firm growth persistence. It highlights the importance of conditional analysis and it shows how to address it, allowing to assess how much persistence remains in the data, once firm growth dynamics is cleaned from the underlying confounding factors. Also, the proposed framework makes a crucial step forward, from a purely qualitative and possibly misleading analysis of TPMs towards formal inference. We applied the new framework to analyze persistence in relative market success as proxied by sales growth, but the same analysis can be easily adapted to study persistence using other proxies of firm size, such as employment or fixed assets. More generally, the proposed methodology could apply in any area of research in firm dynamics, not confined to firm growth studies, as a tool to investigate the extent of intra-distributional persistence in key dimensions of firm performance.

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Appendix

A Number of firms by 2-digit industries

Our choice to compute CQTPMs and associated mobility indexes by 2-digit sectors, is driven by the coverage of ORBIS data across the different countries. In Tables 8-12 we report the number of firms available to compute transitions across yearly growth rates in each country, by 2-digit industries. Coverage varies by country (see also Table 1 in main text). For France, Italy and Spain, at least some of the 2-digit sectors show enough firms to further disaggregate the analysis, e.g. computing CQTPMs and indexes at the level of 3-digit industries. This is not the case for the UK and Germany data, which show considerably lower coverage. In order to keep the same level of sectoral disaggregation across all the countries examined, we choose 2-digit sectors. In any case, as we mention in the conclusions, an interesting direction for future research, in case larger dataset are available, would be to examine more disaggregated definition of industries. Our approach is flexible in this respect.

Table 8: France: # of firms available to compute TPMs by sector/transition

NACE	**[2011/2012]**	**[2012/2013]**	**[2013/2014]**	**[2014/2015]**	**[2015/2016]**	**[2016/2017]**
10	11,243	11,756	9,295	7,871	5,857	4,700
11	1,234	1,255	1,097	1,021	875	803
13	1,167	1,147	982	849	676	560
14	953	952	853	743	596	483
16	2,507	2,494	2,087	1,795	1,279	1,008
17	663	664	656	622	528	450
18	3,516	3,451	2,750	2,254	1,732	1,376
20	1,321	1,328	1,241	1,164	1,025	938
21	219	227	225	227	223	221
22	2,021	1,987	1,847	1,720	1,391	1,201
23	2,134	2,111	1,797	1,590	1,262	1,093
24	488	483	458	435	394	350
25	9,347	9,419	8,231	7,337	5,510	4,516
26	1,281	1,278	1,181	1,084	925	800
27	1,050	1,077	999	918	767	675
28	2,645	2,664	2,409	2,192	1,795	1,562
29	981	993	906	828	703	616
30	315	315	306	271	259	239
31	1,580	1,549	1,172	991	742	598
32	3,393	3,361	2,541	1,999	1,499	1,211

Table 9: Germany: # of firms available to compute TPMs by sector/transition

NACE	**[2011/2012]**	**[2012/2013]**	**[2013/2014]**	**[2014/2015]**	**[2015/2016]**	**[2016/2017]**
10	904	1,128	1,581	1,757	1,669	1,431
11	160	187	299	353	344	300
13	231	294	470	586	526	450
14	132	147	206	240	220	183
16	464	606	1,241	1,628	1,560	1,405
17	231	261	377	477	456	371
18	694	901	1,416	1,582	1,446	1,243
20	476	578	822	987	958	806
21	154	192	250	280	258	212
22	620	811	1,310	1,692	1,628	1,410
23	584	732	1,238	1,467	1,365	1,112
24	318	406	551	697	668	564
25	2,115	2,738	4,964	6,481	6,252	5,646
26	842	1,049	1,681	2,002	1,896	1,623
27	568	707	1,111	1,425	1,379	1,189
28	1,663	2,016	3,269	4,205	4,047	3,545
29	258	297	410	511	469	396
30	126	145	231	271	255	228
31	242	309	602	773	748	649
32	696	935	1,506	1,767	1,623	1,343

Table 10: Italy: # of firms available to compute TPMs by sector/transition

NACE	**[2011/2012]**	**[2012/2013]**	**[2013/2014]**	**[2014/2015]**	**[2015/2016]**	**[2016/2017]**
10	6,828	7,225	7,626	8,047	9,861	11,045
11	1,068	1,108	1,157	1,237	1,349	1,424
13	3,179	3,288	3,406	3,507	3,904	4,126
14	3,699	3,840	4,029	4,293	5,423	6,346
16	2,748	2,903	3,016	3,157	3,893	4,321
17	1,558	1,607	1,644	1,694	1,813	1,881
18	2,888	2,987	3,117	3,245	3,678	3,976
20	2,470	2,582	2,649	2,742	2,903	2,994
21	421	436	447	460	473	492
22	4,108	4,257	4,399	4,562	4,867	5,069
23	4,716	48,58	5,001	5,143	5,666	5,976
24	1,433	1,467	1,514	1,558	1,654	1,712
25	17,558	18,291	19,115	20,029	22,715	24,331
26	3,030	3,128	3,216	3,347	3,549	3,659
27	3,509	3,635	3,771	3,910	4,225	4,427
28	10,228	10,614	10,971	11,398	12,151	12,610
29	1,101	1,120	1,160	1,197	1,304	1,361
30	1,143	1,192	1,203	1,264	1,405	1,509
31	3,421	3,558	3,686	3,883	4,460	4,810
32	3,061	3,182	3,320	3,488	4,387	5,075

Table 11: Spain: # of firms available to compute TPMs by sector/transition

NACE	**[2011/2012]**	**[2012/2013]**	**[2013/2014]**	**[2014/2015]**	**[2015/2016]**	**[2016/2017]**
10	8,153	8,279	8,484	8,742	8,811	8,686
11	1,991	2,047	2,152	2,308	2,429	2,403
13	2,028	1,976	2,006	2,043	2,071	2,033
14	1,668	1,641	1,681	1,718	1,748	1,703
16	3,251	3,161	3,168	3,283	3,288	3,194
17	912	913	925	947	966	944
18	4,599	4,529	4,558	4,684	4,706	4,585
20	2,155	2,171	2,229	2,295	2,355	2,302
21	279	286	281	297	295	293
22	2,338	2,357	2,390	2,426	2,469	2,469
23	3,680	3,566	3,585	3,607	3,673	3,551
24	1,179	1,167	1,191	1,204	1,218	1,208
25	11,761	11,621	11,682	11,990	12,199	12,021
26	847	866	881	919	946	913
27	1,110	1,101	1,124	1,132	1,148	1,128
28	3,633	3,695	3,756	3,831	3,930	3,861
29	1,027	1,027	1,051	1,098	1,120	1,086
30	334	341	340	357	375	372
31	3,346	3,243	3,188	3,237	3,257	3,163
32	1,601	1,595	1,666	1,742	1,784	1,764

Table 12: UK: # of firms available to compute TPMs by sector/transition

NACE	**[2011/2012]**	**[2012/2013]**	**[2013/2014]**	**[2014/2015]**	**[2015/2016]**	**[2016/2017]**
10	827	899	922	964	976	1,016
11	157	172	165	169	180	194
13	215	212	224	216	200	198
14	160	155	159	153	145	142
16	213	209	210	205	199	189
17	237	258	261	258	255	247
18	447	428	419	420	377	348
20	558	573	595	606	615	617
21	222	231	228	238	247	255
22	555	564	593	602	582	600
23	262	260	261	263	248	241
24	246	259	264	266	257	263
25	1,192	1,214	1,246	1,249	1,184	1,170
26	604	609	630	629	627	631
27	476	484	489	505	489	491
28	853	867	875	861	853	835
29	252	254	260	265	268	281
30	188	190	198	201	198	199
31	249	255	262	254	245	241
32	1,311	1,335	1,387	1,383	1,341	1,312

B Country-level CQTPMs, all years

For completeness, we report the country-level CQTPMs computed by averaging the sectoral CQTPMs conditional on size computed in the different countries for each transition. As one can see in Figures from 5 to 9, these CQTPMs are very similar to those computed for the 2016/2017 transition and reported in the main text. In particular, transition probabilities are relatively higher in the corner cells, suggesting some more persistence coupled with bouncing, anti-persistent effects in the top and bottom deciles of firm growth.

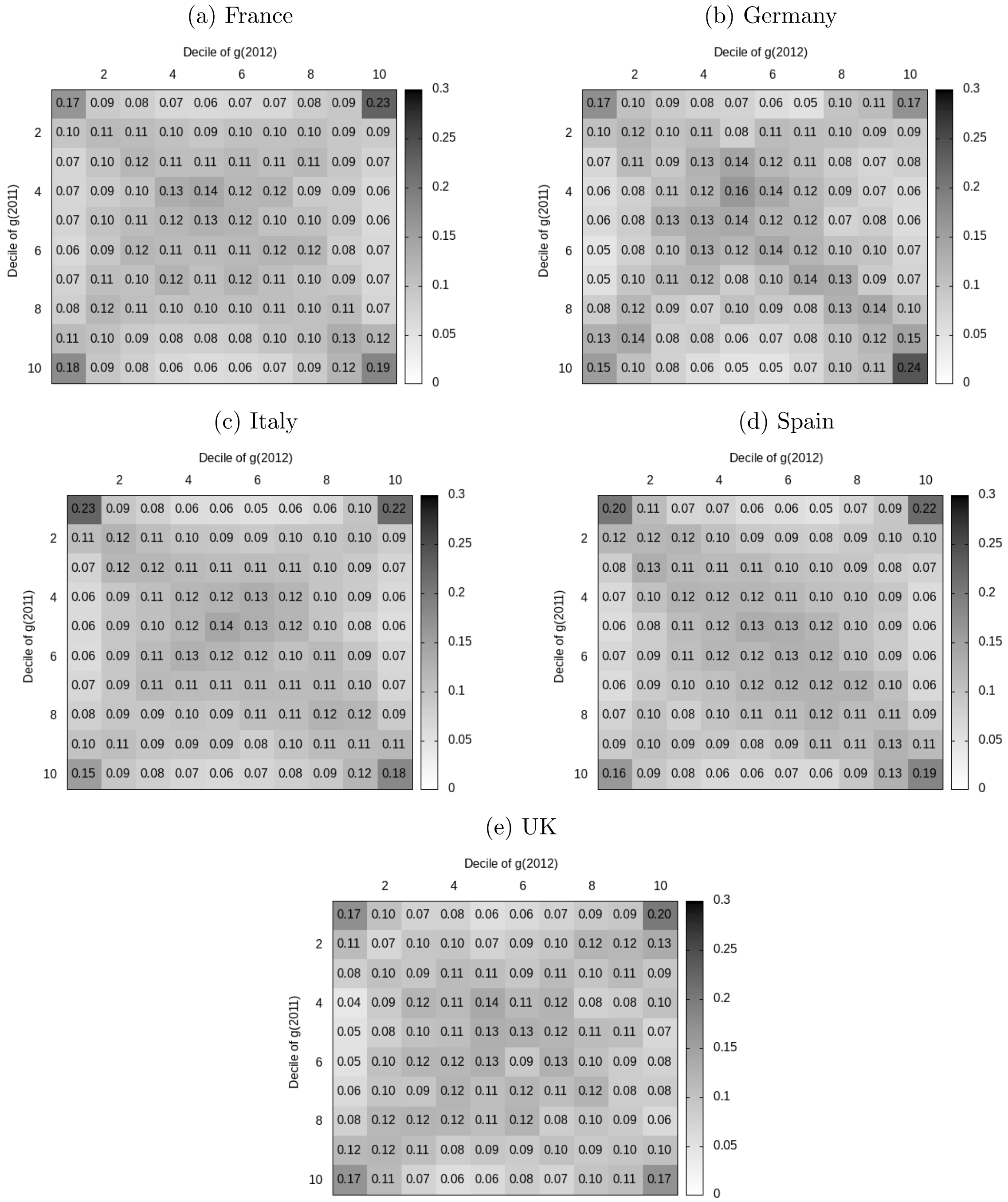

Figure 5: CQTPMs across growth rates deciles – selected countries, 2011/2012 transition.

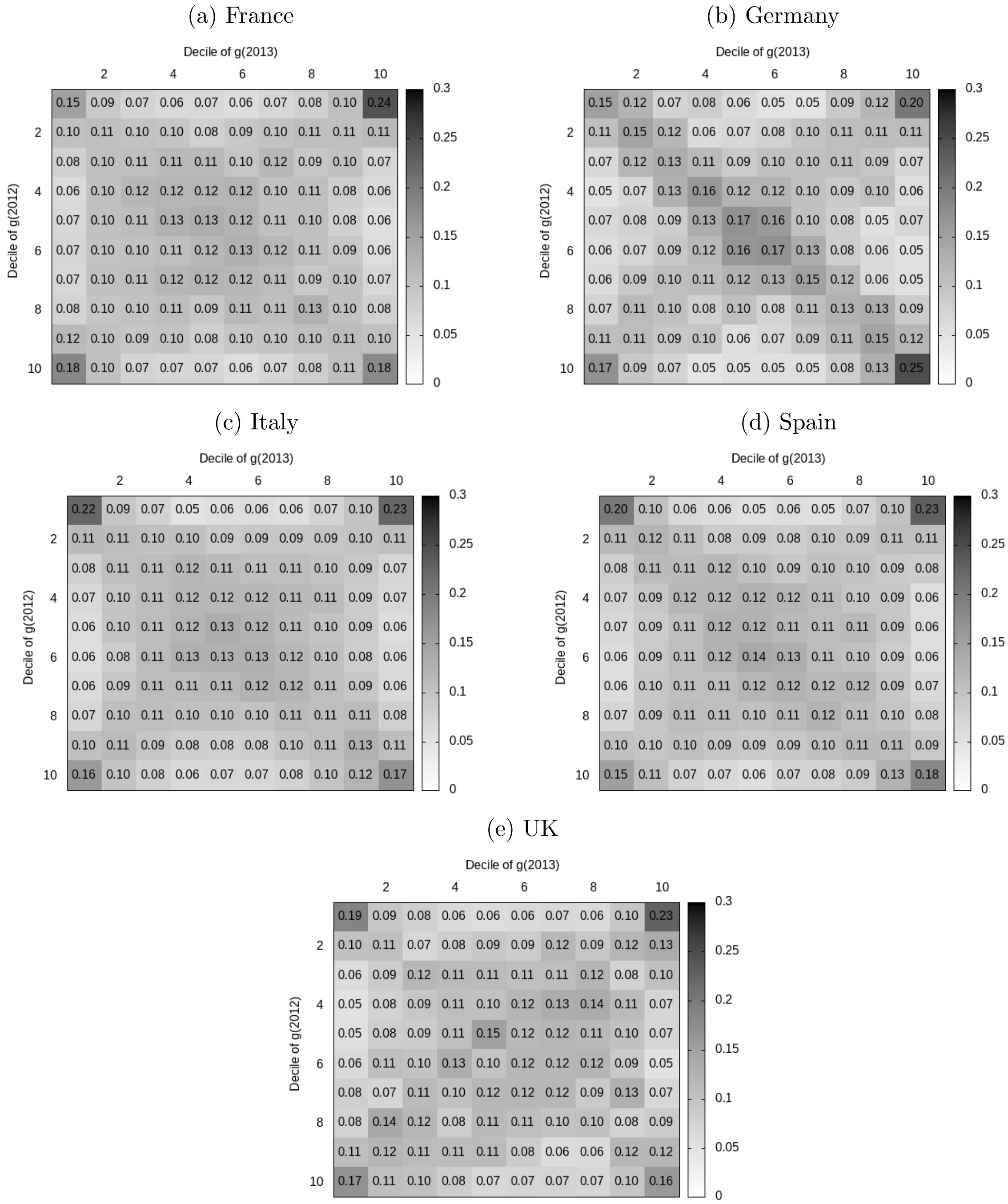

Figure 6: CQTPMs across growth rates deciles – selected countries, 2012/2013 transition.

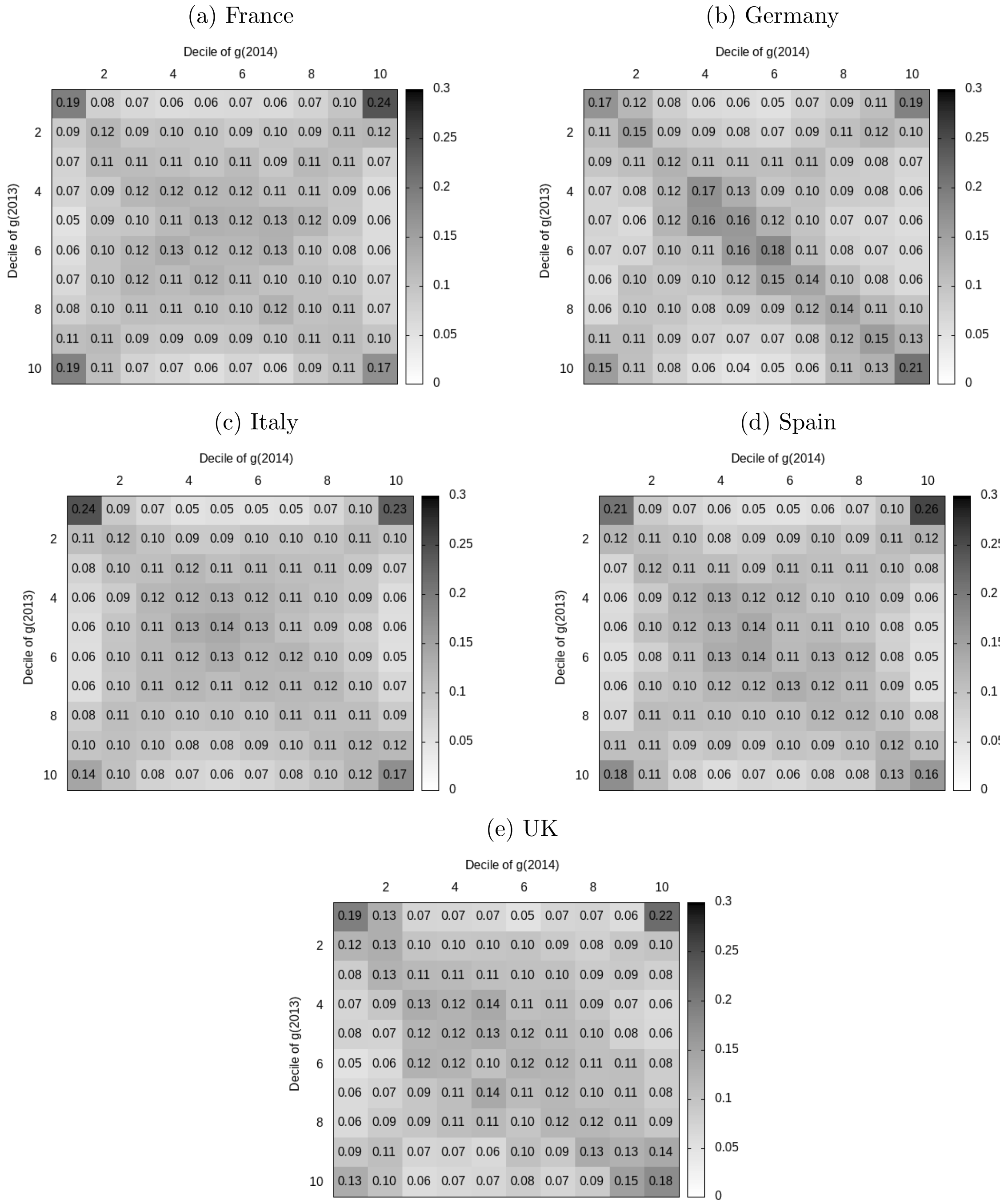

Figure 7: CQTPMs across growth rates deciles – selected countries, 2013/2014 transition.

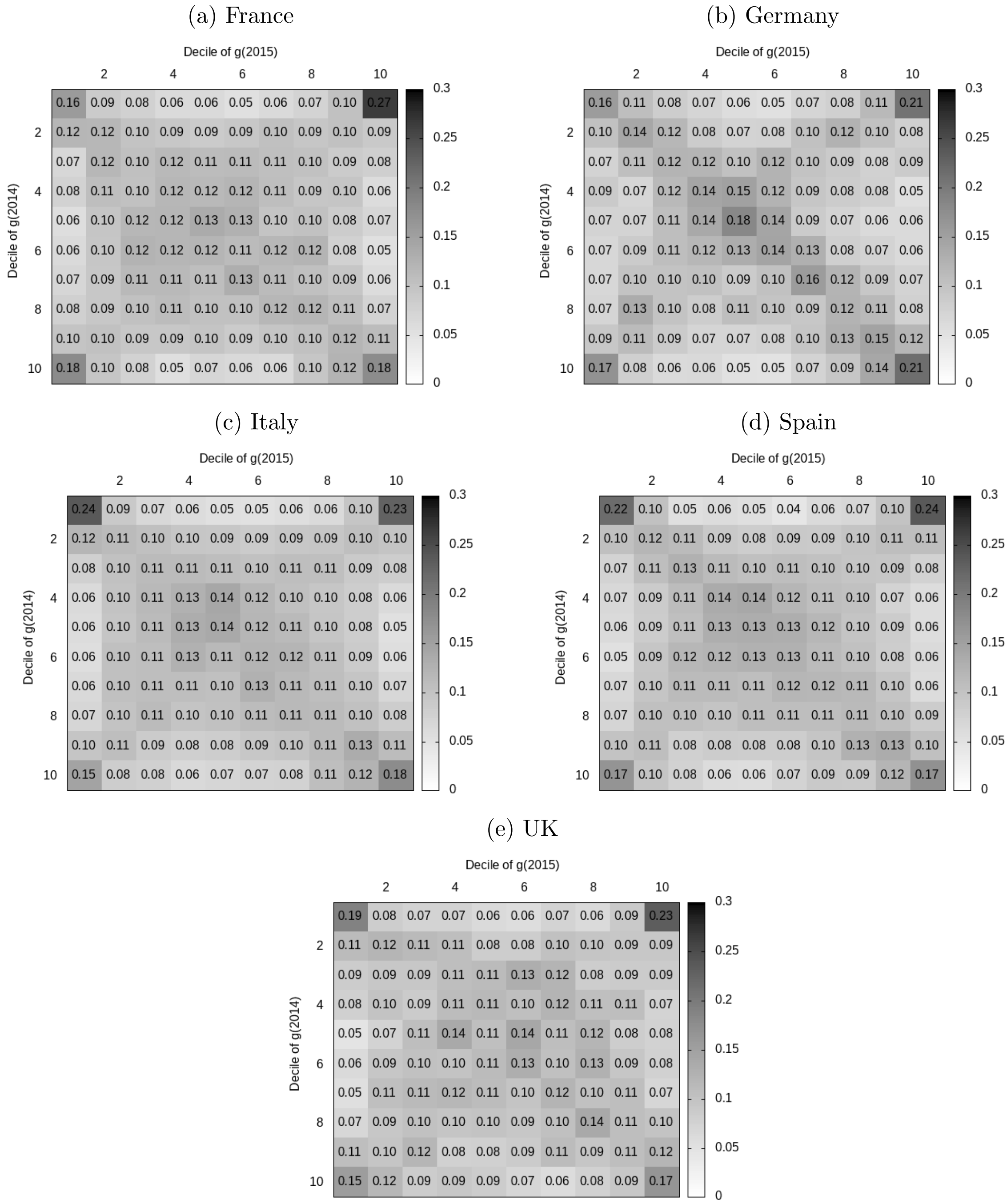

Figure 8: CQTPMs across growth rates deciles – selected countries, 2014/2015 transition.

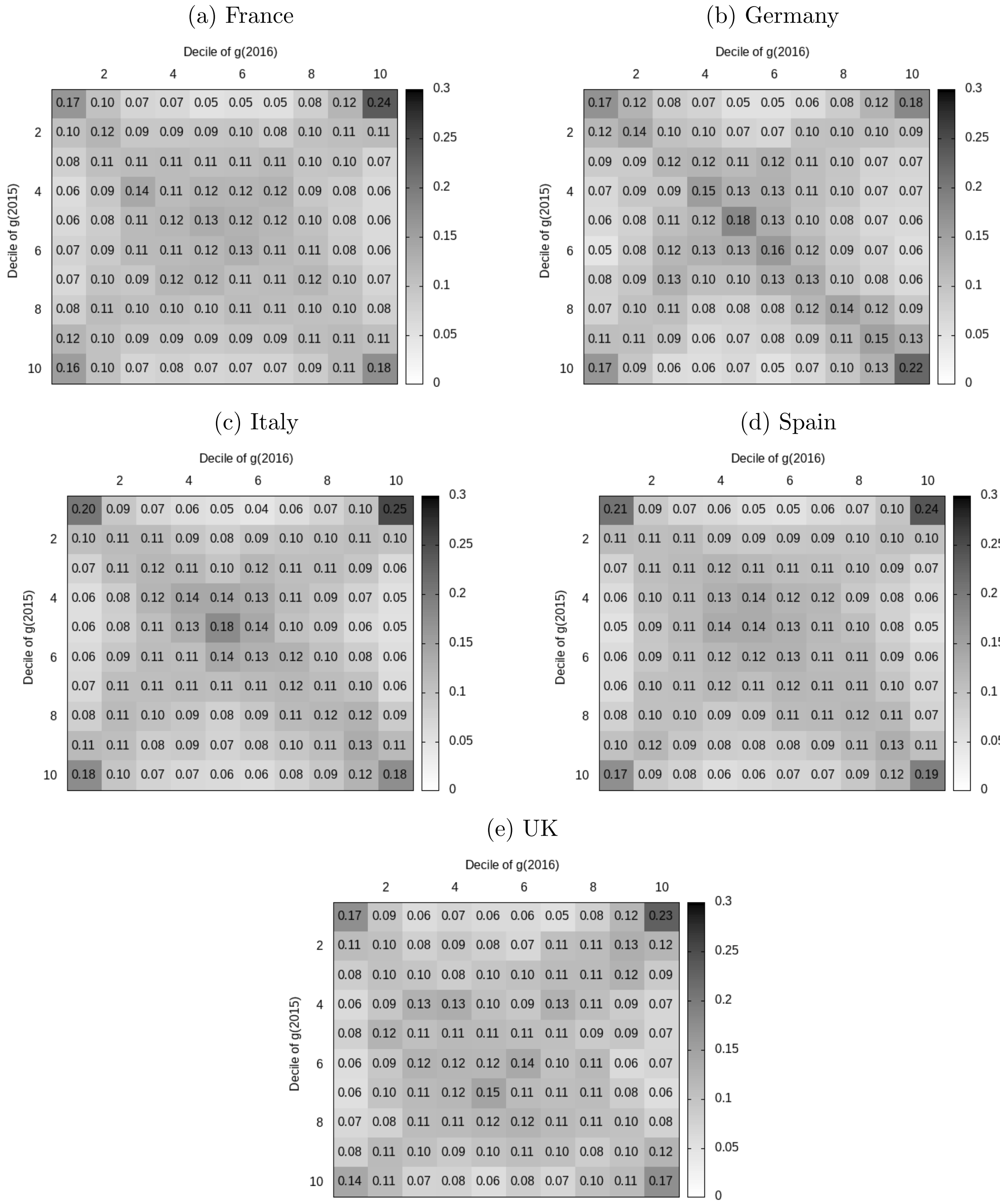

Figure 9: CQTPMs across growth rates deciles – selected countries, 2015/2016 transition.

C Homogeneity Test

In this work we compute and report CQTPMs referring to separate 1-year transitions over the years covered by the data. Computing CQTPMs on data pooled over different transitions is legitimate only if the same underlying process governs all the transitions. As noted in the main text, we verified that this is not the case in our data, via a standard χ^2 test of homogeneity.

Let $n_{i,j,t}$ be the number of observations in state i at date $t-1$ and in state j at date t and $n_{i,t}$ the number of observations in state i at date $t-1$. The estimated transition probability at date t reads $P_{i,j,t} = n_{i,j,t}/n_{i,t}$ while the transition probability estimated pooling the observations over two consecutive dates is $P^{(2)}_{i,j,t} = (n_{i,j,t} + n_{i,j,t+1})/(n_{i,t} + n_{i,t+1})$. For each country and each date we compute the quantity

$$Q_2(t) = \sum_{\tau=t}^{t+1} \sum_{i=1}^{Q} \sum_{j=1}^{Q} n_{i,\tau} \, \frac{(P_{i,j,\tau} - P^{(2)}_{i,j,t})^2}{P^{(2)}_{i,j,t}} \ . \tag{14}$$

Under the hypothesis of homogeneity, $Q_2(t)$ follows a χ^2 distribution with $Q(Q-1)(T-1)$ degrees of freedom, where Q is the number of states (10 in our case, since we use deciles) and $T = 2$. The p-values obtained under the null are reported in Table 13. The hypothesis of homogeneity cannot be rejected at a significance level of 0.01 only in the case of French firms. We also perform the test over all transitions at once, considering the quantity

$$Q_6 = \sum_{\tau=1}^{6} \sum_{i=1}^{Q} \sum_{j=1}^{Q} n_{i,\tau} \, \frac{(P_{i,j,\tau} - P^{(6)}_{i,j})^2}{P^{(6)}_{i,j}} \ , \tag{15}$$

where the pooled transition matrix is now defined as $\hat{P}^{(6)}_{i,j} = (\sum_{t=1}^{6} n_{i,j,t})/(\sum_{t=1}^{6} n_{i,t})$. Under the hypothesis of homogeneity, Q_6 has the same distribution of $Q_2(t)$ but with $T = 6$. The p-values obtained under the null foe the different countries are reported in the last row of Table 13. Also in this case the hypothesis of homogeneity is rejected at a confidence level of 0.01 in all countries apart France.

Table 13: Homogeneity test

Compared TPMs	France P-value	Germany P-value	Italy P-value	Spain P-value	UK P-value
[2011/2012] [2012/2013]	0.021	0.000	0.003	0.000	0.011
[2012/2013] [2013/2014]	0.118	0.000	0.326	0.001	0.000
[2013/2014] [2014/2015]	0.916	0.000	0.154	0.000	0.001
[2014/2015] [2015/2016]	0.988	0.000	0.000	0.042	0.000
[2015/2016] [2016/2017]	0.540	0.000	0.000	0.017	0.000
All TPMs	0.339	0.000	0.000	0.000	0.000

D Analysis of 3-year growth rates

Having access to a limited number of years in the data (the period 2010-2017), in our main analysis we consider transitions across yearly growth rates, in order to maximize the number of transitions examined. As a robustness check and also as an example of the flexibility of our approach, we here examine 3-years growth rates.

Table 14: 3-year growth rates - Mobility indexes and Test of the null of independence

	Transition	France	Germany	Italy	Spain	UK
Shorrocks	$[g_1/g_2]$	0.970***	0.955***	0.962***	0.968***	0.966***
		(-16.418)	(-11.227)	(-32.412)	(-22.084)	(-9.490)
	$[g_2/g_3]$	0.970***	0.949***	0.954***	0.955***	0.972***
		(-13.590)	(-22.084)	(-40.062)	(-30.842)	(-7.688)
Bartholomew	$[g_1/g_2]$	0.348***	0.333***	0.343***	0.353***	0.343***
		(-14.202)	(-11.837)	(-28.507)	(-12.747)	(-9.239)
	$[g_2/g_3]$	0.348***	0.333***	0.334***	0.334***	0.352***
		(-11.968)	(-20.911)	(-39.482)	(-31.036)	(-5.649)

Notes: Shorrocks and Bartholomew mobility indexes (standardized values in parenthesis) computed on CQTPMs, by country and transition. Transition measured over 3-year growth rates, defined as: g_1=s(2013)-s(2011), g_2=s(2015)-s(2013), g_3=s(2017)-s(2015). Asterisks refer to an F-test of the null that the CQTPMs exhibit independence, implemented as an assessment of the distance between observed standardized indexes and their expected value under the null (1 for Shorrocks; $0.3\bar{6}$ for Bartholomew). Significance level: $^{***}p < 0.01$.

For each firm, we define the three growth rates g_1=s(2013)-s(2011), g_2=s(2015)-s(2013), and g_3=s(2017)-s(2015). Then, we compute the CQTPMs associated to the transitions g_1/g_2 and g_2/g_3, following the same steps outlined in the main analysis (that is, with $Q = 10$ and conditional to country, sector and quintile of initial size). Table 14 reports mobility indexes computed aggregating by country. In line with results in the main text (cf. Table 4), negative values of standardised indexes reveal that there is more persistence than under the null of independence. The null is in fact strongly rejected in all countries and transitions, for both the indexes.