On Some Problems of Using the Human Development Index in Economic History

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Abstract
The Human Development Index (HDI) has attracted increasing interest among economic historians during the last 30 years. This paper provides a theoretical framework that shows that the HDI is equivalent to a paternalistic social welfare function: this implies that all alternative HDI formulas used by economic historians are a representation of their ethical systems. The problem is neither the choice of the dimensions included in the HDI, nor the choice of the weighting scheme, but the lack of consistency with standard economic theory. A key consequence is that with HDI, ‘anything goes’. Using Italy 1861-2016 as a case study, we show how given the same data, and identical choices for the dimensions and weights defining the HDI, the interpretation of the Italians’ living standard long-run evolution is entirely driven by the analyst’s preferences. We conclude speculating on possible solutions to reconcile the use of HDI to assist historical analysis.

JEL codes: N01, N3, O15

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1. Introduction

Economic historians interested in the evolution of the living standards are used to consider a variety of indicators, both monetary and non-monetary, which are often characterized by diverging, apparently inconsistent trends. A prominent example is the debate on the living standards of the working class during the British industrial revolution (Taylor 1975): real wages rose (how rapidly is still debated) but a number of other dimensions deteriorated (the condition of women and children, urban crowding, pollution, the consumption of leisure time). Another example is the US ‘antebellum puzzle’, when rapid industrialization and modern economic growth during the antebellum decades, was accompanied by a decline in nutritional intake and heights of the male US population (Steckel and Floud 1997). Similarly, during the interwar years, two global conflicts and the Great Depression led to historically-low growth rates in per capita GDP, but at the same time, longevity, leisure and income inequality improved at unprecedented rates (Gallardo-Albarrán 2019). These are only a few examples where economic historians are challenged by the problem of accounting for the multidimensionality of the standard of living, and by the empirical intricacies that come with its measurement.

One strategy to deal with the complexity of the multivariate nature of historical evidence is to adopt a ‘dashboard’ approach (Ravallion 2012b), where the pros and cons of each piece of evidence are discussed, costs and benefits of each indicator are identified and weighed in, and an evidence-based interpretation comes out as a final, necessarily subjective synthesis of the analysis. Composite indices, such as the
Human Development Index (HDI), introduced by the United Nations Development Programme in the early 1990s, represent a second, alternative strategy. The idea behind composite indices is to reduce the multidimensionality stemming from the use of multiple indicators to a scalar and then track its evolution over time. In particular, the need of ‘objective’ analytical tools, able to provide scholars with non-ideological answers to the aforementioned puzzles has prompted economic historians to pioneer the application of composite indicators of wellbeing – the HDI (Crafts 1997a, 1997b; Costa and Steckel 1997), but also the Biological Standard of Living (equally aimed at “quantify the quality of life as a complex, multidimensional entity”: Komlos 2009: 342) and, more recently, by applying utility-based frameworks borrowed from economics (Jones and Klenow 2016; Gallardo-Albarrán 2019). In the same vein, as part of an effort to go beyond the GDP (Land and Michalos 2018), composite indices have been used to investigate the long-run dynamics of development (e.g., Prados de la Escosura, 2022), as well as by large-scale international comparisons such as How Was Life? (van Zanden et al 2014, and in particular Rijpma 2014).

Recently, prompted precisely by the release of Prados de la Escosura (2022), Jan Luiten van Zanden (2022) noted the need for a careful consideration of the implications of using a synthetic index, specifically in intertemporal comparisons. After an in-depth analysis of the machinery underlying the HDI and its application to historical data, van Zanden’s review resolves into a trenchant conclusion: “Prados de la Escosura has created a highly subjective view of the evolution of the global
standard of living in the period since 1870.” In this paper, we contribute to this debate with a new analytical framework, which is illustrated with an HDI-based reconstruction of well-being in Italy from 1861 to the present day.

The main theoretical finding of our paper is that the HDI can be interpreted not only as a social welfare function, a point already noted by Fleurbaey (2018), but as a *paternalistic* social welfare function (Graaf 1957). This point has two main implications. First, the choice of the indicators – the Gross Domestic Product (GDP), life expectancy at birth, education, political and civil rights, etc. – and the choice of the aggregation rule (which formula to pick for the HDI) is a purely subjective decision of the analyst. This is uncontroversial and empirically relevant. The second implication is more subtle, and goes way beyond the accusation of arbitrariness of the choice of the variables and/or of the weighting system. To establish that the HDI is a *paternalistic* SWF implies that HDI is a welfare measure totally disconnected from individual preferences (Engerman 1997). In other words, the HDI (any of the several available HDIs) cannot be derived by aggregating individuals’ utilities (or well-being measures), unless one imposes on individuals the same preferences of the analyst (Foster and Shneyerov 2000), which means implicitly that individual’s preferences do not matter. This is exactly what the economic historian does when he applies HDI to the data: he acts as the *pater*, and induces an arbitrary ordering directly over a set of social indicators. The nature of the HDI as a paternalistic SWF is not an economist’s theoretical crux: it helps economic historians to identify how
serious is the problem of adopting composite indices of development to objectively ‘solve’ debates on living standards.

While some of the issues discussed above are well known to economists (Fleurbaey and Blanchet 2013; Ravallion 2012a, 2012b), our paper is the first to focus on inter-temporal comparisons, and to conclude that the HDI should be used as an ordinal, rather than cardinal measure: the HDI should be used only to rank countries over space at a point in time, or to rank a single country across time, at odds with most recent applications in the economic history literature.

To elaborate further, we introduce a formula of the HDI based on a constant elasticity of substitution (CES) function. It is well-known that a CES function is regulated by a single parameter, the elasticity of substitution, determining its curvature, that is the extent to which different variables (here the HDI dimensions) can be substituted one for another: in our context, the CES function is applied to the HDI and the elasticity parameter controls over the extent to which GDP and life expectancy, say, are substitutes, and this applies to any pair of its dimensions (education, political and civil rights, etc.). This CES-parameter has nothing to do with usual object of disagreement among scholars working on human development - such as the choice of the dimensions entering the HDI, or the choice of the weight that each dimension receives within the HDI. The choice of the elasticity of substitution uniquely determines a specific formula of the HDI, and is driven simply and solely by the ethical system of the historian. In this sense, the choice of the CES parameter (i.e., of any specific HDIs) reveals the economic historian’s paternalistic preferences and
makes it explicit the trade-offs that come by composing different standard of living indicators into a single measure.

The analytical framework outlined in the paper allows us to come up with an encompassing formula that embeds all the HDIs proposed in the literature. For instance, we show that if the elasticity of substitution is assumed to be infinite (i.e., the variables defining the HDI are perfect substitutes), then we obtain the original HDI put forward by the UN; if the elasticity of substitution is set equal to 1 (imperfect substitution), then we obtain the so called ‘hybrid HDI’ (Gidwitz et al. 2010), the most popular HDI in economic history literature. We then use Italy as a case study to illustrate the ultimate consequence of the paternalistic nature of the HDI: by changing the elasticity of substitution parameter (but not the weighting system, nor any other choice underlying the calculation of the HDI), it is possible to obtain very different temporal trajectories for the HDI. We show, in short, that with the HDI ‘anything goes’. Again, the case of Italy is paradigmatic: even for the Fascist Ventennio, twenty years in which civil and political rights drastically declined, it is possible to reach opposite interpretations (‘well-being of Italians happily improved’ versus ‘well-being decreased dramatically’).

What solutions or alternatives to the HDI? The pars construens of the analytical framework advanced in this paper is admittedly thin: one strategy is that scholars come to an agreement on a specific value (or a subset of values) for the elasticity of substitution underlying the HDI, and agree to stick to it. Does the economic historian have a way to support his choice for a zero elasticity versus an infinite elasticity, or
any values in between? Can he persuade other scholars about the use of convex transformations? Or, as mentioned by van Zanden (2022), can we conceive of “international agreements to limit the impact of the preferences of individual scholars”? This is a possibility that should not be ruled out. A second strategy is to carry out extensive sensitivity analysis on the results – a task greatly facilitated by the CES framework. In the end, however, the real question for economic historians is whether it is preferable to engage and discuss about restrictions of a parametric space (which value should we use for the elasticity of substitution of the HDI components? which indicators? which transformation for each indicator? which goalposts? which weighting system?), or to deal with the often conflicting relationship among well-being dimensions using a more traditional approach.

2. Composite Indices in Economic History

Looking back at the literature of the 1960s and 1970s, one realizes that considerable effort has gone into replacing or supplementing GDP as an indicator of socio-economic development (Hicks and Streeten 1979: 572). Some authors proposed to adjust the GDP, to account for the monetary value of aspects of human development it neglects, such as pollution or longevity. Usher (1973), for instance, proposed a method to adjust GDP per capita for longevity – a route pursued also by Crafts (1997a: 313-318), taking into account both mortality and leisure – while Williamson (1984: 162) proposed a revised version of this last index to correct for the endogeneity of secular improvements in mortality and income. More recently,
Gallardo-Albarrán (2019) and Gallardo-Albarrán and De Jong (2021) investigated the evolution of living standards in interwar Europe and industrial revolution Britain using the utility-based framework developed by Jones and Klenow (2016).

Other scholars chose an alternative route in devising composite indices that, instead of adjusting GDP, aggregated several indicators of development into a single number. After earlier attempts such as the Physical Quality of Life Index (PQLI) introduced by Morris (1979), the United Nations Development Program (UNDP) launched the HDI in 1990. In its original formulation, the HDI was defined as a simple arithmetic mean:

\[
\text{HDI} = \frac{1}{3} I_E + \frac{1}{3} I_S + \frac{1}{3} L_Y
\]

where the terms on the right-hand side stand for life expectancy at birth \((I_E)\), literacy \((I_S)\), and income \((L_Y)\). Each component is normalized between 0 and 1 as follows:

\[
I_x = \frac{(x_t - x_0)}{(x - x_0)}, \quad \text{where } x \text{ and } x_0 \text{ stand for the time invariant maximum and minimum value of variable } x, \text{ respectively (sometimes referred as “goalposts”).}^1
\]

The HDI in equation (1) found simultaneous applications on both shores of the Atlantic, in parallel debates on living standards and industrialization – the so-called

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1 The notation \(L_y\) signals that per capita GDP \((y_t)\) is transformed by applying the natural logarithm to \(y_t, y\) and \(y_0\), to convey the idea of the diminishing returns of income for wellbeing. Initially, UNDP (1990, 12-13) adopted an “average official poverty line income” for nine industrial countries as \(y\), but later reports increased this threshold up to $75,000 (UNDP 2016, 198). The concave transformation has also been expressed in different forms (see UNDP 1994, 108). As noted by Kelley (1991, 317), the choice of any specific transformation is “necessarily arbitrary”.

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‘antebellum puzzle’ in the U.S. (Komlos 2012), and the ‘quality of life’ in Industrial Revolution Britain (Taylor 1975). In 1997, Richard Steckel and Roderick Floud edited an NBER volume, *Health and Welfare during Industrialization*, where a wide array of contributors expressed confidence in the fact that composite indicators would strengthen our understanding and interpretation of history. Engerman (1997: 33) emphasized the advantages offered by these new (composite) indices of welfare, including the PQLI, the HDI, and the Dasgupta-Weale Index (DWI), an index including the dimensions of political and civil rights.

Other authors expressed awareness of the analytical limitations of these tools. Costa and Steckel (1997: 71), for example, stressed that “of particular concern in economic history is the choice of indicators and the selection of maximum and minimum values”; similarly, in the monograph’s epilogue Steckel and Floud (1997: 437) shared their concern about the use of composite indices – a “debatable method” in their words.

In the same year, 1997, Nicholas Crafts led the way to the use of the HDI, in his investigation on the British ‘quality of life’ debate. Crafts (1997a) estimated both the HDI and “its most ambitious cousin”, the DWI, for six benchmark years between 1760 and 1850.² He showed how the steady increase in composite indices was at odds with the gloomy evidence based on height. The comparison between Britain’s HDI- and DWI-based achievements at 1860 with those of eleven other countries,

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² Crafts used a slight variation of equation 1, a formula first proposed in UNDP (1994).
called into question Britain’s ‘leadership’ based on GDP: inter-country rankings differed markedly depending on the measure used. Crafts (1997a: 634) concluded that composite indices could “be even more important for economic historians than for contemporary development economists”. In a second paper, Crafts (1997b: 301) produced estimates of the HDI for 16 industrialized countries, arguing that HDIs offered “a new angle on comparisons of economic progress in different economic eras”, new with respect to those based on real GDP per person.3

Two decades later, while it failed to replace GDP, HDI has become a standard welfare measure in economic history (Prados de la Escosura 2021). It has earned a place in the historiography of every continent – Prados de la Escosura (2013) constructed a series for Africa, Astorga et al. (2005) for Latin America, where Jaramillo-Echeverri et al. (2019) and Devereux (2020) contributed to the historiography of, respectively, Colombia and Cuba, – and in authoritative textbooks (Broadberry and O’Rourke 2010; Persson 2010; Broadberry and Fukao 2021).

Since its first appearance, the HDI has undergone several modifications, often in response to criticism from academia.4 The main reason for dissatisfaction with the original formula in equation (1) is that it implicitly assumes perfect substitutability

3 The index was proposed with two alternatives: together with the one adopted in Crafts (1997a), a second HDI was computed normalizing income, $I_Y$, along the same lines as the non-monetary components, with $y$ set at the US per capita GDP in 1992.

between arguments. In equation (1), one year less of life expectancy is perfectly compensated by an increase of equal magnitude in the schooling index (Desai 1991). Paradoxically, the human development of a modern industrialized economy may be made equivalent to the degree of human development of a population with a zero life expectancy, as long as its citizens – who have not even had the time to go beyond the cradle – are sufficiently educated or wealthy. Perfect substitutability is incompatible with the idea that the components of the index are essential dimensions of wellbeing: by definition, that which is essential cannot be replaced (Sagar and Najam 1998). The latest and most important revision of the HDI was carried out for its twentieth anniversary (UNDP 2010). On that occasion, it was decided to change equation (1) by introducing a geometric mean instead of the simple arithmetic mean:

\[
HDI = \left( I_E^{\frac{1}{3}} \cdot I_S^{\frac{1}{3}} \cdot L_Y^{\frac{1}{3}} \right)
\]

At the same time, GDP was replaced by Gross National Income (GNI), and literacy by an unweighted arithmetic average of children’s expected years of schooling and adults’ average years of schooling (Prados de la Escosura 2021: 949). The change in the functional form of the HDI was acknowledged by economic historians, who however preferred to maintain ‘old’ indicators (literacy, GDP, and life expectancy) (Prados de la Escosura 2015a: 224). The resulting “Hybrid HDI” (Gidwitz et al. 2010) is probably the most popular version currently in use among economic historians.
A notable exception is Prados de la Escosura (2015a). He took up an idea of the Indian development economist Nanak Kakwani, according to which development indicators should reflect the greater difficulty of improvement “as the standard of living reaches progressively higher limits” (Kakwani 1993: 308). To implement this idea, Kakwani proposed a convex transformation of social indicators: 

\[ F_x = \left( \frac{\ln(x - x_0) - \ln(x - x_t)}{\ln(x - x_t)} \right). \]

Applying Kakwani’s transformation to the non-monetary components of the Hybrid HDI, Prados de la Escosura (2015a) proposed a new index:

\[ HIHD = F_E^{\frac{1}{3}} \cdot F_S^{\frac{1}{3}} \cdot L_Y^{\frac{1}{3}} \]

Equation (3) attributes increasing value to marginal increases of life expectancy and education, often costlier to achieve, but maintains the traditional decreasing marginal benefits of income, guaranteed by \( L_Y \). More recently, Prados de la Escosura (2021) proposed an “augmented” HDI – the AHDI – that includes a fourth dimension, political freedom, with equal weight. Due to the availability of long-run estimates for average years of schooling (Lee and Lee 2016), in this later work Prados de la

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5 More precisely, Kakwani (1993) introduced the following parametric “achievement function”:

\[ \left( x - x_0 \right)^\varepsilon - \left( x - x_t \right)^\varepsilon \right) / \left( x - x_0 \right)^\varepsilon. \]

The limit of a linear transformation of this function, as \( \varepsilon \) approaches 0, coincides with the expression in the text and is fully consistent with the axiomatic structure proposed by the author.
Escosura (2021) adopts years of schooling for the education component, while previous versions of the HIHD had maintained the old set of variables\(^6\).

Back to the HDI literature, Amendola et al. (2017), concerned by the arbitrariness of both convex and concave transformations, proposed a new version of the index that aggregates directly the three normalized components using a geometric mean, which basically corresponds to a Cobb-Douglas utility function:

\[
HDI = I_E^{1/3} \cdot I_S^{1/3} \cdot I_Y^{1/3}
\]

A similar version had been computed for Latin American countries by Bértola et al. (2012): actually, after a discussion of pros and cons of both the convex and concave transformations \(L_x\) and \(F_x\), Bértola and coauthors estimated eight alternative HDIs, one for each combination between these options – including the possibility of adopting \(F_x\) only on one of the two non-monetary indicators. The indices proposed by Amendola et al. (2017) and by Bértola et al. (2012) are subject to the interpretative limitations of all other HDIs, but do have the virtue of greater simplicity (due to the absence of any non-linear transformation in the original variables). We shall return to this in Section 4 (Table 1).

\(^6\) A relatively different route was adopted by Rijpma (2014): to discuss the long-run evolution of nine wellbeing indicators since 1820, reconstructed under the OECD project *Howe Was Life?*, the author devices two composite indices: first, he takes an arithmetic, equally weighted average (as in the ‘Old’ HDI of equation 1) of the normalized indicators, to which “no further transformations were performed” for the sake of simplicity; in alternative, to our knowledge for the first time in historical applications, he proposed an indicator based on a latent variable (factor) model.
No clear preference for any of the HDI formulas has emerged among economic historians. In reviewing the literature on interwar Europe, Gallardo-Albarrán (2019: 57-58) notes that HIHD overturns the optimistic interpretations of Millward and Baten (2010) based on the Hybrid HDI, puts aside the ‘newest’ HDI, and turns to the construction of a new index, eventually ‘vindicating’ the optimistic view. A similar strategy was followed by Rijpma (2014), who even rejected the geometric average, going back to the arithmetic mean. Gallardo-Albarrán and De Jong (2021) compare their results to those obtained with the Old HDI by Crafts (1997b). Finally, while Prados de la Escosura (2015a) argues for a convex transformation of non-monetary variable, departing from to the development economics literature, other authors agree on avoiding transformations, even in the case of GDP (Amendola et al. 2017; Rijpma 2014; Bértola and Ocampo 2012). As a result, the economic historian preparing her class on the long-run evolution of wellbeing, in the world or in a single country, would have hard times choosing which indicator to follow. The choice is not a trivial one: as discussed in the next section, different HDIs end up producing very different economic histories.

3. Italian Living Standards through the Lenses of the HDIs

Italian economic historians also explored the use of the HDI, with the aim of achieving a better understanding of the long-run dynamics of welfare in the country. Brandolini and Vecchi (2013) compared the long-run evolution of Italian living standards to a selection of other countries by means of the HIHD, while Felice and
Vasta (2015) constructed new regional estimates of the Hybrid HDI to investigate the roots of the Southern Question. Amendola et al. (2017) focused on the difficulties of adopting the HDI in interpreting Italian development, and based their analysis using the HDI in equation (4). In general, the fact that different authors used different versions of the HDI prompts two questions. First, do different aggregation rules lead to different results? Second, did the HDI succeed in identifying the welfare trend of the Italian population during the last 150 years?

In Figure 1, adopting the new evidence on literacy, longevity and per capita GDP (Vecchi 2017), we calculate four alternative HDIs for Italy, following the formulas reviewed in section 2: the Old HDI (equation 1), the Hybrid HDI (equation 2), the HIHD devised by Prados de la Escosura (2015a) (Equation 3), and the Cobb-Douglas HDI in equation (4), which we shall refer to as CD-HDI. Even a cursory glance at the curves in Figure 1 reveals that the series differ greatly – not only in levels, but also in trends. How should we interpret these differences? Which index is the most ‘correct’?

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7 In order to obtain yearly estimates, education indicators have been linearly interpolated. Despite the availability of average years of schooling, we preferred – in line with most of the literature – to stick on the indicators adopted in the Hybrid HDI. Of course, the choice between these indicators (that, in historical applications, should not necessarily follow the arguments adopted in development and policy debates) adds another dimension of uncertainty in selecting the ‘right’ historical HDI; in this paper, however, we will restrict ourselves to those rising from different aggregation formulas.
Consider the difference between the HIHD (blue, dashed line) and CD-HDI (red, broken line). They proceed hand in hand until World War II, when they diverge in both trend and level. The reason is the different transformations of the components: all possible transformations are at work here, linear, concave, and convex transformations ($I_x$, $L_y$ and $F_x$, respectively). Within the HIHD, in fact, monetary and non-monetary dimensions are treated in opposite ways. While an increase of average income is ‘discounted’ by the logarithm, as in the Old HDI, improvements in education and longevity are magnified by the convex transformation $F_x$. Both
assumptions might be reasonable in historical terms, but they have to be recognized as fully discretionary choices. The issue is that while formulas in Figure 1 do not provide the ‘objective’ answer looked for by economic historians, they are also hidden from view, and hard to assess for readers.

**Figure 2. Italy and the OECD, 1870 – 2007**

![Graph showing human development index for Italy and the OECD, 1870–2007.](image)

Source: Authors’ calculations based on Prados de la Escosura (2010).

Figure 2 elaborates more on the consequences of this last argument, by comparing Italy’s performance (dashed lines) to the OECD average (solid lines): the choice of formula changes not just the level of the index (the geometric mean adopted in the HIHD systematically shifts index values down compared to the arithmetic mean of
the Old HDI), but also changes its dynamics. Regarding levels, Crafts (1997b) noted that the 1870 values of the OECD countries were comparable with those of developing countries today (around 0.4). In contrast, using a geometric formula, the OECD countries of 1870 would fare worse than the Central African Republic (0.352), the country with the lowest HDI in UNDP (2016). Turning to the dynamics, according to the Old HDI, Italy converged with the OECD throughout most of the last century, with an acceleration during the economic miracle of the 1950s, achieving near parity by the 1970s. If we use the HIHD, instead, the lesson to be drawn is the exact opposite: Italy failed to converge for almost 130 years, and managed to recover and align with OECD standards only in the last twenty years. Contrary to the interwar European case discussed by Gallardo-Albarrán (2019: 57-58), this result does not depend on the trade-off between improvement in longevity and schooling in the midst of the Great Depression: even if both GDP and the non-monetary components increased in the long-run, the formulas would produce different convergence stories.

Figures 1 and 2 offer the opportunity to discuss a second issue of great importance. Graphs comparing the HDI series for various countries over time have been created, typically, to study growth rates and convergence patterns between the various areas.8

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8 Crafts (1997a: 310) used the HDI to measure “the speed of development in different eras”; Boyer (2007) discusses the “convergence of living standards”; Prados de la Escosura (2010: 850) asked “whether the human development gap between the ‘core’ and the ‘periphery’ deepened over time” and, eventually, commented on the “absolute gap” and the “rate of variations” (Prados de la Escosura 2015a); for Astorga et al. (2005: 775) the wellbeing of Latin Americans “almost doubled between
This interpretation is clearly based on the *numerical value* of the index. According to the creators of the HDI, however, this is inappropriate. The HDI was created explicitly to *rank* the *relative* performances of the various countries *at a given moment* in time (UNDP 1993: 110). Anand and Sen (1994: 8) did not overlook the possibility of constructing a historical series of the index, but came to the conclusion that “no special significance is attached to the absolute value of the index, the entire analysis being conducted in terms of the ranking of countries relative to one another”.

This interpretation is fully consistent with the idea that the HDI is a Social Welfare Function that allows to order societal outcomes (Fleurbaey 2018).

To be consistent with the purposes of the original creators of the HDI, the index should thus be interpreted as a purely *ordinal* index, which can be used to create a ranking.  

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1900 and 1939, and more than doubled to 1980”; according to Millward and Baten (2010: 253) “HDI showed signs of convergence within Europe during the interwar period”; for Baines et al. (2010: 399) “the average HDI score for Europe rose by almost 30 percent between 1950 and 2003”; Bértola and Ocampa (2012: 41-43) discuss the “narrowing gap” between Latin America and developing country, and the different “rate of advance” implied by different HDIs.

In the economic history literature, we find explicit acknowledgment of this fact only in Baines et al. (2010: 399), according to whom the HDI is a “relative”, or “comparative” measure of development; more implicitly, Bértola and Ocampa (2012: 43) present their HDIs as “relative indices” of human development. In both cases, they also rely on absolute variations of the index (see footnote 8). Herrero et al. 2012: 250, quoted by Prados de la Escosura 2021: 959) note that the use of the geometric mean gives the index “a true cardinal dimension and then allows performing comparisons on how much the human development has changed”. In fact, they show that an HDI based on the geometric mean satisfies the “ratio consistency axiom”, which means that the relative values of the indices of two social states does not depend on the value of a common component. Their argument does not exclude,
This consistency requirement poses, however, serious limits to its use in economic history. When looking at the progress made by a country over time, as in Figures 1 and 2, the HDI – in any of its variants – tells us that the 1950s were better than the interwar years, which were mostly better than the 19th century. However, we cannot tell by how much, nor compare the different speeds at which the HDI increased, since the very definition of a growth rate is based on a cardinal interpretation of the HDI. For most of their history as a unified country, Italians have improved their standards of living. Although not completely mundane as a historical insight, this is unfortunately all that Figure 1 would allow us to say.\textsuperscript{10} Turning to international comparisons, according to Figure 2, we can simply conclude that Italians have always experienced a lower degree of wellbeing, on average, than their contemporaries in northern Europe or in North America. We cannot say by \textit{how much}, and thus replicating (as for instance in Prados de la Escosura 2015a), the convergence analysis carried on by means of the GDP: when indices are ordinal ones, the vertical distance between the two curves in Figures 1 and 2 has no meaningful interpretation. The cardinal interpretation of the HDI, however, is by far the dominant one in economic history. While not intrinsically incorrect, it testifies to the

\textsuperscript{10} More interesting, from the interpretative side, is the exercise of decomposing the HDI into contributions made by each of the three components: see Amendola et al. (2017: 470) and Prados de la Escosura (2021: 968-969).
distance between its practical application in economic historical analysis and its intellectual origins in Sen’s capability approach.

4. The HDI as a Paternalistic Social Welfare Function

This section provides a conceptual framework for composite indices, and in particular the HDI: some of our arguments are not new to the economic literature, but are worthy of reexamination when transposed in the historical setting pertaining to economic historians.

Two alternative interpretations of the HDI are possible: i) an interpretation consistent with Sen’s (1985) “capability approach”, whereby aggregating health, education and income dimensions is a way of empirically capturing the complex relationship between functionings and capabilities; ii) a welfarist interpretation, where health and education represent an extension of the dimensions of individual well-being otherwise encapsulated by the dimension of GDP alone.

The first interpretation seems most coherent with the use of the HDI in economic history. Prados de la Escosura (2018) notes that the HDI originates precisely from the attempt to operationalize Sen’s capability approach.11 In fact, there are a number of criticisms of the ability of the HDI index to capture the full scope of the methodology proposed by Sen (Sagar 1997). According to Fleurbaey and Blanchet (2013), the HDI index is only “a pale reflection of the general and ambitious

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11 See also UNDP (1999) and Haq (2003).
methodology proposed by the capability perspective”. Offer (2003, 2006) notes that although Sen has influenced the development of HDI, he has never embodied its approach in any metrics, precisely because “in keeping with Liberal values, he has not privileged any particular good. Even under indigence it was necessary to respect individual priorities.” In the same theoretical framework as Sen, Offer notes that the components “do not lend themselves to aggregation, indexation, or a focal-point summary figure”.

The HDI is a measurement tool based on a theory of social choices characterized by an ambiguous relationship with methodological individualism (Robeyns 2017) and utilitarian consequentialism (Scanlon 2001; Qizilbash 2022), the two pillars of modern economic theory. The possibility of a consistent methodological comparison with the main traditional welfare indicators generally adopted to describe the trajectory of modern economic development, such as the GDP, would therefore not be possible – an argument raised, in economic history literature, by Gallardo-Albarrán (2019: 58). Felice (2016: 975), stressed that the indicators included in the index are among the “remarkable inconsistencies” with the capability

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12 Ravallion (2012b: 2) distinguishes between two broad types of welfare indicators: a first (including the same GDP) in which “the choices of the component series and the aggregation function are informed and constrained by a body of theory and practice from the literature”; and a second, where “the analyst identifies a set of indicators that are assumed to reflect various dimensions of some unobserved (theoretical) concept”, and “neither the menu of the primary series nor the aggregation function is predetermined from theory and practice, but are “moving parts” of the index—key decision variables that the analyst is free to choose, largely unconstrained by economic or other theories intended to inform measurement practice.” The HDI clearly belongs to the second type.
approach: literacy, now abandoned by the HDI (and the AHDI) was “the only indicator easily understandable in terms of capabilities”. An additional concern arises from the application of the HDI to long-term historical data, which assumes that there is a stable relationship between functionings, capabilities and the dimensions of well-being embodied in the index itself, which is not easy to justify from a historical point of view.\(^{13}\)

The second, welfarist, interpretation, although partially at odds with the intellectual origins of the HDI, is the most widespread interpretation (even if often implicit). This is illustrated by the fact that all economic historians, when interpreting their HDI estimation exercises, (a) make use of the classic tools for the standard value theory, such as marginal substitution rates between index components, marginal rates for changes in the index itself and the elasticities in relation to the components, and (b) assess their consistency with individual preference schemes considered to be more or less plausible.\(^{14}\) The inconsistency here is to embrace both the capability approach (at the conceptual level) and the welfarist approach (at the interpretative level).

The fundamental issue is whether the HDI is consistent with the standard value theory. Economic historians are well aware of the subjectivity of the weighting

\(^{13}\) This issue holds true independently from the fact that the weights between the selected dimensions vary over time, as discussed below.

\(^{14}\) This point, discussed further below, was raised by Ravallion (2012a), and then taken up again by Klugman et al. (2011) and Ravallion (2011).
system embedded in any composite indicator (Brandolini and D’Alessio 2009; Decancq and Lugo 2013), but this is an old critique and has nothing to do, as we will show, with our main argument, namely the consequences of the paternalistic nature of the HDI.

It is possible to combine the HDI with modern economic theory by assimilating the HDI to a paternalistic social welfare function (Graaff 1957; Mas-Colell et al. 1995; Yang 2018). When Costa and Steckel (1997) pioneered the use of the HDI to appraise living standards in antebellum USA, they explicitly mentioned social welfare functions: “Economic historians must understand that the HDI is a retrospective index of welfare; which asks how and when modern levels of welfare were attained” (p. 36). Economists know, from Arrow’s (1951) impossibility theorem, that there are no democratic decision rules that can be used to aggregate individual preferences and that satisfy a minimal set of consistency criteria. Sen (1999) observed that dictatorship would avoid inconsistencies in social ordering systems. In this context, the HDI is not an exception: Arrow rules out the possibility that economic historians could come up with an HDI-based ordering of two societies by aggregating individual preferences, unless they use their own subjective judgments, that is, unless they play dictator.

A simple solution to produce social rankings is via Bergson-Samuelson social welfare functions (SWF). Social welfare functions allow the economic historian to rank social alternatives according to a dictatorial criterion and starting from
individual orderings. In the rest of this section we illustrate how the use of SFWs can lead to appreciate the details underlying any histories based on the HDI\(^{15}\).

Assume that individual preferences are defined over the three variables – longevity, schooling, and income – and that they can be described by a standard utility function:

\[
    u_i = u_i(e_i, s_i, y_i)
\]

where the index \(i\) refers to individuals, and \(e_i\), \(s_i\) and \(y_i\) denote life expectancy, educational attainment and income, respectively. A SWF can be defined as follows:

\[
    W = W(u_1(e_1, s_1, y_1), u_2(e_2, s_2, y_2), \ldots, u_n(e_n, s_n, y_n))
\]

where \(W(\cdot)\) is a real valued function that maps individual utilities into real numbers. The shape of the function \(W(\cdot)\) reflects the ethical system of the dictator, and is defined independently of individual preferences.

One might be tempted to interpret the HDI as a SWF. In this perspective, the choice of a specific functional form of the HDI would reflect the ethical system of the economic historian. This is not the case. The arbitrariness imposed by the HDI is stronger than that implied by a proper SWF. To see this, write the HDI as follows:

\[
    HDI = HDI(E, S, Y)
\]

where \(E\) stands for life expectancy, \(S\) for schooling, and \(Y\) for income (e.g., GDP). This shows that the index does not depend on the wellbeing of individuals

\[^{15}\text{This is also the approach adopted by Fleurbaey (2018).}\]
\[ u_i(e_i, s_i, y_i) \], but rather on aggregate indicators \((E, S, Y)\), which in turn can be assumed to depend on individual-level variables: \( E = E(e_1, e_2, \ldots, e_n) \), \( S = S(s_1, s_2, \ldots, s_n) \), and \( Y = Y(y_1, y_2, \ldots, y_n) \). The HDI can therefore be written as:

\[
(6) \quad HDI = HDI(E(e_1, e_2, \ldots, e_n), S(s_1, s_2, \ldots, s_n), Y(y_1, y_2, \ldots, y_n))
\]

Equation (6) shows that the HDI does not qualify as a SWF, the former being defined on social indicators, the latter on individual preferences. Both equations (5) and (6) ultimately depend on individual levels of longevity, but the definition of an SWF requires that each and every individual in the society comes up with an ordering of possible outcomes based on \( e, s \) and \( y \), and then individual orderings are aggregated by the SWF.\(^{16}\) In contrast, equation (6) does not require any individual ordering. This difference explains why, following Graaff (1957), we can interpret the HDI as a \textit{paternalistic} social welfare function (PSWF) – an argument recently advanced by Yang (2018).\(^{17}\) Here the role of the \textit{pater} is played by the economic historian, who

\(^{16}\) One way to make (5) and (6) equivalent is to assume linear individual preferences and an additive \( W(\cdot) \). This is a special case of a Benthamite (or cardinal) social welfare function. On the difference between aggregation at the individual vs. country level, see Atkinson et al. (2002) and Brandolini and D’Alessio (2009). Another option is to adopt a “symmetric geometric aggregation rule”, for individual preference and over aggregated variables, to exploit the Foster and Shneyerov (2000) path independence property. In this case the HDI in equation (6) would not depend on the fact that that aggregation is first carried out over individuals or over variables. However, this “symmetry” restriction on the aggregation rule is equivalent to impose to individuals the analyst’s own preferences, a more sophisticated way to say that individual preferences do not matter.

\(^{17}\) Technically, the HDI is a social welfare function that does not satisfy the \textit{non-paternalism} property, which prescribes that “in the expression of social preferences only the individual preferences matter. The planner does not have direct preferences on the final alternatives” (Mas-Colell et al. 1995: 825).
arbitrarily chooses not only the shape of the social welfare function, but also the implicit system of individual preferences (when one exists).

The fact that the HDI can be interpreted as a PSWF is crucial to our argument, as it formally establishes that any HDI-based welfare ordering is entirely dictated by the economic historian’s own preferences. Once clarified that the HDI is a PSWF, then to say that the HDI is higher in A than in B is no more to say that the economic historian would choose A rather than B, if allowed to make the choice (Graaff 1957: 5). Irrespective of its specific formulation, the HDI is not an objective measure of human development, that could help the historian overcome his ‘preferences’ in evaluating whether, say, during the Industrial Revolution the improvements in material living standards compensated for the worsening of sanitary conditions. Alternative HDI formulas available in the literature do not reflect more or less innovative ‘technologies’, from which one can easily choose the ‘best’; they correspond to different sets preferences or ethical systems – another point already grasped by Costa and Steckel (1997). The choice between, for instance, the Old HDI and the HIHD, is one between different understandings of living standards – in this case, those of Crafts and Prados de la Escosura. After agreeing over the right measure of real wages, to challenge the ‘optimistic view associated with Sir John Clapham and T. S. Ashton’, a modern epigone of Eric Hobsbawm (1963) would not be satisfied by including non-monetary dimensions of wellbeing, but would argue over the right formula to aggregate these dimensions, since alternative formulas embody more ‘optimist’ and ‘pessimist’ views.
Even so, an interesting question remains to be answered: what are the defining characteristics of these ethical systems? In what ways does Prados de la Escosura’s ethical system differ from Crafts’? The marginal rates of substitution (MRS) provide a possible answer (Brandolini and D’Alessio 2009). In the HDI context, the MRS of life expectancy $E_t$ with respect to per capita GDP $Y_t$ is the amount of dollars that one (the economic historian) has to give up when increasing life expectancy by one year, in order to keep the HDI unchanged. To all intents and purposes, the MRS is the ‘exchange rate’ or the relative importance of the population’s average life expectancy compared to average income. Table 1 shows the MRS implied in each of the formulas proposed for the HDI (equations 1-4).

### Table 1. Marginal rates of substitution (MRS) for different HDI specifications

|               | GDP vs. Life expectancy $|MRS_{e_t/y_t}|$ | GDP vs. Schooling $|MRS_{e_t/s_t}|$ | Life expectancy vs. Schooling $|MRS_{e_t/s_t}|$ |
|---------------|-------------------------|-------------|-----------------|---------------------|
| **Old HDI**   | $y_t(\ln y_t - \ln y_0)$ | $y_t(\ln y_t - \ln y_0)$ | $s - s_0$        |
| (eq. 1)       | $e - e_0$               | $s - s_0$               | $e - e_0$        |
| **Hybrid HDI**| $y_t(\ln y_t - \ln y_0)$ | $y_t(\ln y_t - \ln y_0)$ | $(s - s_0)$      |
| (eq. 2)       | $e_t - e_0$             | $s_t - s_0$             | $(e_t - e_0)$    |
| **HIHD**      | $y_t(\ln y_t - \ln y_0)$ | $y_t(\ln y_t - \ln y_0)$ | $(s - s_0)[\ln(\ln(\ln s - s_0) - \ln(s - s_0))]$ |
| (eq. 3)       | $e - e_t$               | $s_t - s_0$             | $(e - e_t)[\ln(\ln(\ln s - s_0) - \ln(s - s_0))]$ |
| **CD-HDI**    | $(y - y_0) / (e - e_0)$ | $(y - y_0) / (s - s_0)$ | $(s - s_0) / (e - e_0)$ |
| (eq. 4)       |                         |                         |                  |

The MRSs facilitate the understanding of which HDI formula is more consistent with the reader’s preferences. Moreover, since the MRS is a relative magnitude, a sort of relative price, it means that it has a cardinal interpretation even if one does not share a cardinal interpretation of the HDI: it makes perfect sense to carry out comparisons
of different MRSs both in levels and in changes over time. In fact, all the HDIs adopted in the literature imply changing ‘exchange rates’ over time. Figure 3 shows the development over time of the MRS for life expectancy with respect to per capita GDP (top panel), for schooling and per capita GDP (panel in the middle), and for life expectancy versus schooling (bottom panel) in Italy (1861-2016).

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18 Gallardo-Albarrán (2019: 63) notes that the consumption-equivalent measure of wellbeing devised by Jones and Klenow (2016), drawing on historical information about people’s preferences, makes possible to avoid “two important critiques of the HDI and HIHD”, that is, assuming an arbitrary, and fixed weighting scheme over time.
Figure 3. HDI Marginal Rates of Substitution – Italy, 1861-2016

Life expectancy vs. GDP

Schooling vs. GDP

Life Expectancy vs. Schooling

Source: Authors’ calculations based on Amendola et al. (2017).
To begin with, let us consider the CD-HDI index (red dashed line, equation 4). According to this specific HDI, in 1861 one extra year of life would be worth around $400 (in 2013 Geary-Khamis dollars). The top panel in Figure 3 shows that for as much as a century and a half after unification, the MRS remains close to this value. Other formulations used in the literature show a very different trend. The Old HDI starts from a lower MRS (circa $200), while the Hybrid HDI and the HIHD start from a much higher $1,800 and shoot up in the aftermath of World War II. The Old HDI assumes that in the 2010s a year of life is worth around $2,600 – a value not too far removed from the one assumed by those adopting the Hybrid HDI (about $2,500). The HIHD exceeds all of these: from 1861 the value of one additional year of life rises from $1,894 to $17,641. When defending the convex formulation underlying the HIHD, Prados (2021: 952) did discuss the correlation between increases in education and longevity, and their quality: Figure 3 gives the reader a full grasp of the implication of that choice.

The role of MRSs was discussed by Ravallion (2012a), on the occasion of the introduction of the new HDI formulation: the HDI “puts a higher value to an extra year of life for people in rich countries than poor ones”, with the ”unacceptable implication that rich people, or residents of rich nations, are worth more than the poor” (p. 206). Ravallion’s observation naturally applies to inter-temporal comparisons. As Figure 3 shows, different HDIs attribute different weights to life expectancy in different periods of Italian history: this evaluation may be legitimate, but it should be made transparent. To argue that “for rich countries, the high value
of longevity in terms of income simply means that per capita income contributes negligibly to increasing capabilities” (Prados de la Escosura 2021: 959) does require to tackle the question: why should we prefer the rising trend of the HIHD to the less steep, but still rising trend of the Hybrid HDI?19

From an analytical point of view, there seems no way to assess which MRS is ‘better’, or ‘more appropriate’. One could be tempted to find a way to measure historical preferences over these trade-offs, perhaps following Jones and Klenow (2016) approach. Gallardo-Albarrán (2019) revealed preference is for the Hybrid HDI over the HIHD. However, given that the HDI is a paternalistic social welfare function, one can argue that Gallardo-Albarrán’s is just one of a plurality of equally legitimate evaluations. Bértola and Ocampo (2012) and Bértola et al. (2012) provide different possibilities corresponding to as many HDI formulas; by so doing, however, we are back to square one and end up losing the benefit of aggregating different indicators in one single number.20

It is important to stress that while it might be safe to assume that scholars working with historical HDIs are aware of the ‘paternalism’ implied in their analysis, others

19 The same holds true for the idea that “Although the convex transformation of the indicators of longevity and education dimensions mitigates the difference between these bounded variables and unbounded variables such as GDP per capita, it does not put them on a level playing field, and some form of compression of the income dimension of human development is required to make it comparable to its social dimensions” (Prados de la Escosura 2021: 955).

20 Prados de la Escosura (2021: 963), while presenting in a table selected values for a number of alternative indicators, concludes that “the different specifications for an AHDI share common trends”, and that his preferred AHDI “results in an intermediate position among the alternative options”.
might not be fully aware of the subjectivity that comes with the analysis. In the next section, we introduce a specification of the HDI based on the constant elasticity of substitution (CES) utility function, that allows us to more consistently reveal and discuss the ethical judgments hidden in composite indices formulas.

5. Historical HDIs: Anything Goes

In this section we reformulate the HDI as a constant elasticity of substitution (CES) function. This allows us to control the degree of substitutability between the HDI components through a single parameter, the elasticity of substitution, and sheds new light on the implications of using different HDI.\footnote{One might be tempted to further generalize the HDI formula by allowing for different elasticities of substitution among the arguments of the HDI. Unfortunately, Uzawa (1962) proved that if the elasticities of substitution between every pair of arguments are to be held constant, then the elasticities must be identical.} Using notation consistently with previous sections, the generalized HDI can be written as follows:\footnote{Mas-Colell et al. (1995) introduced the CES function as an instance of a generalized utilitarian social welfare function (see example 22.c.4 p. 828-29). However, in the present context, equation 7 generalizes a SWF that violates the non-paternalism property, i.e. a PSWF. Fleurbaey (2018) also adopted the CES function to describe the alternative formulas of the HDI.}

\begin{equation}
    HDI_{CES} = \left[ \alpha_1 \left( \frac{\sigma-1}{\sigma} E_t \right) + \alpha_2 \left( \frac{\sigma-1}{\sigma} S_t \right) + \alpha_3 \left( \frac{\sigma-1}{\sigma} Y_t \right) \right]^{\frac{1}{\sigma-1}}
\end{equation}

where \( \sigma \) represents the elasticity of substitution, \( \alpha_i \) the weight attributed to component \( i \) of the HDI, \( \alpha_i \geq 0 \) for every \( i \) and \( \sum \alpha_i = 1 \). In what follows, we...
assume that $\alpha_i = \alpha_j \forall i, j$, that is, we treat symmetrically all the arguments of the HDI.

In equation (7), the parameter $\sigma$ plays a crucial role. As $\sigma$ approaches infinity, life expectancy, schooling and income become perfect substitutes and equation (7) reduces to the Old HDI (equation 1). As $\sigma$ approaches 1, $HDI_{CES}$ converges to the Hybrid HDI (equation 2) and, with slight modifications, to the HIHD proposed by Prados de la Escosura (2015a) (equation 3). When $\sigma$ approaches 0, then $HDI_{CES} = \min\{E_t, S_t, Y_t\}$, that is, we obtain a new Leontief-like HDI specification, never considered in the literature: in this case the components of the HDI are perfect complements, a characteristic that fits well with the idea that they capture the essential dimensions of wellbeing. A number of intermediate cases can be obtained by varying $\sigma$ between 0 and infinity.\(^{23}\)

Unfortunately, equation 7, even if simple and of immediate interpretation, is not sufficiently flexible to encompass exactly all the historical HDIs proposed in the literature. This is because these HDIs introduce specific achievement functions, i.e.

\(^{23}\) Zambrano (2014, 2017) follows an axiomatic approach to characterize different specifications of the HDI. He finds that, i) in contrast to multiplicative forms of the HDI, the additive HDI does not satisfy either the “subsistence axiom”, or the “capabilities growth independence axiom”, a stronger form of independence with respect to the “independence axiom” satisfied by the multiplicative HDI. All these axioms can be associated to different degrees of substitutability and are consistent with the role played by the parameter $\sigma$ in equation (7). An advantage of our approach is that the parameter $\sigma$ is a continuous variable with an intuitive interpretation, a drawback is that we need to impose the CES functional form for the HDI.
transformations of the original, elementary indicators. A general $HDl_{CES}$ formulation that encompasses all the HDIs used in the literature is as follows:

$$\begin{align*}
HDl_{CES}(\sigma, \varepsilon, \eta) &= \left[ \sum_{i=1}^{K} \alpha_i \left( \frac{(x_i^t - x_i^0)^\varepsilon - (x_i^t - x_i)^\varepsilon}{(x_i^t - x_i^0)^\varepsilon - 1} \right) a_i \right]^{\frac{\sigma - 1}{\sigma}} \\
+ & \left( 1 - \sum_{i=1}^{K} \alpha_i \right) \left[ \frac{(y_t)^\eta - (y_0)^\eta}{(y)^\eta - (y_0)^\eta} \right]^{\frac{\sigma - 1}{\sigma}} \\
&= \left[ \sum_{i=1}^{K} \alpha_i \left( \frac{x_i^t - x_i^0}{x_i^t - x_i} \right) \right]^{\frac{\sigma - 1}{\sigma}} \\
&+ \left( 1 - \sum_{i=1}^{K} \alpha_i \right) \left[ \frac{y_t - y_0}{y - y_0} \right]^{\frac{\sigma - 1}{\sigma}}
\end{align*}$$

(8)

The notation in equation (8) is consistent with that used in previous sections; the parameter $\varepsilon \in (0, 1]$ describes the degree of convexity of the achievement functions for the $K$ components other than GDP, and the parameter $\eta \in (0, 1]$ regulates the degree of concavity of the achievement function for GDP.\(^{24}\) In short, equation (8) identifies a family of HDIs characterized by the parameter vector $(\sigma, \varepsilon, \eta)$.

Table 2 below shows how the HDIs proposed in the economic history literature can be obtained from equation (8) by choosing the parameter set $(\sigma, \varepsilon, \eta)$ appropriately. The first four rows refer to the indices described in the previous sections. The basic HDI formula assumes perfect substitutability among the HDI components, and the achievement functions are linear with the exception of the GDP component. This implies $\sigma = \infty$, $\varepsilon = 1$ and $\eta = 0$. The Hybrid HDI is based on the same assumptions about the achievement functions but allows for Cobb-Douglas type imperfect substitutability among the components, corresponding to $\sigma = 1$. The HIHD

\(^{24}\) The adjustment coefficients $\alpha_i = \left( x_i^t - x_i^0 - 1 \right) / \left( x_i^t - x_i^0 \right)$ are necessary to guarantee the exact convergence of the $HDl_{CES}(\sigma, \varepsilon, \eta)$ with the HDIs proposed in the literature.
introduces a variation on the achievement functions for the non-GDP components by assuming, other things being equal, $\varepsilon = 0$. The CD-HDI assumes linear achievement functions for all the components, which means $\varepsilon = \eta = 1$. The fifth row identifies a case that deserves particular attention, as it assumes the complete absence of substitutability ($\sigma = 0$), with all the possible combinations of the parameters $\varepsilon$ and $\eta$. In this case, the HDI, as observed above, takes the shape of a Leontief type social welfare function that captures, in a utilitarian framework, the ethical system proposed by Rawls (1971). Under this specification, all the social indicators included in the HDI are supposed to be essential dimensions of wellbeing for which a trade-off cannot be defined: as a consequence, the pattern of the HDI is entirely driven by the most deficient dimension of wellbeing. The last rows capture the residual possibilities characterized by imperfect substitutability and different degrees of concavity/convexity of the achievement functions.
Table 2. A taxonomy of indices within the generalized historical HDI

<table>
<thead>
<tr>
<th>Denomination</th>
<th>σ</th>
<th>ε</th>
<th>η</th>
<th>Preferences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Old HDI (eq. 1)</td>
<td>∞</td>
<td>1</td>
<td>0</td>
<td>almost perfect substitutes</td>
</tr>
<tr>
<td>Hyrid HDI (eq. 2)</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>HIHD (eq. 3)</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>CD-HDI (eq. 4)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>Cobb Douglas</td>
</tr>
<tr>
<td>Rawlsian HDI</td>
<td>0</td>
<td>[0,1]</td>
<td>[0,1]</td>
<td>perfect complements</td>
</tr>
<tr>
<td>Imperfect substitute</td>
<td>(0,∞)[1</td>
<td>[0,1]</td>
<td>[0,1]</td>
<td></td>
</tr>
<tr>
<td>Perfect substitute</td>
<td>∞</td>
<td>1</td>
<td>1</td>
<td>linear preferences</td>
</tr>
</tbody>
</table>

Figure 4 below plots the $HDI_{CES}$ series that we obtain by varying the elasticity of substitution $\sigma$ and the parameters $\eta$ and $\varepsilon$. The figure contains all the HDIs illustrated in Figure 1, but also two examples of other indices based on the ‘Rawlsian’ and ‘imperfect substitute’ parameterization respectively. It is evident that a fairly wide range of patterns results from this exercise: while most readers will find perfect complementarity an extreme assumption, they will have harder time deciding why the imperfect substitutability of the orange series should not be preferred to, say, the one implied by the Hybrid HDI, whose geometric average was motivated by that very idea.
Figure 4. The CES-HDI in Italy, 1861-2017

![Graph showing the CES-HDI in Italy from 1861 to 2017](image)

Source: Authors’ calculations.

Within the family of HDIs encompassed in equation (8), it is not difficult to find an ethical system, that is a specific triad for the parameters \( (\sigma, \epsilon, \eta) \), which can generate any trajectory for the HDI, for the same underlying elementary indicators. ‘Any trajectory’ is not used here in mathematical sense: it cannot be claimed that all possible HDI trajectories can be produced by means of equation (8), but under fairly general conditions, a suitable choice of the parameters \( (\sigma, \epsilon, \eta) \) leads to any suitable HDI-based story. Anything goes. Indeed, as we see in Figure 4, a simple change in the parameter \( \sigma \) dramatically changes our perception of an established result in Italian economic history, such as the ‘benevolence’ of Liberal Italy (1861-1911) in terms of human development (Toniolo 2003); different parameters radically change
our perception on the trade-off between GDP and non-monetary indicators during the interwar decades (Gallardo-Albarrán 2019; Gabbuti 2020).

It is important to note that the ‘anything goes’ result does not depend on the weights ($\alpha_i$) used for the components of the HDI, that attracts most of the attention in the literature. What matters is their substitutability regulated by the parameter $\sigma$ that, in turn, reveals the analyst’s preferences. The fact that $\sigma$ can take on, at least theoretically, any value greater than zero (and that, as we have discussed, the theoretical indeterminacy of this index does not suggest any obvious, shared solution to this problem) implies a fundamental ‘indeterminacy’ in the HDI. This indeterminacy suggests the HDI is unable to deliver a history of human development based on transparent and non-discretionary criteria. There are many, virtually infinite, stories that can be told according to the ethical judgments of those who build the HDI index. Economic historians relying on HDIs are back where they were before the introduction of composite indices; the heated debates over historical standards of living have been replaced with more technical, and less transparent, discussions over parameters and functional forms.

We used the expression *anything goes*, and this deserves a clarification. This kind of indeterminacy is limited by the fact that the standard components of the HDI are, for most countries, highly positively correlated.\(^{25}\) This is a well-documented empirical

\(^{25}\) Also in the economic literature, many authors have pointed out the redundant nature of the additional information incorporated in the human development index with respect to the single GDP component; see Srinivasan (1994) and Brandolini and D’Alessio (2009). Ogwang (1994) and Ogwang
regularity: Rijpma (2014) shows that for most OECD country this holds true since the second half of the 19th century, due to medical technological innovations, and this is indeed the period covered by Prados de la Escosura (2015: 222), noting that 1870 marks the begin of “large scale improvements in health, helped by the diffusion of the germ theory of disease since the 1880s, and mass education” in “Western Europe and the European offshoots”. An immediate consequence of the fact that life expectancy at birth and education co-move with per capita GDP is that, irrespective of the elasticity of substitution, most of the time, and most of all, in the long run we are not able to reverse the trend of the historical HDI. Surely, the different degree of substitution embodied in the formula brings down the HDI in years characterized by sharp GDP contractions – the 1930s, and most notably, the decade of stagnation inaugurated by the 2008 recession, the most severe in the history of Italy (Baffigi 2015), but also the shorter-term falls after the oil shock of 1973 and the currency crisis of 1992. On the other hand, as we have seen in Figures 1 and 4, the different changes implied by different MRS may lead to different relative positions between different historical periods. Considering these long-run correlation between the underlying components of the index, however, our results amount to saying that the ranking by years is unequivocally determined by the index, and is independent of $\sigma$.

and Abdou (2003), adopting a principal components analysis, showed that using a single indicator (life expectancy) would result in almost the same country ranking, “without loss of too much information”.
Given the ordinal interpretation of the HDI pursued in this paper, this is reassuring, but makes the construction of these indicators of little use.

Apart for earlier periods (as the industrial revolution investigated by Gallardo-Albarrán and de Jong 2021), a possible rehabilitation of the historical HDIs relies on the introduction of new indicators, not so closely correlated with per capita GDP. While enriching the concept of wellbeing inherent in the HDI, as proposed by several scholars, this would practically enlarge the number of conceivable trajectories of the HDI that can be sustained by an appropriate ethical system. Strong candidates for this inclusion are political and civil rights. On one hand, these are a obviously important dimensions of human development, already explored by Dasgupta and Weale (1992): despite the unavoidable difficulties in estimating similar indicators, the importance of this inclusion is even more relevant in inter-temporal, long-run comparisons. Indeed, the estimation of the Dasgupta and Weal Index (DWI) was pursued by Crafts (1997b); more recently, Prados de la Escosura (2021) “augmented” his HIHD to include this fourth component. On the other hand, the emergence of the Fascist regime marks a clear discontinuity in this metric, making Italy a perfect candidate to test for the reliability and interpretation of historical HDIs in evaluating historical, troubling trade-offs between uncorrelated dimensions of human wellbeing – the task they were meant to pursue when first introduced in the discipline. This ‘test’ is the object of the next section.
6. The ‘Gramsci Test’: The HDI in Troubled Times

It comes as no surprise that economic historians consider political and civil rights crucial in the intertemporal assessment of living standards. The creators of the HDI argued that “human development is incomplete without human freedom” (UNDP 1990: 15), but they ended up limiting the aggregation to education. The choice was not an accident: it was put forward by Paul Streeten, a distinguished development economist, but also a refugee from Austria who had fought in Sicily against Nazi-Fascist forces (Haq 1995: 61). Streeten (1994: 236) argued in favor of measuring and including political rights: they are “so important” – he wrote – “that no trade-off should be possible”. On the other hand, the volatility of such indices and the inevitable subjectivity in quantifying such indicators represent a weakness. Crafts (1997b: 621-622) acknowledged the last argument, but also noted that Dasgupta and Weale’s idea of including this dimension echoed radical stances in the standard of living debate, such as Thompson (1963).

The case of Italy makes clear how the inclusion or exclusion of political and civic liberties significantly alters the historical evolution of wellbeing. It is hard to reconcile the evolution of Figure 1 in the years 1922-1943 with the definition of human development as “enlarging people’s choice” (UNDP 1997: 15). After a wave of violence against political opponents, in October 1922 Benito Mussolini was appointed Prime Minister following the so-called ‘March on Rome’. Within a few years, his coalition government evolved into a dictatorship that restricted in many ways the scope of citizens’ free will. Aside from the killings of opposition leaders,
and the illiberal laws abolishing the freedom of the press, banning workers’ strikes, and installing a one-party system by 1926, the Fascist regime restricted many civic liberties. The infamous Race Laws (1938), that expelled Jewish citizens from public education and employment and prohibited mixed marriages, were only the most extreme examples because Fascist rule restricted many aspects of individual freedom, for instance by banning internal migration in the 1930s.\footnote{For a concise history of civil and political rights in Italy, see Amendola et al. (2017: 475–479) and the literature mentioned there.} As recalled by the same Sen (2003: 1244), in 1926 Antonio Gramsci – one of the most important Italian intellectuals, and a leader of the Communist party – was sentenced to twenty years imprisonment: in his final address, the public prosecutor stressed that the state had to “prevent this brain from functioning” (Sen 2003: 1244). Still, all versions of the HDI adopted in the economic history literature, as shown in Figure 1, increase in this period, driven by the slow but steady increases in education and life expectancy.

While no indicator of political rights can be considered as better than the others, two sources have gained growing consensus (Hogstrom 2013). The first is Freedom House, a US non-governmental organization that provides annual estimates of the Political Rights and Civil Liberties indicators, proposed by Taylor and Jodice (1983: 58-68), for almost all polities in the world from 1972. Each year, a score between 0 and 4 is assigned to 25 questions on various dimensions of political life and civil rights, resulting into an index conveniently scaled between 0 and 100. Amendola et al. (2017: 475-479) constructed historical series of these two indicators to estimate
the DWI for Italy from unification to the present. A second alternative is the ‘Polity2 Indicator’, provided by the Polity IV Project (Marshall et al. 2017). The index evaluates political institutions (such as the openness of elections and checks on the executive branch) and assigns a score between -10 and +10 (-6 and +6 being the thresholds for dictatorship and democracy, respectively). Prados de la Escosura (2021) relied on the Liberal Democracy Index proposed by Coppedge et al. (2022).

Figure 5 illustrates the evolution of political and civil rights in Italy: all the indicators show similar trends, and agree on the timing of the major reversal experienced by Italians in the interwar years. The inclusion of an indicator of civil and political rights into historical HDIs, allows us to capture Sen’s idea of “development as freedom”, but also to introduce a dimension non systematically and positively correlated with the others (Amendola et al. 2017).
Figure 5 – Freedom Indices for Italy, 1861-2016

Source: Authors’ elaborations on Amendola et al. (2017), Coppedge et al. (2022), and Marshall et al. 2017. Note: the Polity2 score has been rescaled.

In the light of our methodological discourse, what matters here is the discontinuity, rather than which index is more ‘correct’. For this reason, in Figure 6 we show alternative versions of the CES-HDI augmented including the indicator constructed by Amendola et al. (2017).
Figure 6 – Development and Freedom in the CES-HDI: Italy, 1861-2016

![Diagram showing the development and freedom in the CES-HDI for Italy, 1861-2016.]

Source: Authors’ calculations.

Figure 6, indeed, shows that, once again, anything goes. The two curves are based on the same dataset, they adopt the same dimensions in defining human development, and use the same weighting scheme: yet, they tell opposite stories. The CES-HDI variant depicted by a solid line assumes a high elasticity of substitution, and delivers the message that the interwar years saw a steady improvement in human development, despite the fascist dictatorship; the dashed line variant, which assumes a low elasticity of substitution, instead shows a dramatic deterioration – historically unparalleled outside of wartime – over the same period. Depending on the parameter
the ethical system of the analyst – any history can be told. It could be tempting
to rule out the solid line as ‘obviously wrong’, but this does not mean that we could
find a $\sigma$ we all agree with. In the absence of a defined link with standard economic
theory, there is no guidance on choosing this parameter. Indeed, extending the HDI
to include freedom makes the point even more clear: in cases, such as the rise of the
Nazis and the end of the Weimar Republic, where authoritarian dictatorships were
the outcome of a democratic election, it would be hard to justify the choice of the
index in the light of historical ‘revealed preferences’ that clearly conflict with those
of most modern observers, who would arguably agree with Streeten and impose very
low, if any, elasticity of substitution with respect to the freedom component. The
estimation of MRS and the adoption of the CES-HDI would allow economic
historians to have a transparent conversation on similar choices, highlighting the
contribution of each component of the HDI. That said, it is not clear how this might
lead to any conclusive agreement.

7. Conclusion

Faced with the challenge of going beyond the traditional, one-dimensional analysis
based on GDP, development economists of the 1990s produced the HDI, a composite
index that was easy to calculate and communicate. Economic historians were quick
to adopt this idea, began to use the HDI and to experiment with it by adding to its
dimensions, and trying out specifications others than the original formula. In this
paper, we have made two arguments, one theoretical, and one empirical, which highlights some problems of using the HDI in economic history.

Following Graaff (1957), Mas-Colell et al. (1995) and, more recently, Yang (2018) and Fleurbaey (2018), we showed that any HDI can be interpreted as a paternalistic social welfare function. Consequently, the HDI is unsuitable for producing social rankings transparently derived from individual rankings. Moreover, we highlighted that, according to its inventors’ original interpretation, and with the paternalistic social welfare function interpretation the HDI is an *ordinal* indicator, not cardinal. This implies that economic historians willing to use the HDI within this framework, can rely on a tool suitable to *rank* different countries at a point in time, or different years of a given country’s history, but not suitable for calculating growth rates of economic variables, nor to carry out convergence analysis. This represents a clear limitation to most research situations in the field of economic history.

A second problem has been identified and discussed within the empirical investigation of Italian living standards during the last 150 years. We have shown that (almost) any ranking between two societies can be established by choosing an appropriate specification of the HDI. To clarify how general is this result, we have introduced a family of HDIs based on the constant elasticity of substitution (CES) function. This tool, despite its conceptual simplicity, is a powerful one. We have shown that almost any conclusion can be reached by choosing the CES-HDI elasticity of substitution $\sigma$ appropriately. We can conclude, for instance, that welfare increased or decreased during the years when Fascism was in power, keeping
constant both the choice of the variables entering the definition of the HDI and the weighting scheme. In other words, anything goes.

If the use of the HDI aimed to resolve the challenge of the trade-offs implied by the multidimensional nature of social wellbeing, then the battle can be considered lost. The HDI only allows economic historians to rank wellbeing consistently with the specific value system associated with the any particular HDI specification; in our framework, by choosing a particular parametrization of the CES-HDI. The formal link established with paternalistic SWF suggests that the HDI does not tell us about the data, nor about history, but it tells us about the historian.

What implications can we draw from our analysis? What solutions can be outlined to deal with the interpretive limitations of the human development index?

A first possible solution is to rely to the approach, inspired to Williamson (1984), Voth (2001), and Jones and Klenow (2016), also explored by Gallardo-Albarrán (2019) and Gallardo-Albarrán and de Jong (2021). This framework is fully consistent with standard economic theory, but unfortunately is unable to include difficult-to-value dimensions, such as freedom and political rights. The price of consistency lies in the limited ability to include the very dimensions of well-being that the human development index aims to capture.

A second solution is a more radical one and consists of abandoning the analytical perspective of the human development index, going back to the so-called “dashboard approach”. As Ravallion (2012b) put it, “recognizing the multidimensionality of
development goals does not imply that we should be aggregating fundamentally
different things in opaque and often questionable ways. Rather it is about explicitly
recognizing that there are important aspects of development that cannot be captured
in a single index.”

Finally, a third way, as also recently suggested by van Zanden (2022), is to open a
debate to reach a consensus about the dimensions of well-being that should be
included in the human development index and the functional form that should be
adopted to aggregate these dimensions – following the examples developed before,
we should reach a scholarly, or broader societal agreement over the $\sigma$ that better
reflect the consensus over the trade-off between political rights and other dimensions
of well-being in Fascist Italy, or among monetary and non-monetary aspects of
well-being in the Industrial Revolution. Such a consensus would certainly not make
it possible to overcome the interpretive limitations related to the subjective and
paternalistic nature of the index but, similar to what has already been experienced in
the Maddison project, it would make historical data more comparable over time and
space.

The strategy of proceeding in open order, where each scholar persists in defending
his or her own specification of the human development index would certainly lead
to a worse outcome than any of the alternative approaches outlined above.
References


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