AGGREGATE DEMAND, INSTABILITY, AND GROWTH*

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Abstract: This paper considers a puzzle in growth theory from a Keynesian perspective. If neither wage and price adjustment nor monetary policy are effective at stimulating demand, no endogenous dynamic process exists to assure that demand grows fast enough to employ a growing labor force. Yet output grows persistently over long periods, occasionally reaching approximate full employment. We resolve this puzzle by invoking Harrods's instability results. Demand grows because it follows an explosive upward path that is ultimately limited by resource constraints. Downward demand instability is contained by introducing an autonomous component to aggregate demand.

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Why do modern economies grow? Since the pioneering work of Solow (1956), almost all mainstream economists would answer this question by invoking the supply side: growth arises from expansion in the supply of inputs and improvements of technology. But how do we know that aggregate expenditure will grow to employ supply-side resources? Mainstream growth theory provides a simple, although usually implicit, answer: because growth theory is the domain of the "long run," demand considerations fade into the background. Short-run sticky wages or prices may cause Say's Law to fail over some horizons, but nominal variables eventually adjust to clear markets and supply creates its own demand over the long periods relevant for the concerns of growth theory. In this sense, demand growth is assumed to be automatic over the long run and is therefore ignored, a characteristic of the famous neoclassical synthesis.

While nominal rigidity is considered the *sine qua non* of Keynesian macroeconomics in mainstream interpretations, Keynes himself did not believe that nominal adjustment would cure the problem of inadequate demand. He argued that falling nominal wages and prices would likely *magnify* the problem of under-utilized resources (see chapter 19 of the *General Theory*). This perspective is manifest in modern efforts to avoid deflation. In this case, we argue that it is inappropriate to assume that demand growth endogenously accommodates to supply in macroeconomic growth models. While supply growth is undoubtedly *necessary* for long-term expansion, it may not be *sufficient*.

The observation that demand growth is not automatic, however, suggests a puzzle. Aggregate output statistics for developed countries over long sweeps of time show persistent growth. Although there are periods when economies operate below potential, sometimes for an extended time, measures of resource use such as the unemployment rate approach levels consistent with full utilization, at least momentarily. If aggregate demand does not automatically adjust to potential output, as assumed in the neoclassical synthesis, what is there about the dynamics of demand growth that generates persistent expansion over long horizons that occasionally employs most resources?

To explore this question, we construct a dynamic model of demand growth in the Keynesian tradition. We begin with a simple model that reproduces the basic results discovered by Harrod (1939). Thus our approach must address two well-known and important issues that question the empirical relevance of Harrod's growth model. First, the steady state of this model is dynamically unstable. The model predicts explosion or collapse should actual growth deviate slightly from what Harrod calls the "warranted" path. Second, the steady-state path of the model does not generate full employment over any horizon, except by coincidence, if the demand-determined steady-state "warranted" rate of growth happens to equal the "natural" growth rate of the labor force and productivity.

These issues have been taken up in an extensive literature over the years, which we selectively survey in the next section of the paper. We offer a new, and remarkably simple, approach. Instead of rejecting instability as empirically unrealistic, we show how instability, contained by modifications of the basic Harrod model, can resolve the puzzle described above. The downward direction of short-run instability endogenously switches from contraction to expansion if part of aggregate demand does not depend on the state of the business cycle. In other words, the presence of some autonomous demand—acyclical government spending, for example—contains short-run instability. Positive instability is constrained through a different mechanism. When unstable demand rises enough to fully employ the economy's resources, the supply side becomes the binding constraint on growth. Once this happens, the system will push up against the neoclassical growth path emphasized in mainstream theory. But this path can be *fundamentally unstable,* a result never considered in mainstream theory. When full employment is reached, the system's dynamics can start a new cumulative process that pushes the economy below the neoclassical growth path, until the autonomous demand floor (or a positive demand shock) once again turns the path upward. In our approach, Harrod's instability, rather than being interpreted as empirically unrealistic, becomes the engine of demand dynamics that exhibit secular but unstable growth.

This perspective has important implications for how we understand growth in modern economies. The model generates several different qualitative patterns for growth over long periods of time. It is possible that demand is so robust that it continually pushes up against supply constraints. It is also possible that autonomous demand limits positive growth prior to the economy reaching full employment and the actual growth path never reaches full employment. In the pattern most consistent with the behavior of modern developed economies in recent decades, however, economic growth and employment are almost always constrained by the demand side. Nonetheless, the system occasionally approaches full employment. Although the occasional points of full employment are transitory and unstable, they constitute a sequence of cyclical peaks of the long-term path. The outer envelope of these peaks will demonstrate *persistent growth* if resources grow and technology improves over time. It is the peaks of the growth process only that are constrained by supply. This result provides a kind of synthesis between mainstream growth theory and realistic Keynesian macroeconomics, one that does not rely on the weak price adjustment-demand link to relegate demand to the shadows in the "long run" relevant for growth models.

Following the literature review in the subsequent section, we revisit Harrod's results and explore the interpretation of his key instability result in section 2. We conclude that the basic Harrod model is a natural way to model demand dynamics and that the model predicts generic instability that does not depend on particular parameter values. We discuss how resource constraints and the presence of autonomous demand contain growth instability in section 3. The results show that the model generates a corridor that contains demand growth between a resource-determined ceiling and a floor that evolves through time in a way that is similar to the equilibrium output arising from a simple *static* Keynesian multiplier model. Section 4 describes the basic qualitative growth patterns that emerge from the model. The concluding section presents a variety of links between the approach pursued here and issues in economic dynamics that merit further research.

1. Motivation: Literature on Demand and Growth

In traditional growth theory, there is no role for demand. In the foundational Solow (1956) model, for example, long-run growth occurs at full employment. The possibility that nominal adjustment may not restore demand to full-employment levels, and therefore that demand constraints might bind beyond the short run, is not considered by Solow or in the massive mainstream growth literature that his paper spawned, including "endogenous growth" theory (see Aghion and Howitt, 1998). In this sense, mainstream growth theory does not prove the irrelevance of Keynesian demand problems in the long run. As Skott (1989, page 21) writes, "[b]y imposing full employment, the neoclassical economists had effectively assumed away most of the problems which worried Harrod."

The possibility that nominal flexibility does not endogenously restore demand to a supply-driven path was discussed in detail in the *General Theory*, chapter 19, and this issue has been developed in a small, but persistent, strand of literature. The key insight is that wage and price adjustment cannot solve problems of insufficient *aggregate* demand. Wages and prices will likely decline for particular firms or in particular sectors with under-utilized resources. But any rise in spending directed toward some part of the economy due to a change in relative wages or prices comes at the expense of demand in other sectors. Generalized deflation (or disinflation relative to the previous trend of prices) could have favorable real balance effects that push aggregate demand back toward full employment. But these stabilizing effects can be offset by (1) destabilizing effects of expectations on real interest rates, (2) redistribution from high-spending borrowers to low-spending debtors, or (3) deteriorating conditions in credit markets that curtail spending.[1] Thus, in theory, nominal adjustment may or may not restore demand to absorb potential output. Caskey and Fazzari (1992) develop an empirically calibrated model that incorporates these effects and find that the destabilizing channels dominate the stabilizing channels, a point amplified by Fazzari, Ferri, and Greenberg (1998) and Palley (2008). This work and recent historical experience strongly suggest that wage and price adjustment will not play the stabilizing role that eliminates demand constraints according to the mainstream neoclassical synthesis perspective.[2] In this case, Say's Law fails not just in the short run but over a longer horizon as well, and the question of whether supply expansion is a sufficient condition for economic growth becomes central.[3]

The idea that demand conditions affect long-term growth has been a persistent theme in heterodox Keynesian research. Kregel (1980) reviews earlier literature; more recent contributions are discussed by Foley and Michl (1999) and the volume edited by Setterfield (2010). This paradigm begins with the seminal contribution of Harrod (1939) which is closely related to the model discussed in the following section. He presents a simple, intuitive approach to model how demand evolves over time to target a desired

[1] Keynes and his followers discussed all of these channels. In more modern literature, these three destabilizing channels have been emphasized, respectively, by Delong and Summers (1986), Tobin (1975), and Caskey and Fazzari (1987).

[2] The lost decade(s) in Japan provide perhaps the most relevant modern example. Furthermore, the obvious desire of monetary authorities around the world to avoid nominal deflation suggests that they have, at least implicitly, accepted the basic argument that falling prices are not stabilizing. While mainstream "New Keynesian" models continue to emphasize nominal rigidity as the reason that demand may fall short of potential output, much practical research has largely abandoned nominal adjustment as the solution to insufficient demand. In "New Consensus" models, the Taylor Rule or some form of inflation targeting creates a "visible hand" of monetary policy that replaces nominal adjustment as the central mechanism for eliminating the real effects of the demand. Recent experience with both the "zero bound" for nominal interest rates and the inability of less conventional "quantitative easing" policies to initiate sustained recovery in the U.S. and elsewhere from the Great Recession call into questions the effectiveness of monetary policy in this regard. Dutt (2010, p. 229) raises related questions about the effectiveness of monetary policy in generating full employment growth.

[3] Palley (1996) also motivates the importance of demand effects on growth and the Harrod model by referring to the "incapability" of wage adjustment to restore demand to a supply-determined path.

capital-output ratio (or, equivalently, a desired rate of capacity utilization). Despite the intuitive appeal of its basic dynamic structure, however, the Harrod model, "is generally accepted to contain an anomaly or a problem, viz. the 'knife edge'" (Kregel, 1980, page 97; also see Bortis, 1997, page 134, Palley, 1996, and Skott, 1989, chapter 6, and 2010). Simply put, the steady-state "warranted" growth path of Harrod model is unstable; any disturbance from steady state causes the growth path to explode without bound or collapse toward zero, an empirically unrealistic prediction.

A number of authors have modified the Harrod model to address this knife-edge instability. Shaikh (2009, p. 464), for example, proposes "a simple and sensible dynamic adjustment process" that "renders the warranted path perfectly stable." Skott (1989, 2008) also discusses mechanisms that can make growth models with a Harrod-like investment function stable in the long run. The insights from this work are important. Nonetheless, instability of the "warranted path" remains a persistent feature in Keynes-Harrod growth models.[4] In the approach developed here, we embrace the knife-edge property of the original Harrod model as a key feature of the economy that helps to explain long-term growth. This approach stands in contrast to the conclusion reached in much other research that the instability of Harrod's warranted path is an empirically unrealistic result that suggests a flaw in the model (Skott 1989 is an exception). We draw motivation from the contributions of Hicks (1950) and Minsky (1959, 1982). These models are locally unstable but contain globally explosive dynamics by introducing ceilings and floors. While this technique has been mainly applied within a business cycle perspective (see Ferri and Greenberg, 1989, and Ferri and Minsky, 1992), we believe that

[4] Skott (1989, page 104) writes that Harrod's instability argument is "vindicated" despite features of an extended model that may contain local instability of the warranted path in global limit cycles

it can also add to our understanding of the role of aggregate demand over longer horizons.[5]

Much recent heterodox growth literature has followed an alternative path to Harrod model based on the approach originated by Kalecki. This research has pursued a wide variety of important issues, especially the impact of income distribution on growth. The Kaleckian models, however, have a problematic implication. The long-run rate of capacity utilization is not determined by a target derived from firm behavior. Rather, steady-state capacity utilization emerges from other features of the model, and firms are assumed to adjust their behavior to accept the equilibrium utilization rate that arises from the model solution. Some authors have put forward mechanisms to facilitate this adjustment. For example, firms may adjust their target rate of utilization in response to recent experience if such experience differs from the utilization rate firms would choose for technological or strategic reasons (see Dutt, 2010, page 235, for example). Or firms may accept a range of utilization rates as "normal" (Lavoie, 2010, p. 144), which allows other factors to affect the long-run utilization rate, as long as it remains within the normal range. Nonetheless, the investment decision is ultimately made by the firm, and we therefore assert that the firm's objectives should be the starting point for understanding investment dynamics. That is, the target utilization rate should be chosen by firms, consistent with Harrod's approach.[6]

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[5] A different approach has been suggested by Aoki and Yoshikawa (2007), who also emphasize the link between demand and growth.

[6] For an overview of this issue and many further references, see Blecker (2002), Skott (2010) and Lavoie (2010). Skott and Zipperer (2012) compare growth models empirically and find that the implications of the Kaleckian models are not supported by the data.

2. Harrod's Model Revisited

We begin with the most basic of Keynesian models. Output (Y_t) is determined by aggregate demand (AD_t), which consists of consumption (C_t) and investment (I_t):

$$Y_t = AD_t = C_t + I_t \tag{2.1}$$

We do not consider inventory dynamics and international trade in this paper. The effect of including an autonomous component of demand, such as government spending and resource constraints on production is discussed in section 3.

As in Harrod, consumption is proportional to income. Because we want to trace the actual, rather than the equilibrium, path of demand and output through time, we assume that period *t* consumption depends on the expectation of period *t* income. It is convenient to represent these expectations in the form of a growth rate, so that

$$C_t = (1-s)(1+Eg_t)Y_{t-1} \tag{2.2}$$

where *s* is the constant propensity to save and Eg_t is the expectation of output and income growth between period *t-1* and *t*, conditional on information available at *t-1*.[7] While specifications such as equation (2.2) are often criticized in the mainstream literature because they lack optimizing "microfoundations," this specification leads to simple analytical results and links clearly to the literature started by Harrod.[8] We note that equation 2.2 can incorporate persistence into the consumption decision, as emphasized in

[7] Expectation errors could cause the *ex post* saving-income ratio to deviate from the target *s*. Harrod assumes this possibility away by specifying current consumption as a function of current income. Because current income depends on current consumption, Harrod's model generates an equilibrium path for demand and output, with the simple Keynesian multiplier resolved within each period. Our approach makes it easier to follow the dynamics of demand and output through time, but results are similar to those of Harrod.

[8] A linear consumption rule such as equation (2.2) can be derived from the first-order conditions for intertemporal optimization of a representative consumer with a reasonable utility function as in Aghion and Banerjee (2005, page 12).

life-cycle and permanent income models of consumption, because the expected growth rate may depend on longer lags of income or other variables.

To model investment, we first make assumptions about the firm's production technology. As in Harrod, productive capacity is a linear function of the stock of capital.[9] We assume that investment in period t becomes effective in period $t+1$. Let v^* denote the desired capital-output ratio for firms in the economy. Then the desired capital stock in $t+1$ is

$$K_{t+1} = v^* EY_{t+1} = v^*(1+Eg_t)^2 Y_{t-1}. \tag{2.3}$$

Assume that capital depreciates at rate δ (Harrod assumed $\delta = 0$). To reach this target in $t+1$, given that the law of motion for capital is

$$K_t = K_{t-1}(1-\delta) + I_t, \tag{2.4}$$

investment is

$$I_t = v^*(1+Eg_t)^2 Y_{t-1} - (1-\delta)K_t. \tag{2.5}$$

Note that K_t is the capital stock at the beginning of t that arises from past investment spending. Therefore, as is the case with consumption, investment in t is determined only by variables dated in earlier periods. Since gross investment cannot be negative, the investment entering into equation (2.1) is the maximum of 0 and the I_t given in equation (2.5). This constraint is ignored in the following analytical discussion, but it is incorporated into the simulation model discussed in later sections.

Output in t is given by

[9] This technology has also been resurrected in recent "AK" models of economic growth (see Aghion and Banerjee, 2005).

$$Y_t = C_t + I_t = (1-s)(1+Eg_t)Y_{t-1} + v^*(1+Eg_t)^2 Y_{t-1} - (1-\delta)K_t. \tag{2.6}$$

The output growth rate is $g_t = (Y_t / Y_{t-1}) - 1$ and we equation 2.6 divide by Y_{t-1} to obtain

$$1 + g_t = (1-s)(1+Eg_t) + v^*(1+Eg_t)^2 - (1-\delta)\frac{K_t}{Y_{t-1}}. \tag{2.7}$$

Equation 2.7 gives the law of motion for growth, conditional on expected growth.[10] All variables on the right side of 2.7 are predetermined. This equation provides the foundation for the dynamic analysis to follow.

The Warranted Rate

We can solve equation 2.7 for the steady-state growth rate, which is the growth rate that makes the actual growth rate in period *t* equal to the expected rate.[11] Steady state also requires that the capital-output ratio, $v_t = K_t / Y_t$ equal the target level v^*.[12] Denote the steady state growth rate by g^*, and substitute g^* for Eg_t and v* for v_t. From 2.7, we obtain the steady state:

$$\begin{aligned}1 + g^* &= (1-s)(1+g^*) + v^*(1+g^*)^2 - (1-\delta)\left[\frac{K_t}{Y_t}\right]\left[\frac{Y_t}{Y_{t-1}}\right] \\ &= (1-s)(1+g^*) + v^*(1+g^*)^2 - (1-\delta)v^*(1+g^*)\,. \\ &\Rightarrow g^* = \frac{s}{v^*} - \delta\end{aligned} \tag{2.8}$$

This steady-state solution is what Harrod calls the "warranted" rate of growth. If firms and consumers expect the warranted rate to prevail, the dynamic law of motion delivers

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[10] A fully specified law of motion would require assumptions about how expectations evolve. This issue is discussed below.

[11] See Sen (1970), Fazzari (1984) and Palley (1996) for similar interpretations of the warranted rate. Puu, Gardini and Sushko (2005) call this equilibrium a fixed growth point.

[12] These steady-state conditions parallel the requirements for equilibrium in both flows and stocks discussed by Hicks (1965, chapter 11).

an actual growth rate that validates expectations. If there is no exogenous change in expectations, growth should continue at this rate. But what happens if there is such a change?

Consider a shock to expectations. We can understand the source of instability in this model by looking at the derivative of actual growth (*g*) with respect to *Eg,* using the temporary law of motion, equation (2.7),

$$dg_t / d(Eg_t) = \left[2v^*(1+Eg_t)\right] + (1-s) \tag{2.9}$$

If this derivative is greater than one, a shock to expectations is magnified by the system dynamics. The derivative exceeds one if

$$Eg_t > \frac{s}{2v^*} - 1 \tag{2.10}$$

Along the warranted path, $Eg_t = g^* = (s/v^*) - \delta$, so the condition in 2.10 is always satisfied at the warranted growth rate. That is, a disturbance in the expected growth rate along the warranted path always generates a deviation in the actual growth rate away from the warranted rate *larger* than the size of the disturbance. This result is central to the generic instability of the model.

The mathematics behind equations 2.9 and 2.10 is straightforward, but the economic intuition is somewhat subtle. A casual look at the derivative given by equation 2.9 might suggest that instability of the actual growth rate could be overcome in ways familiar from static Keynesian models. The first term in brackets is an accelerator effect: higher expected growth induces more current investment, more current demand, and more actual growth. The second term in brackets is the positive effect of expected growth on consumption that also raises current demand and actual growth. Weaken the accelerator (reduce v^*) or raise the saving rate (increase s) and the impact of a shock to expected growth on actual growth gets smaller for a given level of expected, consistent with Keynesian intuition. But such changes *cannot* make the warranted path stable because we cannot determine the stability of the warranted path by evaluating condition 2.10 at a *given* level of expected growth. Changes in the demand parameters v^* and s that appear to stabilize demand dynamics have the opposite effect on the warranted growth rate that must be achieved along the steady-state path. For example, while raising s dampens the consumption multiplier, it also requires a higher warranted rate of growth since steady-state investment will have to be a higher share of output to absorb the extra saving. Although higher s raises the right side of the inequality in 2.10, it also raises the relevant growth rate for evaluating the stability of the warranted path on the left side of 2.10, and by a larger amount. This result is key to understanding why the warranted path is unstable.

Expectations and Instability

The path of the system following a disturbance to the warranted path depends on how expectations evolve. The derivative in equation 2.9 exceeds one for all values of Eg_t above g^* and a large interval of values below g^*.[13] Thus, if the expected growth rate deviates from the warranted rate, actual growth will deviate from the warranted rate by an even larger amount (in absolute value). Therefore,

[13] For reasonable values of s and v^*, the inequality in 2.10 fails only for large negative expected growth rates. In this case, the model can be stable, but the dynamics do not converge to the warranted rate. Rather, the system goes to a growth rate of negative 1 and zero output.

$$\begin{aligned} &Eg_t > g^* \Rightarrow g_t > Eg_t \\ &Eg_t = g^* \Rightarrow g_t = Eg_t \\ &Eg_t < g^* \Rightarrow g_t < Eg_t \end{aligned} \tag{2.11}$$

To trace out the dynamics of the system out of steady state, we need to specify how expectations evolve. We think of Eg_t as incorporating all relevant information into expectations available at time *t-1*. Suppose that the new information arriving at time *t* about growth relative to the previous expectation (that is, g_t relative to Eg_t) induces a change in the subsequent expected growth rate (Eg_{t+1}) in the same direction as the most recent expectation error. More formally, let the dynamics of expected growth follow the simple rule

$$\begin{aligned} &g_t > Eg_t \Rightarrow Eg_{t+1} > Eg_t \\ &g_t = Eg_t \Rightarrow Eg_{t+1} = Eg_t. \\ &g_t < Eg_t \Rightarrow Eg_{t+1} < Eg_t \end{aligned} \tag{2.12}$$

Under this assumption, equations (2.11) and (2.12) show that the warranted path is *always unstable*. Indeed, because the only non-degenerate steady state is given by the warranted path, the model dynamics are always unstable. Growth rises without bound if growth expectations are disturbed above g^*, and output converges to zero if expectations are disturbed below g^*.[14]

The instability property is generic to the Harrod model structure. It does not depend on parameter values as long as the rule for expectation formation in equation 2.12 holds (see Ferri et al., 2011, for a general discussion). And 2.12 seems reasonable. Suppose that expected growth is 3% in period 1 and actual growth in period 1 is 3.5%.

[14] Also see Kregel (1980), Sen (1970), Fazzari (1984), and Palley (1996) for related interpretations of Harrod instability.

What will happen to expected growth for period 2? If the disturbance to growth is perceived as entirely temporary, perhaps expectations for period 2 would remain at 3%. But if there is just the slightest bit of uncertainty that there may be some new information in the actual 3.5% growth rate that prevails in period 1, expectations for period 2 should rise above 3%. The increase in expected growth may be small, but a small move in the direction of the most recent expectation error is all that is required for instability. That is, if expectations for period 2 rise by a small increment ε, actual growth in period 2 will be even higher than the 3.5% actual rate for period 1, by an increment that *exceeds* ε (this statement is implied by 2.11). The expectation updating rule in 2.12 will therefore be validated by the operation of the actual model dynamics.

3. Containing Instability

Explosive or implosive dynamics do not seem realistic. While we see instability in modern developed economies, growth does not rise to infinity or collapse to zero. Indeed, as discussed above, modern developed economies have demonstrated persistent, but bounded growth over long sweeps of time. It is therefore not surprising that researchers should dismiss the "inconvenient truth" of Harrods's knife-edge steady state, either by rejecting the model or by ignoring dynamics off the warranted path. We propose a different response, one that accepts the basic instability of the model as a robust feature of economic growth, but with additional mechanisms to contain the instability within realistic bounds.[15]

Productive Capacity: The Supply Side as a Ceiling

A natural source for a ceiling on positive instability is a supply-side resource constraint (for a discussion, see Ferri, 1997). The model in section 2 is driven entirely by demand under the implicit assumption that all that households and businesses want to purchase can indeed be produced. But growth in demand at an increasing rate will eventually push the system toward the full employment of its resources. Harrod called the exogenous growth rate of the labor force, possibly adjusted for productivity changes, the "natural" rate of growth, and this growth rate defines a path of potential output that represents full employment of labor. In detail, assume that potential output is initially Y_0.

[15] Our approach is closely related to the ceiling and floor model of Hicks (1950). In a comment that almost directly foreshadows the results of this section, Asimakopulos (1997, p. 299) reviews Harrod's growth model and specifically mentions that upward deviations in the actual growth rate would be contained by a "ceiling" determined by "shortages of productive capacity and labor" and downward movements in growth bounded by a "'floor' which is set by autonomous expenditures."

Labor supply and labor productivity grow at a compound exogenous rate g^N. Then, labor resources will constrain aggregate supply to be no greater than

$$Y_t^s = Y_{t-1}^s(1+g^N) = Y_0(1+g^N)^t, \tag{3.1}$$

and actual output will be the minimum of aggregate demand and aggregate supply.[16] We do not impose capital limitations on supply. There are two reasons for this asymmetry between capital and labor. First, capital is produced while labor is not. The reason for investment as specified above is to accumulate capital to a level appropriate to produce anticipated output. Second, empirical evidence suggests that capital is hardly ever a binding constraint. This is not only because firms invest in capital in anticipation of output growth, but also because firms seem to operate with excess physical capacity. The U.S. capacity utilization rate in manufacturing has averaged not much above 80 percent even in good economic times (much lower in recessions); utilization has not been above 85 percent since the 1970s. A desired rate of capacity utilization below 100 percent can be easily incorporated into the target capital-output ratio v^*. The assumption that capital constraints do not bind restricts the relevance of our model to the moderate growth fluctuations of the kind observed in developed economies over recent years. A different approach might be needed in an exceptional boom when capital constraints bind, but the only historical case in recent U.S. history when such circumstances prevailed in the aggregate was World War 2. Fast-growing developing countries, however, may well

[16] Also see Minsky (1982, page 260). Skott (1989) proposes a limit to instability due to the rise in the strength of labor relative to firms caused by high employment rates. Palley (1996, equation 7) proposes a supply constraint on unstable demand dynamics. In his case, however, the constraint is that actual *growth* is the minimum of demand growth and supply growth. In our case, we restrict the *level* of output to be the minimum of the *level* of demand and supply.

have capital constraints on output and little constraint on labor, which would require a different approach to the one developed here.[17]

The supply ceiling restriction creates a discontinuity in the system dynamics that makes analytical results difficult and uninformative when the actual path hits the ceiling. We analyze the behavior of the model in the neighborhood of the ceiling with simulations. In the simulation model expected growth adjusts adaptively to actual growth according to

$$Eg_{t+1} = (1-\alpha)g_t + \alpha Eg_t \tag{3.2}$$

with a weight (α) of 0.3 on lagged expected growth (adjusted as discussed below). The desired capital-output ratio v^* is 0.6 and the depreciation rate is 0.1. Potential output grows at 3 percent per year and the saving rate is calibrated to produce a 3 percent warranted rate of growth. (We discuss the possibility that the natural and warranted rates of growth differ in section 4.) The system is initialized with the warranted growth rate equal to the expected and actual growth rates, with 4% unemployment of labor.

What happens when a small positive shock to expected growth initiates unstable demand dynamics that push output up against the resource ceiling? As the system approaches the ceiling, actual and expected output are growing faster than the natural rate of supply expansion. When the supply constraint binds, actual growth is limited to g^N and demand falls quickly below the ceiling, and below expected demand. The actual rate is now below the warranted rate. The system bounces off the ceiling onto an unstable declining growth path.

[17] Therefore, the relevant context for our model is what Skott and Zipperer (2012) label "mature" economies in which development has proceeded to the point that there is not an infinitely elastic supply of labor available outside of the modern sector of the economy.

The basic interpretation is straightforward. Upward demand instability can drive demand to a level that fully employs labor resources. But the full employment path is not stable. We shall return to discuss the role of the ceiling and momentary full employment below in the context of the full model.

Autonomous Demand to Contain Downward Instability

In this subsection, we demonstrate that the presence of an autonomous component of demand, one that evolves independently from the state of the economy, is sufficient to contain downward instability in growth. We denote autonomous demand by F_t and consider below the effect of various specifications for the time path of this variable. One can think of F_t as government spending, but it could also include autonomous components of private consumption or investment.[18]

With the addition of F_t, demand (and output, as long as demand is less than potential supply) becomes

$$Y_t = (1-s)(1+Eg_t)Y_{t-1} + v^*(1+Eg_t)^2 Y_{t-1} - (1-\delta)K_t + F_t\,. \tag{3.3}$$

It is clear that a positive value of F_t prevents demand and output from collapsing to zero. The growth rate law of motion, conditional on expectations, is:

$$1+g_t = (1-s)(1+Eg_t) + v^*(1+Eg_t)^2 - (1-\delta)\frac{K_t}{Y_{t-1}} + \frac{F_t}{Y_{t-1}}. \tag{3.4}$$

[18] Minsky (1982, page 260) imposes a similar floor condition. In Hicks (1950) the autonomous demand floor comes from an autonomous component of investment.

Inspection of equation 3.4 shows that if g_t is on a downward path toward negative one, growth must eventually turn positive because as long as F_t does not decline (or at least declines more slowly than Y_t), shrinking Y_{t-1} will eventually cause the final, always positive, term of 3.4 to dominate the determination of growth.[19]

To understand these dynamics more fully, we first consider the conditions under which expectations are realized in the model with autonomous demand. Unless F_t grows at the same rate that equates actual and expected growth, the model will not have a steady state with a non-zero growth rate. This is clear because the ratio F_t / Y_{t-1} in equation 3.4 cannot be constant unless F_t and output grow at the same rate. At any time t, however, we can define a growth rate $\hat{g}_t$ such that

$$Eg_t = \hat{g}_t \Rightarrow g_t = Eg_t. \tag{3.5}$$

Let $v_t = K_t / Y_t$ denote the actual capital-output ratio and define $f_t = F_t / Y_t$. Then 3.4 and 3.5 imply that

$$\begin{aligned}
1+g_t &= (1-s)(1+Eg_t) + v^*(1+Eg_t)^2 - (1-\delta)\frac{K_t}{Y_{t-1}} + \frac{F_t}{Y_{t-1}} \\
1+g_t &= (1-s)(1+Eg_t) + v^*(1+Eg_t)^2 - (1-\delta)(1+g_t)\frac{K_t}{Y_t} + (1+g_t)\frac{F_t}{Y_t} \\
1+\hat{g}_t &= (1-s)(1+\hat{g}_t) + v^*(1+\hat{g})^2 - (1-\delta)(1+\hat{g}_t)v_t + (1+\hat{g}_t)f_t \\
1 &= (1-s) + v^*(1+\hat{g}_t) - (1-\delta)v_t + f_t \\
v^*(1+\hat{g}_t) &= s + (1-\delta)v_t - f_t \\
\hat{g}_t &= \frac{s-f_t}{v^*} + (1-\delta)\frac{v_t}{v^*} - 1
\end{aligned} \tag{3.6}$$

[19] Firm investment behavior will strive to keep the capital stock in line with output, thus the term K_t / Y_{t-1} will not grow without bound as Y_{t-1} declines. If gross investment is bounded below by zero but output continues to shrink the capital stock will decline at rate δ, so if F_t is constant or growing, the final term of 3.3 must eventually dominate.

Note that $\hat{g}_t$ will be time varying in general because f_t and v_t change over time.[20]

Consider the behavior of $\hat{g}_t$ along a negative growth path. First, suppose f_t equals zero as in the basic Harrod model discussed in section 2. The dynamics of $\hat{g}_t$ are then driven entirely by the disequilibrium between the actual and target capital-output ratios (if f_t is zero, v_t is the only time-varying component on the right-hand side of equation 3.6). Because the system is on an unstable negative path, $g_t < Eg_t$ and the capital stock exceeds the target level *ex post*. This pushes v_t and $\hat{g}_t$ upward while actual g_t declines. Because the g_t and $\hat{g}_t$ paths never cross, the downward dynamics of growth are never arrested and output converges to zero. This result is simply a restatement of the downward Harrod instability discussed in the previous section. But if F_t is constant (or growing), f_t grows as Y_t falls, pushing $\hat{g}_t$ downward, other things equal.

Will $\hat{g}_t$ necessarily catch up with g_t? The answer is yes. Suppose for the moment that $F_t = F$ for all time. It is clear that as the economy declines it will eventually reach a minimum level of output, since demand can never fall below F. Thus, even though growth has been negative along the declining path, the actual growth rate must eventually hit zero. If actual growth is zero with negative expected growth, the actual rate exceeds the expected rate and this implies that $\hat{g}_t$ has fallen below actual growth. (If F_t is growing over time, f_t rises even faster, pushing $\hat{g}_t$ down even more quickly. So the demonstration

[20] If the actual and expected growth rates of demand are equal to g^*, and the growth rate of F is set to g^*, we can solve for a steady state such that $f_t = f^*$ and $v_t = v^*$. In this case, equation 3.2 implies that $g^* = [(s - f^*) / v^*] - \delta$ which could be viewed as a generalization of Harrod's warranted rate. But the existence of this steady state depends on the arbitrary assumption that F_t grows at exactly g^*.

that $\hat{g}_t$ catches up with g_t with constant F is sufficient to obtain the same result for rising F_t.)[21]

The state of the model after $\hat{g}_t$ falls below g_t is analogous to the Harrod instability result discussed in section 2. Actual growth rises and output follows a locally unstable path. Instability is clear since the model is the same as Harrod's except for the F_t term in equation 3.4. This term is predetermined and therefore does not affect the derivative of g_t with respect to Eg_t that proves instability (see equations 2.9 and 2.10).

How far can output fall? Clearly the value of F_t is critical to answering this question. Suppose output hits a local minimum value Y_t' at some point along its dynamic path. At this time, the actual growth rate approaches zero, and lagged output equals current output.[22] From equation 3.6 we have:

$$1+g_t=(1-s)(1+Eg_t)+v^*(1+Eg_t)^2-(1-\delta)(1+g_t)v_t+(1+g_t)\frac{F_t}{Y'_t}$$

and since actual growth is zero at Y_t',

[21] Harrod (1939, pages 28-29) recognizes the possibility of a similar phenomenon: "As actual growth departs upwards or downwards from the warranted level, the warranted rate itself moves, and may chase the actual rate in either directions. The maximum rates of advance or recession may be expected to occur at the moment when the chase is successful." In our context the warranted rate that is "chasing" the actual rate is analogous to $\hat{g}_t$ and the maximum rate of "recession" occurs when $\hat{g}_t = g_t$.

[22] Because the model is specified in discrete time, output may reach a minimum when growth is negative but switches to positive in the subsequent period. Nonetheless, the analysis presented here for actual growth of zero provides a lower bound on output just prior to the switch in the sign of actual growth. Furthermore, a growth cycle could occur without actual growth every becoming negative. Instead falling, but positive, growth could switch to rising growth before output hits a minimum. In this case, the analysis in the text represents a non-binding lower bound on actual output just prior to the switch in the direction that actual growth changes.

$$1 = (1-s)(1+Eg_t) + v^*(1+Eg_t)^2 - (1-\delta)v_t + \frac{F_t}{Y'_t}$$

$$Y'_t = \frac{F_t}{s(1+Eg_t) - Eg_t - \left[v^*(1+Eg_t)^2 - (1-\delta)v_t\right]} \tag{3.7}$$

This result for the value of output at (or near) a local minimum of the dynamic business cycle is analogous to a *static* Keynesian multiplier outcome, given the level of autonomous expenditure F_t. The first term in the denominator of 3.7 is the marginal propensity to save out of an increase in lagged income. The bracketed term in the denominator is the marginal propensity to invest out of lagged output.[23] Equation 3.7 shows that the floor on output over time rises with the size of autonomous demand and any consumption and investment demand induced by the presence of autonomous demand. That is, an economy with a high propensity to consume or invest (low s or high v^*, respectively) will have a higher output floor that contains its downside dynamics. Furthermore, for reasonable parameter values (particularly a small value of s) the floor is much higher than autonomous demand itself.[24]

Before leaving the discussion of F_t, we need to consider how the addition of an autonomous component of demand affects the behavior of upward instability. If this component is large relative to total demand, it can constrain upward instability *before* the system reaches the resource-constraint ceiling. The definition of $\hat{g}_t$ from equation 3.6 helps explain this possibility. As the economy is on an upward growth path f_t can decline

[23] To simplify the interpretation of equation 3.6 further, suppose Eg_t is zero and $v_t = v^*$ at Y_t'. Then the right side of 3.6 reduces to $F_t / (s - \delta v^*)$. Because expected growth is zero and capital is at the desired level, the gross propensity to invest out of lagged income is just equal to depreciation.

[24] There is a similarity between this result and the "supermultiplier" model presented by Bortis (1997). The supermultiplier models focus on potential rather than actual output, but autonomous expenditure plays an important role. Bortis (page 153) writes "autonomous expenditures … act as an engine which initiates the production of consumption and investment goods."

if F_t grows more slowly than output. If the autonomous demand is a large share of total demand (f_t is large) this effect can dominate the dynamics of $\hat{g}_t$, so that $\hat{g}_t$ catches up with g_t before output reaches the full-employment limit. In this case, expected growth exceeds actual growth, and expected growth declines. Thus, the presence of autonomous demand can prevent upward instability from pushing the system to full employment. We will discuss the conditions under which this situation arises in more detail in the next section.

4. Patterns of Growth

The model presented in the previous sections leads to three qualitatively different kinds of growth paths that we describe next.

Excess Demand

There is nothing in our model that prevents the floor level of output defined in equation 3.6 from becoming arbitrarily large (let s approach zero, for example). It is therefore possible for the floor on demand to approach and exceed the resource constraint ceiling from equation 3.1. In this case, the resource constraint is always binding, and there is persistent excess demand. Output grows along the supply-determined path. Note that downward instability described in section 2 does not occur, even briefly, because the floor on output is so high.

Equation 3.7 shows that the demand floor exceeds exogenous potential supply if either autonomous demand is high or the static Keynesian multiplier is large. This possibility relates in an interesting way to the classic problem, identified by Harrod, that the steady-state “warranted” growth rate may not be equal to the “natural” rate of supply growth. *If* the warranted path were an appropriate proxy for actual output, then a low warranted rate would suggest indefinitely rising unemployment. But, as we have seen, the warranted rate, if it exists at all, is not a stable equilibrium for the system and therefore should not be viewed as a theoretical proxy for actual growth. Indeed, a low warranted rate corresponds to a low saving rate and a high investment rate (high value of v^*), both of which *raise* the output floor induced by autonomous demand and make the situation of excess demand in which labor resources are persistently fully employed more likely. In this sense, the dynamics of our model exhibit a paradox of thrift (also see Harrod, 1939, pages 30-31).

While persistent excess demand and a full-employment growth path are possible outcomes of our simple model, this situation does not seem to describe empirical growth paths in modern developed countries. Moments of strong full employment seem to be fleeting and generalized excess demand is almost never observed.[25] A detailed explanation for why this is the case is beyond the scope of this paper, but we offer three observations here. First, as the economy approaches a state of excess demand, endogenous forces could lead to an acceleration of the natural rate of growth. Both profit opportunities and sources of funding for investment will expand as the result of strong economic performance associate with high demand. These conditions could spur innovation that raises productivity and relaxes supply constraints. This favorable outcome may be most likely if excess demand is modest so that innovation induced by a high-utilization economy can expand capacity before severe supply bottlenecks or

[25] In the U.S., the only clear situation of generalized excess demand seems to be during World War 2. Although accelerating inflation with very low unemployment in the middle 1960s suggests that the growth path may have approached excess demand.

inflationary pressures arise. A second possibility, likely of relevance in recent decades, is that fear of accelerating inflation as unemployment falls leads to monetary tightening that chokes off demand. Third, the financing of high rates of demand growth could raise financial fragility, as discussed extensively by Minsky (1986). This fragility could curtail strong investment and consumption. The result would be a decline in the demand floor to levels that eliminate excess demand and open the door to downward instability. We leave exploration of these issues for further research.

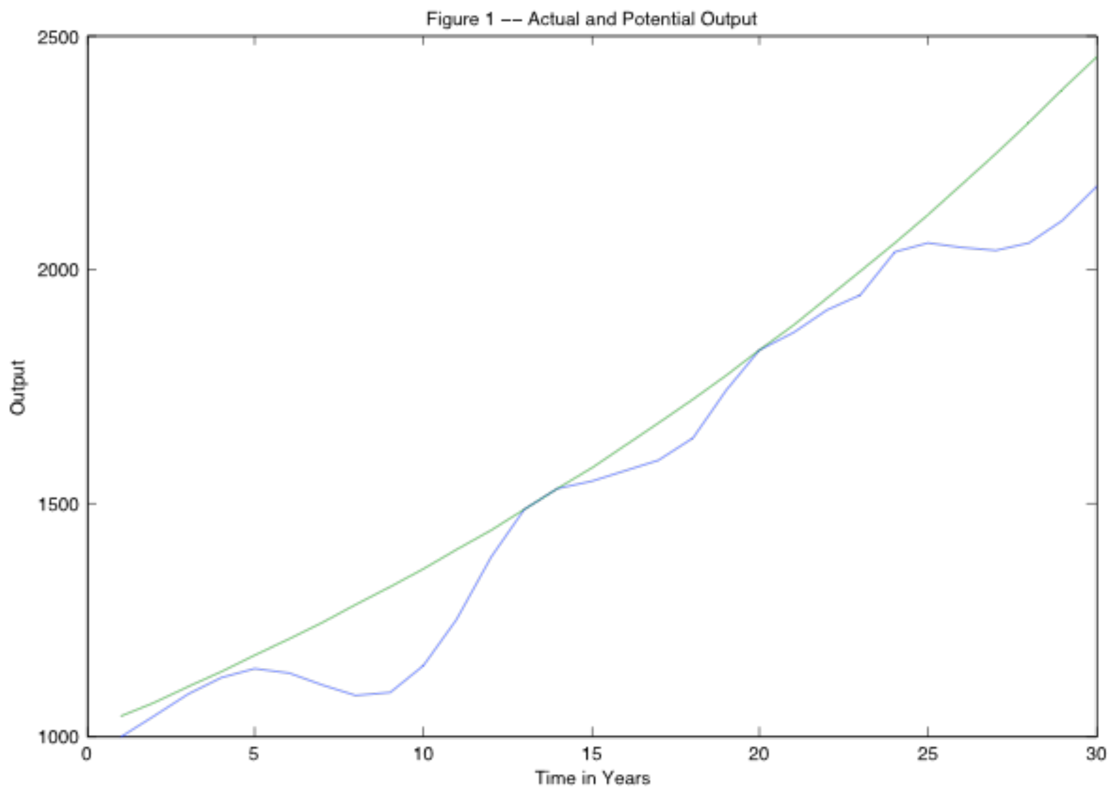
Cyclical Demand-Driven Expansion

A more realistic growth pattern arises when the dynamic floor for output lies below the supply-determined ceiling. In this case, the actual growth path predicted by the model cycles in a bounded range. If both the resource constraint and the autonomous demand variable are growing over time (not necessarily at the same rate), the bounds that contain the actual path of the economy will expand as well. Because the economy occasionally touches the growing resource constraint, actual output will exhibit secular growth, consistent with observed actual growth paths in developed economies, as discussed in the introduction.

This growth path is fundamentally different from results generated by most other growth models. The output path does not converge to a full-employment steady state; it follows an endogenous cyclical path between the floor and ceiling. Full employment constrains growth temporarily for brief moments of time, but demand determines output and growth at almost all points. In addition, the lower bound on the output path is entirely demand determined according to typical Keynesian logic. The floor defined by equation 3.7 rises toward the resource constraint ceiling as autonomous demand becomes larger or as parameters change that cause the consumption and investment induced by autonomous demand to increase.

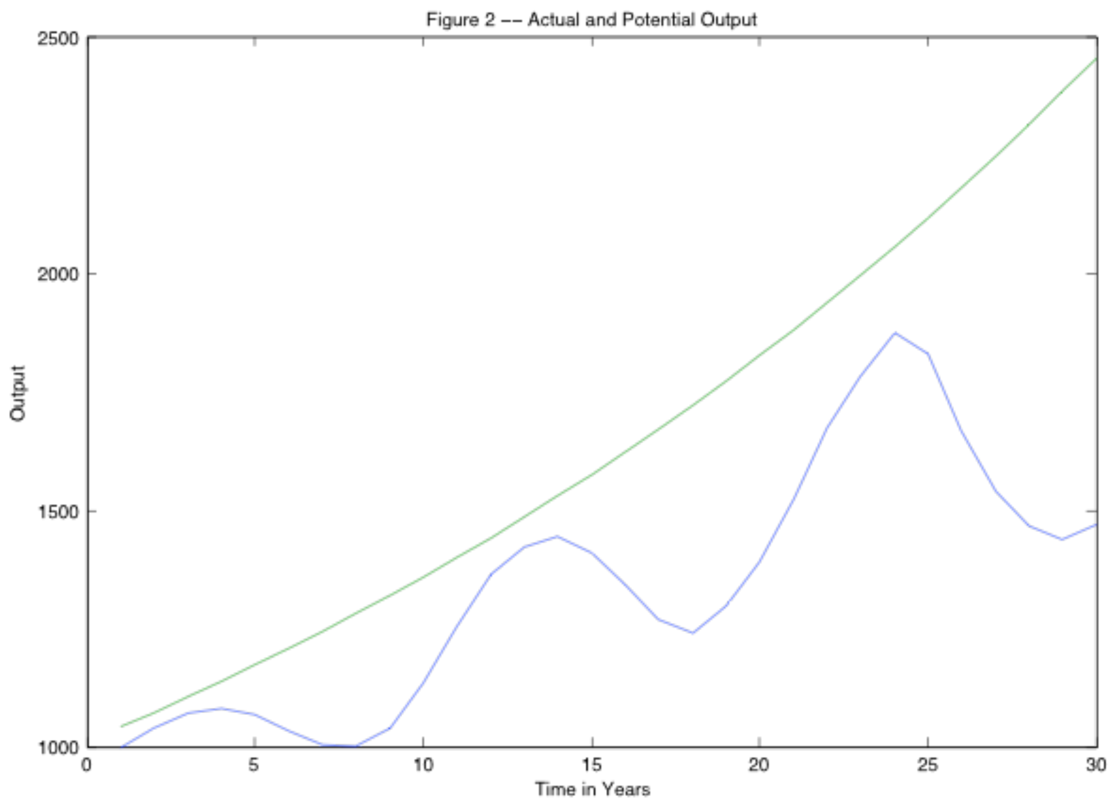
Figure 1 presents a simulated growth path produced by the model. Recognizing that demand will fluctuate for a variety of reasons not captured by the basic model laid out in section 2, we added stochastic shocks to expected growth with a standard deviation of one percentage point per year. The stochastic shocks make the cycles irregular, and hence more realistic than the smooth cycles the model produces in the absence of shocks. Note how output occasionally touches or closely approaches the full-employment potential path, but does not stay there. As the relative share of autonomous demand declines, or as the Keynesian multiplier falls, the gap between floor and ceiling widens, which allows output path to wander further away from the potential path. For example, the average deviation between the potential and actual path in figure 1 is 5.6 percent with autonomous demand initially set to 30 percent of total demand. If autonomous demand is initially 28 percent, the average gap widens significantly to 13.7 percent (holding all else constant, including the sequence of stochastic shocks to expected growth).[26] Thus, the dynamic characteristics of the growth path appear to be quite sensitive to the size of autonomous demand.

[26]The simulation model used to generate figure 1 has the benchmark parameters described earlier in the text. Potential output and autonomous demand both grow at 3 percent per year. The saving rate is calibrated to 0.378, which produces a 3 percent warranted rate of growth. (A warranted rate exists because potential output and autonomous demand are set to grow at the same rate.) Note that the saving rate must be high enough to accommodate both investment and autonomous demand in steady state. If autonomous demand is government spending, it would be appropriate to interpret part of what appears as saving in this model as income taxes imposed at a constant marginal rate.



Cyclical Peaks Below the Potential Path

As discussed in section 3, it is possible that the presence of autonomous demand imposes an endogenous demand ceiling on the growth cycle below the level of the resource-constraint ceiling. Figure 2 shows this situation. The parameter values and shocks are the same as those that generated figure 1 with three exceptions. First, autonomous demand grows at 2 percent, less than the 3 percent growth of resources. Second, the initial share of autonomous demand is higher, 40 percent rather than 30 percent, so that the F_t / Y_{t-1} term in equation 3.3 plays a relatively larger role in the dynamics. Third, to offset the effect of higher autonomous demand on the level of the floor, the saving rate is raised by 10 percentage points. These changes start the demand floor and resource-constraint ceiling at approximately the same place they were for figure 1, but autonomous spending is a bigger share of initial demand and grows more slowly.



The peaks of the cycle do not reach the potential path. The gaps get larger as time proceeds. In this case the growth path is entirely demand determined and there is no endogenous mechanism that drives demand high enough to absorb the economy's resources. (The cycle is also more regular because the endogenous dynamics dominate the stochastic shocks.)

This growth pattern, like the excess demand case discussed above, seems somewhat unrealistic. Modern economies do seem to have occasional moments of full employment. We conclude that the cyclical path that reaches potential, such as the one depicted in figure 1, is the most empirically relevant. But we must recognize that the dynamics in figure 2 remain possible.

5. Conclusion: Demand and Growth

In the model presented here, demand drives growth at almost all points in time. Of course, growth cannot persist indefinitely without expanding supply. But in the most realistic cases produced by our model, the growth path is usually constrained by demand, not supply. That said, supply can determine the upper envelope of the growth path. With some sets of parameter values, the output path occasionally pushes up against resource constraints, but full employment is not a dynamic equilibrium.

The source of secular demand growth is the fundamental upward instability of the dynamics of investment and consumption, as identified decades ago by Harrod. Because our model does not rely on a price mechanism that drives demand toward potential output, it avoids the unrealistic neoclassical synthesis assumption, implicit in traditional neoclassical growth theory, that disinflation or deflation necessarily pushes aggregate demand to full employment levels in the "short run."

This result shifts the interpretation of Harrod's instability. In much of the literature that followed Harrod, the dynamic instability of the model was treated as a theoretical problem, because real-world economies do not have knife-edge properties. For example, Bortis (1997, page 134) rejects "Keynesian growth models of the Harrod type" because they "imply too high a degree of instability." In contrast, instability helps align the predictions of our model with broad empirical facts because it is the source of endogenous demand growth that is necessary to generate long-term growth paths consistent with observed histories of modern economies (also see Skott, 1989 in this regard).[27]

Harrod's instability, however, operates in both directions. It is naturally limited on the high side by resource constraints. To contain downward instability, we introduce an autonomous component of demand, like authors such as Hicks (1950), Minsky (1959, 1982), and Asimakopoulos (1997) before us. We have shown that autonomous demand has a profound effect on the model dynamics. It induces a floor that turns around negative dynamics toward growth. If autonomous demand is a large share of total demand, its presence can also constrain the upward instability, so it is possible that demand and output never reach the full-employment path.

We focus on demand-led dynamics that transmit outcomes in period t to demand in period $t+1$, an approach that leads to the basic Harrod results. By adding autonomous demand, however, the model reveals a feature analogous to the static Keynesian theory. The floor on the growth dynamics at a minimum point of the business cycle is the value of autonomous spending times a multiplier that depends on marginal propensities to spend in a familiar way. In this sense, the model merges aspects of static and dynamic Keynesian theory.

One could interpret these results to infer that demand does not affect the long-term growth rate that occurs between the points when the economy occasionally hits the supply-determined ceiling. From an empirical point of view, the result could hardly be otherwise. If, as a matter of fact, economies occasionally nudge up against full use of

[27] Sources of upward instability beyond the investment dynamics analyzed in this paper could also explain demand growth. For example, commodity booms could be important, particularly for emerging market countries.

their resources, an upper envelope of the growth path that connects two points of full employment will necessarily grow at the rate of expansion of supply during the period between these two points. But we argue that it would be fundamentally misleading to argue from such a result that demand does not matter for growth for several reasons. First, the moments of full employment may be far apart in time (even decades) and *everywhere* in between such moments the growth path is demand determined.[28] Second, the corridor in which the growth path fluctuates is fundamentally determined by demand since its lower bound arises from simple Keynesian multiplier principles. Third, while in our simple model the supply side path is assumed exogenous, it is easy to posit realistic economic channels through which the actual demand-determined performance of the economy away from full employment affects conditions of supply. The quantity and productivity of labor and capital at occasional business-cycle peaks will likely depend on the demand-determined performance of the economy in the normal case in which the system is below full employment. Such effects have been studied in other heterodox growth models (see the approaches surveyed by Dutt, 2010, pp. 236-238, for example) and aspects of neoclassical endogenous growth theory are also likely to be relevant along these lines.

This approach opens a variety of other new directions for future research. The fundamental importance of autonomous demand suggests a critical role for government spending in determining the bounds of economic fluctuations. Indeed, countercyclical

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[28] An anonymous referee points out that one can interpret moments of full employment as exceedingly rare in the U.S., arguably associated in the past 80 years only with major wars (World War 2, Korea, and Vietnam) or the dot.com bubble of the late 1990s. With this view of recent history the model here could be interpreted as optimistic because it predicts occasional moments of full employment endogenously based on private investment dynamics alone. We accept this criticism and propose that reasonable extensions of the model could limit the economy's ability to reach the resource constraint as suggested in the following paragraphs.

fiscal policy could add an additional dynamic relationship to the model that we plan to consider in future work. The significance of autonomous demand also implies the need to further study what might make some share of private demand roughly autonomous. Innovation investment, driven more by new ideas than demand expectations, could induce a autonomous component to investment as well as affecting supply constraints along the lines discussed above.

A particularly intriguing possibility arises from the role of income distribution. Models in the Kaleckian tradition (see the overview and references in section 1 above) emphasize the distinction between "profit-led" and "wage-led" growth. In the former case, the demand stimulus from the effect of a higher profit share on investment raises the growth rate, dominating the dampening effect of a higher profit share on consumption. In wage-led regimes these effects are reversed. Investment behavior linked to income distribution could affect the supply-determined ceiling in more complicated ways then it does in our model. Both consumption and investment also have demand effects on the floor of our model, and income distribution is likely to have a significant effect on the floor. For example, if high-income people have a large share of national income, autonomous consumption may be relatively large, but the saving rate is likely to be larger than in an economy with more equality. These two factors have opposite effects on the dynamic floor of the growth path, and their impact deserves further research.

The simple model presented here abstracts from any consideration of finance. This makes the model less relevant for understanding recent events in a variety of countries. The recent U.S. business cycle did not turn negative at the outset of the Great Recession because the economy had hit full employment. Rather, the downturn was triggered by financial crisis arising from unsustainable household finance. This event, and similar instances of financial crisis as the trigger that turns an economy from expansion to contraction, suggest that financial fragility can impose a growth ceiling that contains unstable upward demand dynamics. This approach was championed by Minsky for decades, and it merits further study in the framework presented here. In addition, the rising importance of finance for consumer spending strongly suggests that consumption dynamics could play a much more important role in demand growth than is the case with the passive income-based consumption specification employed here.[29]

Further extensions of this model could address questions that arise from the perspective of neoclassical theory. We have not discussed price and wage adjustment. It would be straightforward to simply add a Phillips Curve mechanism to explain inflation and inflation expectations, but this feature would not affect the real growth path of the model unless we add a feedback channel between nominal dynamics and real demand or supply. Neoclassical growth models assume such a channel, usually implicitly, because price and wage adjustment push demand to the full employment path through real balance effects. But as discussed above we consider this kind of feedback of little empirical relevance. Indeed, the more likely feedback effect of price adjustment dynamics is that deflation would exacerbate downward demand instability, a channel that deserves exploration in an extension of the model that incorporates debt and finance.

Because our approach hearkens back to Harrod, it is appropriate to conclude this list of extensions by mentioning Solow's (1956) argument that growth instability arises

[29] Hein and van Treeck (2010) and Palley (2010) discuss finance and consumption in the context of demand-led growth. Also see Skott and Ryoo (2007).

from the assumption of fixed coefficient (Leontief) technology. One could explore the effect of endogenous variation in the capital-output ratio (v^*) in our model, although that would also require explaining why v^* should change, for example by adding endogenous dynamics to interest rates and the cost of capital. We are skeptical, however, that endogenous changes in v^* have much to do with empirical growth paths of modern economies. The aggregate capital-output ratio has been remarkably stable over long periods of time.[30]

Harrod provided an intuitive and, from our perspective, robust approach to understanding Keynesian demand dynamics in the immediate aftermath of the publication of the *General Theory*. For a variety of reasons, however, this approach has been mostly relegated to a backwater of growth theory. We contribute to a literature that has developed Harrod's original ideas by reinterpreting one reason that some authors rejected the Harrod model: the knife-edge instability of the model's steady-state growth path. We propose that Harrod's instability, appropriately contained, can help to explain a central characteristic of modern industrial economies: the persistent, but unstable, growth of demand over long sweeps of time.

[30] Also see the discussion in Skott (1989, pages 76-78) that supports this conclusion.

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