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WHAT ECONOMISTS SHOULD LEARN FROM ECONOPHYSICS

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Abstract-We state the usual postulatory approach used by economists and then contrast it with our empirically based discovery of the dynamics of financial markets, where all predictions are calculated from 'the market Green function'. In particular, we predict option prices in agreement with traders' valuations, but without using any nonempirically determined parameters. Both global and local volatility are defined via the noise traders' diffusion coefficient, and a new dynamic definition of 'value' is given. Self-fulfilling prophecies are discussed in the context of complexity.

Keywords: Market dynamics, Markov processes, volatility, option prices, economics and finance

1. Introduction

Economists postulate a model, nearly always stationary (near statistical equilibrium), and try to force fit it to empirical data by a best choice of arbitrary parameters (see, e.g., Chow & Kwan, 1998). They conclude that the data are too hard to fit over long time intervals. One object of this lecture is to explain that economic data are too easy to fit. In particular, finance data are very easy to fit over all observable time scales, and other economic data are far more sparse than finance data.

Journal Information

Neo-classical economic theory (utility maximization) is a falsified model but is still taught in all leading textbooks as if it would describe the ideal standard to be achieved by real markets (the model is used by The World Bank, the IMF, the EU, the US Treasury, and the US Federal Reserve (Stiglitz, 2002)). Another main aim of this work is to make economists aware that they must discard all existing standard texts and start over again, using empirically-based modelling (see also Soros (2000) for a related viewpoint).

By market dynamics we mean excess demand dynamics: $dp/dt=\varepsilon(p,t)$ where $\varepsilon(p,t)$ = excess demand = D(p.t)-S(p,t), and where D(p,t) is demand and S(p,t) is supply at price p and time t. Financial markets suggest stochastic dynamics: price changes on the smallest time scales ($\Delta\tau\approx 1$ sec.) are not predictable, whereas in deterministic dynamics, even in chaotic and complex systems, changes on the smallest time scales are easily predictable due to local integrability.

In stochastic dynamics, excess demand is modelled as drift plus noise. Ignoring for the time being the fact that price and time changes in markets are discrete, excess demand dynamics in finance markets is pretty well described by the stochastic differential equation (sde)

$$dp=prdt+\sqrt{p^2d(p,t)}dB(t) \qquad (1)$$

where r is an interest rate, $p^2d(p,t)$ is the price diffusion coefficient and dB(t) is the Wiener process (dB/dt is white noise, but we use Ito calculus in order to preform coordinate transformations on (1) easily and systematically). The function d(p,t) characterizes the market and must be discovered from the data (McCauley, 2004). If d(p,t) would be constant then the price distribution would be lognormal (and the returns $x=\ln p$ would be Gaussian). But for real markets $d(p,t)\neq$constant and cannot be merely postulated.

Laws of physics are based on the four standard space-time symmetry principles (Wigner, 1967). Are there any corresponding symmetry principles for markets? Only one is known: the 'no arbitrage' condition applied to a single asset in spatially separated markets is a geometric invariance principle analogous to rotational invariance. There are no other known space-time invariance principles for markets.

1.1 Proving 'equilibrium' without dynamics

Economists love to prove that equilibrium 'exists' mathematically in a model, but dynamics is generally ignored. We now illustrate why existence proofs without dynamics are dangerous. The lognormal pricing model is defined by the sde (1) with $d(p,t) = \sigma_p$ = constant. The corresponding price density g(p,t) satisfies the Fokker-Planck partial differential equation (pde)

$$\frac{\partial g}{\partial t} = -r\frac{\partial}{\partial p}(pg) + \frac{{\sigma_p}^2}{2}\frac{\partial^2}{\partial p^2}(p^2 g) \qquad (2)$$

whose fundamental time-dependent solution (Green function) is the lognormal distribution. The condition for statistical equilibrium is solved by the nonnormalizable function $g(p)=Cp^{-2r/\sigma 2}$ but statistical equilibrium is never reached by the (normalizable) lognormal Green function, which instead vanishes as t goes to infinity. Unbounded prices mean that equilibrum can't be attained (due to the continuous spectrum of the Fokker-Planck operator). Price controls, bounds on p(t), produce statistical equilibrium asymptotically via a discrete spectrum. The lognormal model is nonstationary, describing a hypothetical unstable market where the Gibbs entropy increases without bound. In spite of this fact finance theorists still talk about 'equilibrium markets'.

2. The Myth of the Invisible Hand

How would a hypothetical equilibrium market behave empirically? Market equilibrium would require that g(p,t) is asymptotically stationary (t-independent) over an observable market time scale (a week, a month, a year), or equivalently, that all moments of the distribution g(p) must become constants, independent of time t. Statistical equilibrium demands a stationary process asymptotically. In particular, the average drift $\langle pr(p,t)\rangle$ must vanish, guaranteeing that $\langle\varepsilon(p,t)\rangle=0$, and because all higher moments of the distribution must be constant as well, the variance

$$\sigma^2 = \left\langle \Delta p^2 \right\rangle = \left\langle p^2 \right\rangle - \left\langle p \right\rangle^2 = \int_t^{t+\Delta t} \left\langle p(s)^2 d(p,s) \right\rangle ds \qquad (3)$$

must be asymptotically constant for a long enough time interval (t, t+Δt). This condition on the variance is easily tested and is badly violated by real markets, as is the condition for vanishing excess demand. In other words, real markets are far from statistical equilibrium, equilibrium (stationary process) is a completely illegal zeroth order approximation to market reality.

Why should anyone care about equilibrium? Because were equilibrium to hold, then we could take the equilibrium price p* to be either the average or most probable price, and this would yield a t-invariant definition of "value", in agreement with neo-classical economic theory. In the absence of equilibrium there is no t-invariant definition of „value“: we will explain in part 6 that the ideas ‚undervalued' and ‚overvalued' when applied to future asset prices are effectively subjective. Next, we compare the equilibrium predictions with the empirical facts about financial markets.

3. Real financial markets

It 's necessary empirically to study logarithmic returns

$$x(t) = \ln(p(t)/p_o) \qquad (4)$$

rather than prices p(t), price increments δp, or small returns $\delta p/p$, where p_o is a reference price, because the variable x is both additive and units-free. A Markov process (1) is a good zeroth order approximation because it agrees with the efficient market hypothesis (EMH), which simply means that markets are very hard to beat. Using Ito calculus, the sde for x is

$$dx = (r - D(x,t)/2)dt + \sqrt{D(x,t)}dB(t) \qquad (5)$$

where D(x,t)=d(p,t). "Volatility" is defined by the variance of x

$$\sigma^2 = \left\langle \Delta x^2 \right\rangle \approx c\Delta t^{2H} \qquad (6)$$

where H is the Hurst exponent and $\Delta x = x(t+\Delta t) - x(t)$. For stationarity H=0 is required, but real markets yield $H \approx 1/2$ There is much extensive about stationarity in the finance literature). Real markets are nonstationary/unstable, there is no Invisible Hand to produce market stability. Traders, unable to know 'value' (as we explain in part 6), are uncertain and trade often, contributing to nonstationarity and volatility. These are the noise traders, the traders who provide liquidity in normal markets (Black, 1986).

Here's how we constructed our finance market model. Start with an empirical time series x(t) and construct the market density f(x,t), f(x,t)dx=g(p,t)dp, as unmassaged histograms. The empirical distribution is approximately exponential, is far from Gaussian for small to moderate intraday returns x. Discovering the dynamics means discovering the t-dependence of three parameters (γ,ν,δ) defined in McCauley (2004) in the distribution. We used the global volatility $\sigma^2 = \Delta t$ to discover that $\gamma, y = \Delta t^{-1/2}$, yielding

$$f(x,t) = \frac{A_\pm}{\sqrt{\Delta t}} e^{-|x-\delta|/\sqrt{\Delta t}} \qquad (7)$$

Then, we plugged f(x,t) into the Fokker-Planck equation

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial x}(Rf) + \frac{1}{2}\frac{\partial^2}{\partial x^2}(Df) \qquad (8)$$

where R is defined below and solved the 'inverse problem' to find D(x,t). The exponential distribution with x-dependence appearing in the form $x/\sqrt{\Delta t}$ is generated by the diffusion coefficient

$$D(x,t) = 1 + |x - \delta| / \sqrt{\Delta t} \qquad (9)$$

and where

$$\delta = \langle \Delta x \rangle \approx R\Delta t \qquad (10)$$

locates the peak of the distribution, the most probable return. In other words, and this is the main point, *we discovered the form of the noise the market*. We emphasize that the 'local volatility' D(x,t) haracterizes the so-called 'noise traders'.

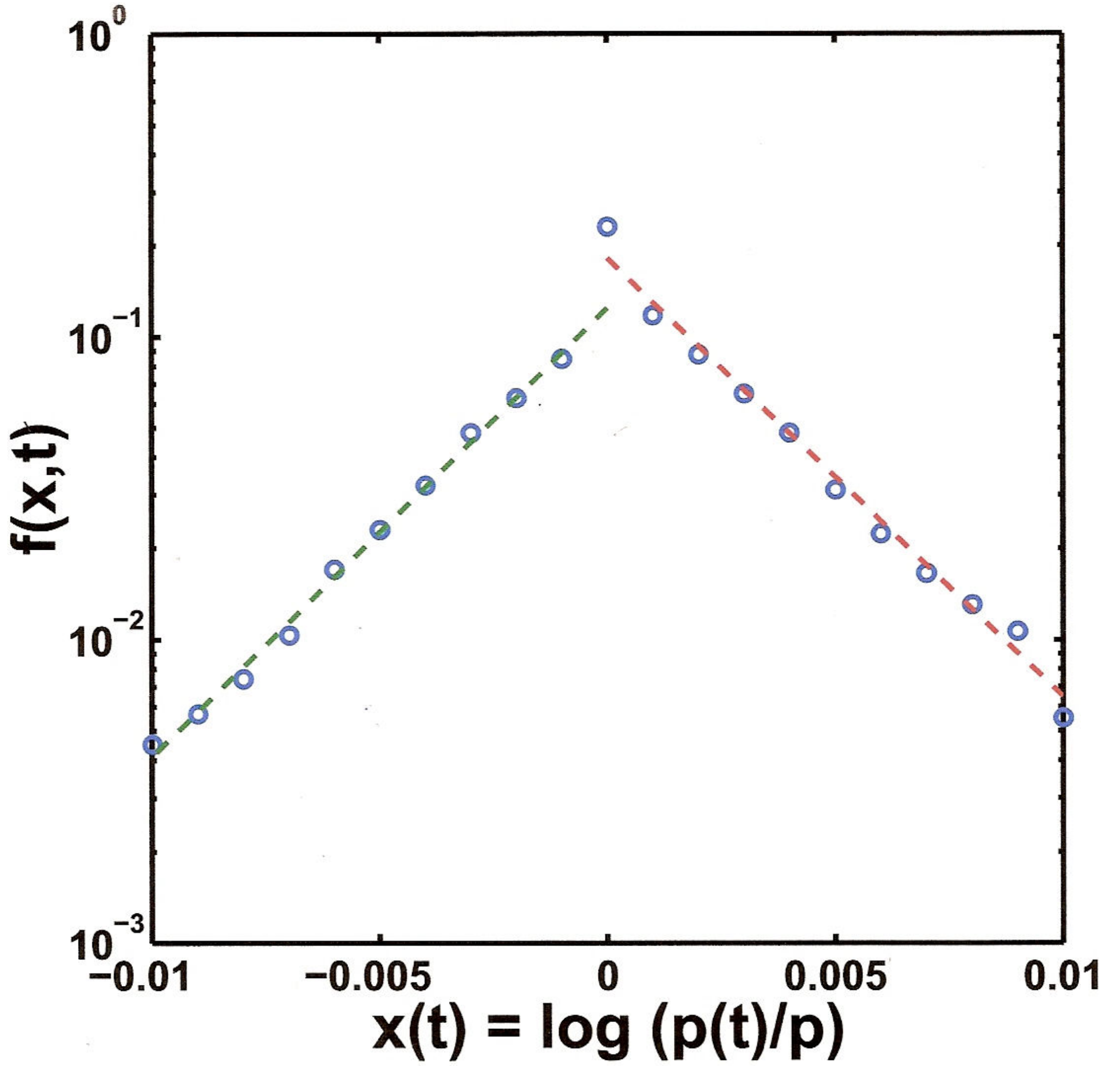

Figure 1. The empirical distribution of financial returns is exponential for small to moderate intraday returns.

4. Volatility and option prices

Quite generally, the average/global volatility is

$$\sigma^2 = \langle \Delta x^2 \rangle = \int_t^{t+\Delta t} ds \langle D(x,s) \rangle = \int_t^{t+\Delta t} ds \int_{-\infty}^{\infty} dx' G(x,t;x',s) D(x',s) \qquad (11)$$

where G(x,t;x',t') is the market Green function satisfying the Fokker-Planck pde (8) and D(x,t) is the 'local volatility', $\sigma^2 \sim D(x,t)\Delta t$ for $\Delta t \ll 1$. The empirical distribution (7) is the market Green function for x'=0, G(x,t;0,0)=f(x,t). Our option pricing prediction, based on the exact formula

$$C(p,K,T-t)e^{r(T-t)} = \langle (p_T - K)\theta(p_T - K) \rangle e^{r(T-t)} = \int_{\ln K/p}^{\infty} (pe^{x_T} - K) G(x_T, T; x, t) dx_T \qquad (12)$$

agrees with traders' prices without using any adjustable parameters (falsifiable model). In (12) C is the call price, T is the expiration time, t is the present time, p is the known price at time t, and K is the strike price at expiration. The reason that we can calculate option prices from the Market Green function is that, with the choice of R(x,t))=r-D(x,t)/2 satisfying the risk neutral hedge condition, the correct 'Black-Scholes' pde is, to within a time transformation, just the backward time Kolmogorov pde corresponding to the market Fokker-Planck pde (8) (McCauley, 2004).

There is nonuniqueness in deducing the t-dependence of the empirical density f(x,t) from the data, but we have the luck that for option pricing the nonuniqueness doesn't matter on a time scale small compared with 100 years. Due to uniqueness in modelling empirical data via infinite precision dynamics, this leads to the viewpoint that the data are *too easy* to fit over long times To be honest, we already know this important lesson from nonlinear dynamics (Chhabra et al, 1989).

4.1 Liquidity, noise traders, and crashes

The essential unstated assumption so far is that we have an adequate 'liquidity bath'. By a normal market we mean the following: A liquidity bath is assumed, meaning that approximately reversible trades are possible via your discount broker in real time over the shortest time intervals (Δt is on the order of a few seconds) on your Imac or PC. This assumption is represented by the noise term $\sqrt{D(x,t)}dB(t)$, which describes the uncoordinated actions of the "noise traders". Noise traders provide the liquidity/entropy in the market. Mathematically seen, noise traders *are* the market ('with measure one'). Noise traders, uncertain about 'value', buy and sell often: a financial market is largely noise because most traders don't have either inside or other knowledge to trade on.

Actually, it was von Neumann who suggested to Shannon to look for market entropy in liquidity. The liquidity/money bath is analogous to a thermal heat bath, but the liquidity bath cannot be described by equilibrium ideas like temperature.

Fat tails do not describe market crashes, fat tails describe large returns that occur during perfectly normal markets. In contrast, a market crash is a *liquidity drought* (the noise traders can't sell because there are no buyers) and is described qualitatively by $R<<0$ and $D(x,t)\approx 0$.

5. Three Easy Pieces

We study the pde

$$\frac{\partial f}{\partial t} = -R\frac{\partial f}{\partial x} + \frac{1}{2}\frac{\partial^2}{\partial x^2}(Df) \qquad (13)$$

with R=constant (Alejandro-Quinones et al, 2004). To satisfy the replicating self-financing hedge condition in option pricing (risk neutral hedge) we need $R=r-D(x,t)/2\neq$constant, but we can take $R \approx$ constant on any time scale small relative to 100 yrs: this is part of the nonuniqueness. Therefore, we can study

$$\frac{\partial f}{\partial t} = \frac{1}{2}\frac{\partial^2}{\partial x^2}(Df) \qquad (14)$$

and then replace x by $x-R\Delta t$. This partial differential equation has nice scaling properties. In order to find out, set $u=x/\sqrt{Dt}$, $f(x,t)=F(u)/\sqrt{Dt}$, and $D(x,t)=D(u)$. The

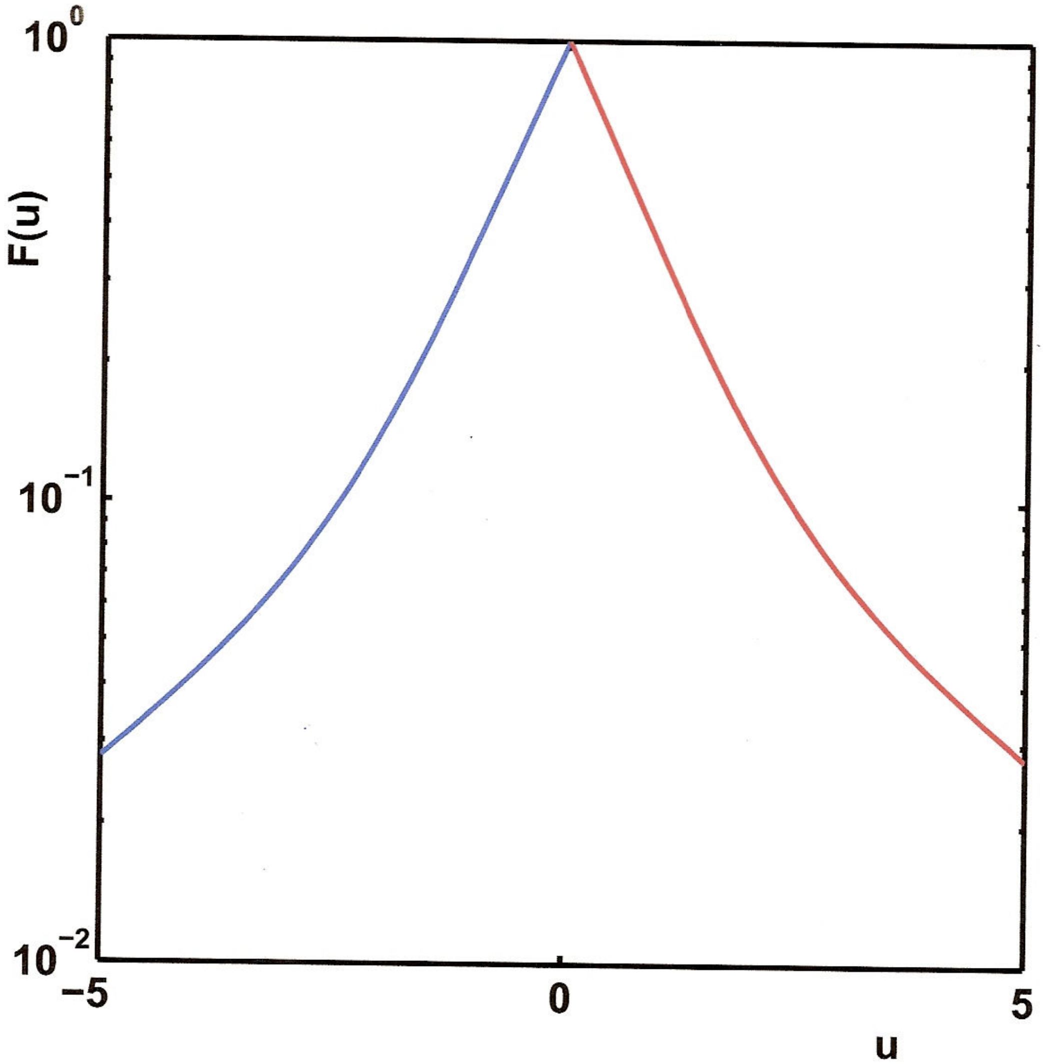

Figure 2 The market distribution is exponential for moderate returns but has fat tails for large returns. The tail exponents are nonuniversal and range from 2 to 7 (see also Dacorogna et al, 2001)

result: given the returns density f(x,t) we can calculate the local volatility D(x,t), and vice/versa, and we can do that analytically for at least three essential cases.

If

$$D(u) = 1 + \varepsilon|u| \qquad (15)$$

then

$$F(u) = Ce^{-|u|((\varepsilon|u|+1)^{\alpha-1})/\varepsilon} \qquad (16)$$

where $\alpha = \varepsilon^{-2}$. As ε increases then the tails of the distribution decay more slowly. The limit ε=0 yields the Gaussian, and ε=1 yields the exponential (7).

Next, if

$$D(u) = 1 + \varepsilon u^2 \qquad (17)$$

then the result is surprising: we can generate (nonstationary!) fat tails

$$f(x,t) \approx |x|^{-\mu}, |x| >> 1 \qquad (18)$$

for all tail exponents $2<\mu<\infty$: the exact solution is

$$F(u) = \frac{C}{(1+\varepsilon u^2)^{1+1/2\varepsilon}} \approx u^{-2-1/\varepsilon}, |u| >> 1 \qquad (19)$$

where $\mu=2+1/\varepsilon$. Note that were even one extra higher order term required in (17) to generate fat tails, then the empirically observed tail exponent μ would not determine all the free parameters in D(x,t).

We can generate the observed financial distribution (Fig. 2) via a 'noise trader function'

$$D(u) = 1 + |u| + \varepsilon u^2 \qquad (20)$$

where the tail exponent $\mu=2+1/\varepsilon$ uniquely determines ε. A decisive test of our model would be to measure D(x,t) empirically, which is very hard: Peinke (2001) tried, but his results fail for large returns x because he inadvertently made a small returns approximation. Finally, One sees immediately from (12) that option prices diverge if fat tails are included: *option traders do not and cannot insure against fat tails.*

6. Our new dynamic definition of "value"

Both f(x,t) and D(x,t) have extrema at $x=\ln p_c/p_o=\delta$ where $p_c=p_oe^{\delta}$ is the most probable price, and p_o is the initial most probable price. The price p_c defines the 'consensus price' and so represents the most widely agreed upon "value" of the asset at time t. This is our non-neo classical definition of value. However, the peak $\delta \approx R\Delta t$ of f(x,t) does not stand still, it can shift suddenly in a market crash and even in normal intraday trading (the expected return R can change suddenly, discontinuously, with sudden changes in noise traders' sentiments). In other words, "value" is very far from a time-invariant idea and depends on what the noise traders believe about an asset at any given time. In particular, 'value' is impossible to know in advance (complexity), we can at best know what value was at different times in the past. This means that notions like 'overvelued' and 'undervalued' are knowable only at the present time or historically, but cannot be predicted any degree of confidence for the future. This viewpoint is completely non-neo classical. Pricing options by using the empirical density (7) rather than the market Green function (still analytically unknown, hard to calculate) means approximating the present observed asset price p(t) by the consensus price.

Fischer Black (1986) was wrong: there is no tendency for price to 'return to value' because market dynamics are unstable, prices always diffuse away from 'value', there being no 'springs' in the market to pull prices back to value. But Soros (1998) was right, *financial markets are dynamically unstable*.

7. Market Complexity

So far, we've discussed nothing but simple stochastic dynamics that generates the historic statistics, so where's the complexity? Predictions based on past statistics hold so long as there are no basic market shifts, or 'surprises'. Surprises generated by UTM (universal Turing machine) dynamics and undecidability were discussed by Moore (1990, 1991) and at the 2001 Geilo School (Skjeltorp and Viscek, 2002). Insurance companies assume that tomorrow will be statistically like yesterday. This assumes that the noise traders never change their diffusion coefficient D(x,t), never change their noisy behavior/psychology. This assumption will fail in an unknown way at some unknown time in the future.

There are also *self-fulfilling expectations* that are not merely a repetition of past statistics, but represent the creation of something new. Examples of self-fulfilling expectations are communism via dictatorship (regulatory extremism) and globalization via deregulation (free market extremism). Feyman contrasted nonthinking nature with socio-economic phenomena and pointed out that the latter are very different from physics because wishful thinking can be made into reality by acting on it. But just as Turing said of numbers and arithmetic, we can assert about physics that mathematical laws of nature are beyond human invention, convention, and intervention, whereas all market phenomena are human-made, are invented by human will and actions. Here, we connect with Wigner (1967): a single space-time invariance principle (the assumption that arbitrage is to zeroth order impossible between spatially separated markets) is inadequate to pin down time-invariant mathematical laws of motion. Our market distribution is only a model, not a fixed market law, and will fail when the noise traders eventually change their habits enough that (21) no longer describes the market.

Neo-classical economics, the theoretical basis for globalization via deregulation that assumes that perfect knowledge of the future on the part of all traders, is a falsified model (McCauley, 2004) is *mathematized ideology*. There is neither simple uncertainty nor complexity in that ideology. Nor can the model be relaxed to make it perturbatively realistic in any sense: instead of approximately perfect knowledge (vanishing entropy), real markets reflect large and ever increasing entropy

$$S(t) = -\int_{-\infty}^{\infty} f(x,t)\ln f(x,t)dx \qquad (22)$$

due to liquidity. Far from randomness and other simplicity, a very few traders do not generate noise but also do not behave predictably. E.g., George Soros (1998) defeats self-fulfilling expectations of opponents (e.g., the Bank of England) by generating *surprises* (or psychological tricks). Soros (1998, 2000) tries to describe how traders behave and discusses self-reference and the Cretan Liar with in light of Gödel's incompletelness theorem. However, it's not clear to this writer that Soros does anything more complicated that to play winning poker with an adequate bankroll (he avoids the gamblers' ruin).

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