Coordination of Expectations in Asset Pricing Experiments

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We investigate expectation formation in a controlled experimental environment. Subjects are asked to predict the price in a standard asset pricing model. They do not have knowledge of the underlying market equilibrium equations, but they know all past realized prices and their own predictions. Aggregate demand for the risky asset depends upon the forecasts of the participants. The realized price is then obtained from market equilibrium with feedback from six individual expectations. Realized prices differ significantly from fundamental values and typically exhibit oscillations around, or slow convergence to, this fundamental. In all groups participants coordinate on a common prediction strategy.

Expectations play an important role in economics. The decisions of economic agents are based upon their expectations and beliefs about the future state of the market. Through these decisions, expectations feed back into the actual realization of the economic variables. This expectations feedback mechanism seems to be particularly important for financial markets. For example, if many traders expect the price of a certain asset to rise in the future, their demand for this asset increases, which, by the law of supply and demand, will lead to an increase of the market price. This self-confirming nature of expectations is typical for speculative asset
markets and it illustrates that the “psychology of the market” may be very important. A theory of expectation formation is therefore a crucial part of modeling economic and in particular financial markets.

It is hard to observe or obtain detailed information about individual expectations in real markets. One approach is to obtain data on expectations by survey data analysis, as done, for example, by Turnovsky (1970) on expectations about the Consumers’ Price Index and the unemployment rate during the post-Korean war period. Frankel and Froot (1987) use a survey on exchange rate expectations and Shiller (1990) analyzes surveys on expectations about stock market prices and real estate prices. However, since in survey data research one cannot control the underlying economic fundamentals or the information that the forecaster possesses, it is hard to measure expectation rules in different circumstances.

An alternative approach is to study expectation formation in an experimental setting. In this article, we report the findings of a laboratory experiment about expectation formation in a simple asset pricing model. In this experiment, we ask the participants to give their expectation of next period’s price of an unspecified risky asset. Submitting predictions is the only task for the participants. They do not have knowledge of the underlying market equilibrium equation, but they know all the past realized prices and, of course, their own predictions. Their earnings are inversely related to the prediction error they make. Given the price forecast of a participant, a computer program computes the associated aggregate demand for the risky asset, and subsequently the market equilibrium price. The realized price thus becomes a function of the individual forecasts. Our experiment is designed to obtain explicit information about the expectations of participants in such a controlled expectations feedback environment.

An important advantage of the experimental approach is that the experimenter has control over the underlying fundamentals. In our experiment, economic fundamentals are constant over time. Participants have perfect information about the mean dividend and the interest rate, and could use this information to compute the (constant) fundamental price. A second advantage of our experimental approach is that we get explicit information about individual expectations. Since in our setup there is no trade, our data are not disturbed by speculative trading behavior, or by changes in the underlying demand and/or supply functions of the participants. Prior to the experiment the only unknown to the experimenters is the way subjects form expectations. Hence, our experimental approach provides us with “clean” data on expectations.

Finance is currently witnessing an important shift in research emphasis, according to some even a paradigmatic shift, from a modeling approach with perfect, rational agents to a behavioral finance approach with “boundedly rational” agents using simple “rule of thumb” trading strategies. The
psychology of investors plays a key role in behavioral finance, and different types of psychology-based trading and behavioral modes have been identified in the literature, such as positive feedback or momentum trading, trend extrapolation, noise trading, overconfidence, overreaction, optimistic or pessimistic traders, upward or downward biased traders, correlated imperfect rational trades, overshooting, and contrarian strategies. Some key references dealing with various aspects of investor psychology include, Cutler, Poterba, and Summers (1990), DeBondt and Thaler (1985), DeLong et al. (1990a, 1990b), Brock and Hommes (1997, 1998), Gervais and Odean (2001) and Hong and Stein (1999, 2003), among others, see, for example, Shleifer (2000) and Hirshleifer (2001) for extensive surveys and many more references on behavioral finance. Individual expectations about future asset prices play a key role and are intimately related to these different behavioral modes. Our experiments may be viewed as an attempt to classify individual forecasting rules. We will present experimental evidence for various of these behavioral modes, in particular for correlated imperfect rational forecasting due to trend extrapolation and overreaction. We will also investigate how far this behavior deviates from perfect rationality and to what extent individual forecasting strategies are “irrational.”

Our main experimental findings are the following. Realized experimental asset prices differ significantly from the (constant) fundamental price. We observe different types of behavior. In some groups the price of the asset converges (slowly) to the fundamental price and in other groups there are large oscillations around the fundamental price. For some groups these oscillations have a decreasing amplitude and prices seem to converge to the fundamental price slowly; in other groups the amplitude of the oscillations is more or less constant over the duration of the experiment or even increasing and there is no apparent convergence.

We are particularly interested in the individual prediction strategies used by the participants. Analysis of the predictions reveals that the dispersion between prediction strategies is much smaller than the forecast errors participants make on average. This indicates that participants within a group coordinate on a common prediction strategy. Although participants make forecasting errors, they are similar in the way that they make these errors. Estimation of the individual prediction strategies shows that participants tend to use simple linear prediction strategies, such as naive expectations, adaptive expectations, or “autoregressive” expectations. Again, participants within a group coordinate on using the same type of simple prediction strategy. We also find evidence for trend extrapolation and overreaction. This behavior is consistent with momentum trading and positive short run correlation in asset returns.

Surprisingly little experimental work focusing on expectation formation has been done. Williams (1987) considers expectation formation in an experimental double auction market that varies from period to period by
small shifts in the market clearing price. Participants predict the mean contract price for four or five consecutive periods. The participant with the lowest forecast error earns $1.00. In Smith, Suchanek, and Williams (1988) expectations and the occurrence of speculative bubbles are studied in an experimental asset market. In a series of related papers, Marimon, Spear, and Sunder (1993) and Marimon and Sunder (1993, 1994, 1995) have studied expectation formation in inflationary overlapping generations economies. Marimon, Spear, and Sunder (1993) find experimental evidence for expectationally driven cycles and coordination of beliefs on a sunspot two-cycle equilibrium, but only after agents have been exposed to exogenous shocks of a similar kind. Marimon and Sunder (1995) present experimental evidence that a “simple” rule, such as a constant growth of the money supply, can help coordinate agents’ beliefs and help stabilize the economy. Although all these papers are clearly related to our work, they cannot be viewed as pure experimental testing of the expectations hypothesis, everything else being constant, because in all these cases dynamic market equilibrium is affected not only by expectations feedback but also by other types of human behavior, such as trading behavior. A number of other laboratory experiments focus on expectation formation exclusively. Schmalensee (1976) presents subjects with historical data on wheat prices and asks them to predict the mean wheat price for the next five periods. In Dwyer et al. (1993) and Hey (1994) subjects have to predict a time series generated by a stochastic process such as a random walk or a simple linear first-order autoregressive process. The drawback of the last two papers is that no economic context is given. Kelley and Friedman (2002) consider learning in an Orange Juice Futures price forecasting experiment, where prices are driven by a linear stochastic process with two exogenous variables (weather and competing supply). The main difference with our approach is that in the last three papers expectations feedback is ignored.

In our experiment we have explicitly accounted for this expectations feedback, which we believe to be very important for many economic environments, and especially for financial markets. Finally, Gerber, Hens, and Vogt (2002) recently studied a repeated experimental beauty contest in which participants in each period place either a buy or a sell order. Prices are determined by total market orders and noise. Although this is a positive feedback system like in our experiment, they do not measure expectations explicitly and their experimental environment is more stylized. Similar to our results, a high level of coordination is found.

The article is organized as follows. Section 1 describes the design of the experiment and Section 2 discusses the underlying asset pricing model. Section 3 presents an analysis of the realized asset prices, whereas Section 4 focuses on the individual prediction strategies. Concluding remarks are given in Section 5.
1. Experimental Design

In financial markets traders are involved in two related activities: *prediction* and *trade*. Traders make a prediction concerning the future price of an asset, and given this prediction, they make a trading decision. We designed an experiment that is exclusively aimed at investigating the way subjects form predictions. We solicit predictions from the subjects about the price of a certain asset for the next period. Given these predictions the computer derives the associated individual demand for the asset and subsequently the market clearing price (i.e., the price at which aggregate demand equals aggregate supply). Each subject therefore acts as an advisor or a professional forecaster and is paired with one trader, which may be thought of as a large pension fund. The subject has to make the most accurate prediction for this trader and then the trader (i.e., the computer) decides how much to trade. The earnings of the subjects in the experiment are inversely related to their prediction error.

The experiment is presented to the participants as follows. The participants are told that they are an advisor to a pension fund and that this pension fund can invest its money in a risk free asset (a bank account) with a risk free gross rate of return \( R = 1 + r \), where \( r \) is the real interest rate, or it can decide to invest its money in shares of an infinitely lived risky asset. The risky asset pays uncertain dividends \( y_t \) in period \( t \). Dividends \( y_t \) are independently and identically distributed (i.i.d.) with mean \( \bar{y} \). The mean dividend \( \bar{y} \) and interest rate \( r \) are common knowledge. The task of the advisor (i.e., the participant) is to predict the price of the risky asset. Participants know that the price of the asset is determined by market equilibrium between demand and supply of the asset. Although they do not know the exact underlying market equilibrium equation they are informed that the higher their forecast, the larger will be the fraction of money invested in the risky asset and the larger will be the demand for stocks. They do not know the investment strategy of the pension fund they are advising or the investment strategies of the other pension funds. The participants are not explicitly informed about the fact that the price of the asset depends on their prediction or on the prediction of the other participants. They also do not know the number of pension funds or the identity of the other members of the group.

The information for the participants is given in computerized instructions. Comprehension of the instructions is checked by two control questions. At the beginning of the experiment the participants are given two sheets of paper with a summary of all necessary information, general information, information about the stock market, information about the investment strategies of the pension funds, forecasting task of the financial advisor and information about the earnings. The handout also contains information about the financial parameters (mean dividend and risk free rate of return) with which an accurate prediction of the fundamental
price can be made. Finally they are given a table from which they can read, for a given forecast error, their earnings.1

In every period \( t \) in the experiment, the task of the participants is to predict the price \( p_{t+1} \) of the risky asset in period \( t + 1 \), given the available information. This information consists of past prices of the risky asset \( p_t, p_{t-1}, \ldots, p_1 \) and the participant’s own past individual predictions \( p_{ht}^e, p_{ht-1}^e, \ldots, p_{h1}^e \), where \( p_{ht}^e \) is the price participant \( h \) expects for period \( t \). Notice that the participants have to make a two period ahead forecast for \( p_{t+1} \), since \( p_{t-1} \) is the latest price observation available. Subjects are told that their price forecast has to be between 0 and 100 for every period. In periods 1 and 2 no information about past prices is available. At the end of period \( t \), when all predictions for period \( t + 1 \) have been submitted, the participants are informed about the price in period \( t \) and the earnings for that period are revealed. Figure 1 shows an English translation of the computer screen the participants are facing during the experiment. On the screen the subjects are informed about their earnings in the previous period, total earnings, a table of the last 20 prices and the corresponding predictions, and a time series of the prices and the predictions.

The earnings of the participants consist of a “show-up” fee of 5 Euro and of the earnings from the experiment that depended upon their forecasting errors. The number of points earned in period \( t \) by participant \( h \) is given by the (truncated) quadratic scoring rule

\[
e_{ht} = \max \left\{ 1300 - \frac{1300}{49} (p_t - p_{ht}^e)^2, 0 \right\},
\]

where 1300 points is equivalent to 0.5 Euro. Notice that earnings are zero in period \( t \) when \( |p_t - p_{ht}^e| \geq 7.2 \).

An experimental asset market consists of 6 participants and a certain fraction of computerized “robot” traders3 (henceforth called fundamentalist traders) and it lasts for 51 periods. A total of 60 subjects (10 groups) participated in this experiment. Subjects (mostly undergraduates in economics, chemistry, and psychology) were recruited by means of announcement on information boards in university buildings, and via e-mail. The computerized experiment was conducted in the CREED laboratory. It lasted for approximately 1.5 hours and average earnings were 21.46 Euro.

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1 An exact description of the information for the participants as well as a further elaboration of the results presented here can be found in the more extensive working paper version (CeNDEF WP 04-02) of this article at http://www.fee.uva.nl/cendef.

2 Paying participants according to quadratic forecast error is equivalent (up to a constant) with paying them according to risk-adjusted profit of the traders [for details see Hommes (2001)].

3 The use of computerized traders in addition to “active” trading by participants is not uncommon in experimental asset markets. See for example, Bloomfield (1996) and Bloomfield and O’Hara (1999) for experimental markets with computerized informed traders and computerized noise traders.
2. The Price Generating Mechanism

2.1 The asset pricing model

The realized prices are generated by a standard asset pricing model with heterogeneous beliefs. For textbook treatments of this model see, for example, Cuthbertson (1996) or Campbell, Lo, and MacKinlay (1997). There are two assets, a risk free asset paying a risk free gross return \( R = 1 + r \) and a risky asset, with asset price \( p_t \), paying an uncertain dividend \( y_t \) in period \( t \). The dividends are i.i.d. with mean \( \bar{y} \). The fundamental value (i.e., the discounted value of the expected future dividend stream) of the risky asset is then given by \( p^f = \bar{y}/r \) and is thus constant over time.

The asset market is populated by six pension funds and a small fraction of fundamentalist traders, as discussed below. Each pension fund \( h \) is matched with a participant to the experiment and makes an investment decision at time \( t \) based upon this participant’s prediction \( p_{h,t+1}^c \) of the asset price. The fundamentalist traders always predict the fundamental price \( p^f \) and make a trading decision based upon this prediction. Moreover, the fraction \( n_t \) of these fundamental traders in the market is endogenous and depends positively upon the absolute distance between the asset price and the fundamental value.\(^4\) The greater this distance the more these fundamental traders will invest, and the other way around. These fundamentalist traders therefore act as a “stabilizing force” pushing prices

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\(^4\) This is similar to the model discussed in Brock and Hommes (1998) where the fraction \( n_t \) of traders using prediction strategy \( h \) is also endogenous. In their paper this fraction depends positively upon past performance of the prediction strategy.
in the direction of the fundamental price. Their presence therefore excludes the possibility of speculative bubbles in asset prices. DeGrauwe, DeWachter, and Embrechts (1993) discuss a similar stabilizing force in an exchange rate model with fundamentalists and chartists. In the same spirit Kyle and Xiong (2001) introduce a long-term investor that holds a risky asset in an amount proportional to the spread between the asset price and its fundamental value.

The realized asset price in the experiment is determined by market clearing as follows. The amount of shares pension fund $h$ wants to hold in period $t$ depends positively upon their expected excess return $p_{h,t+1}^e + \overline{\gamma} - Rp_t$. This means that an increase in the expected price of the asset for period $t + 1$ of participant $h$ leads to an increase in demand for the asset by the pension fund $h$ in period $t$. The market clearing price in period $t$ is then given as [cf. Campbell, Lo, and MacKinlay (1997, eq. 7.1.4) and Brock and Hommes (1998, eq. 2.7)]

$$p_t = \frac{1}{1 + r} [(1 - n_t)p_{t+1}^e + n_t p^f + \overline{\gamma} + \epsilon_t],$$

where $p_{t+1}^e = \frac{1}{6} \sum_{h=1}^{6} p_{h,t+1}^e$ is the average forecast for period $t + 1$ of the six participants. The current period’s asset price is therefore determined by (average) beliefs about next period’s asset price and an extra noise term $\epsilon_t$, where the latter corresponds to (small) stochastic demand and supply shocks. Note that the realized asset price $p_t$ at time $t$ is determined by the individual price predictions $p_{h,t+1}^e$ for time $t + 1$. Therefore, when traders have to make a prediction for the price in period $t + 1$ they do not know the price in period $t$ yet, and they can only use information on prices up till time $t - 1$.

In the experiment the risk free rate of return, $r = 0.05$, and the mean dividend are fixed such that $p^f = 60$ (with $\overline{\gamma} = 3$) in 7 of the groups and $p^f = 40$ (with $\overline{\gamma} = 2$) in 3 of the groups. Small demand and supply shocks $\epsilon_t$ are independently drawn from $N(0, \frac{1}{4})$. In order to be able to compare the different groups in the experiment, we used the same realizations of the demand and supply shocks for each group. Finally, the weight $n_t$ of the fundamentalist traders is given by

$$n_t = 1 - \exp\left(-\frac{1}{200} | p_{t-1} - p^f | \right),$$

which increases as the price moves away from the fundamental price. Notice that $n_t = 0$ for $p_{t-1} = p^f$. Moreover, given that the fundamental value equals $p^f = 60$ or $p^f = 40$, the weight of the fundamentalist traders is bounded above by $\overline{n} = 1 - \exp\left(-\frac{3}{10}\right) \approx 0.26$. The weight of the other traders is the same for each trader and equal to $(1 - n_t)/6$.

An important feature of the asset pricing model is its self-confirming nature: If all traders have a high (low) prediction the realized price will...
also be high (low). This important feature is characteristic for a speculative asset market: If traders expect a high price, the demand for the risky asset will be high, and as a consequence the realized market price will be high assuming that the supply is fixed.

2.2 Benchmark expectations rules

This section discusses some important benchmark expectations rules in the asset pricing model. In Sections 3 and 4 we will discuss which of these benchmarks gives a good description of the results from our asset pricing experiments. The development of the asset price depends upon the (subjective) expectations of the different trader types. Under rational expectations the subjective expectation \( E_{ht} \) of trader type \( h \) is equal to the objective mathematical conditional expectation \( E_t \) for all \( h \). Given that bubbles cannot occur in our framework this gives \( E_t p_{t+1} = p^f \). Equation (1) then gives

\[
p_t = p^f + \frac{1}{1 + r_t} e_t.
\]

Therefore, under rational expectations \( p_t \) corresponds to independent drawings from the normal distribution with mean \( p^f \) and variance \( (\sigma_e/R)^2 = 100/441 \). The upper left panel of Figure 2 shows the asset price under rational expectations for the realization of the demand and supply shocks that was used in the experiment, when the fundamental value is given by \( p^f = 60 \).

The rational expectations hypothesis is quite demanding. It requires that participants know the underlying asset pricing model and use this to compute the conditional expectation for the future price and that they do not make structural forecast errors. In particular, rational expectations requires knowledge about the beliefs of all other participants. It will only prevail when participants are able to coordinate on the rational expectations equilibrium and all participants forecast the asset price to equal its fundamental value. Notice however that, since participants know the values of \( \bar{y} \) and \( r \), they have enough information to compute the fundamental value and predict it for any period, that is, they can submit \( p^e_{h,t+1} = p^f \) as a forecast, for all \( t \).

Let us now consider asset price behavior when participants use simple forecasting rules instead of rational expectations. The perhaps simplest expectations scheme corresponds to static or naive expectations, where

\[
p^e_{h,t+1} = p_{t-1},
\]

that is, the participant’s prediction for the next price corresponds to the last observed asset price. Under the assumption that all traders have naive expectations the price dynamics reduces to
It can be easily seen that in this case prices will converge to the neighborhood of the fundamental price (see the upper right panel of Figure 2). Moreover, in the absence of any stochastic demand and supply shocks, prices converge monotonically to the fundamental price. This also holds true for another well-known prediction strategy, adaptive expectations, which corresponds to

\[ p_{t+1}^e = \frac{1 - n_t}{1 + r} (p_{t-1} - p^*) + \frac{1}{1 + r} \varepsilon_t. \]

It can be easily seen that in this case prices will converge to the neighborhood of the fundamental price (see the upper right panel of Figure 2). Moreover, in the absence of any stochastic demand and supply shocks, prices converge monotonically to the fundamental price. This also holds true for another well-known prediction strategy, adaptive expectations, which corresponds to

\[ p_{h,t+1}^e = \omega p_{t-1} + (1 - \omega) p_{h,t}^e = p_{h,t}^e + \omega (p_{t-1} - p_{h,t}^e), \]

where \(0 < \omega < 1\). Hence, under adaptive expectations the prediction is adapted in the direction of the last observed price. The weight parameter \(w\) determines how fast predictions are updated. Notice that naive expectations corresponds to a special case of adaptive expectations, where \(w = 1\).
The lower left panel of Figure 2 shows realized prices when agents use the sample average as their forecast, that is,

\[ p_{h,t+1}^e = \frac{1}{t-1} \sum_{j=1}^{t-1} p_j. \]

In that case, equal weight is given to all past observed prices and as a result convergence to the fundamental price is much slower than in the case of naive expectations where all weight is given to the last observation.

We conclude this discussion on simple prediction strategies by looking at the class of linear autoregressive prediction strategies with 2 lags, that is

\[ p_{h,t+1}^e = \alpha_h + \beta_{h1} p_{t-1} + \beta_{h2} p_{t-2}. \] (3)

We will refer to Equation (3) as an AR(2) prediction rule. Notice that the endogeneity of the fraction of fundamentalist traders introduces a nonlinearity in the price generating mechanism (1), even if all prediction strategies are linear. Now assume all participants use rule (3) and let \( \beta_l = \frac{1}{6} \sum_{h=1}^{6} \beta_{hl}, \) for \( l = 1, 2. \) Depending on the values of \( \beta_1 \) and \( \beta_2 \) one can have different types of dynamics, such as monotonic convergence, converging or diverging oscillations, or cyclic behavior.

The AR(2) prediction strategy (3) can be rewritten as

\[ p_{h,t+1}^e = \alpha + \beta p_{t-1} + \delta (p_{t-1} - p_{t-2}), \]

where \( \beta \equiv \beta_1 + \beta_2 \) and \( \delta \equiv -\beta_2. \) Expressed in this way it provides a nice behavioral interpretation. Participants believe that the price will be determined by the last observation (the first two terms on the right-hand side) but they also try to follow the trend in the prices (expressed in the third term): If \( \delta > 0, \) they believe that an upward movement in prices will continue in the next period, whereas if \( \delta < 0 \) they believe that an upward movement in the prices will be (partially) offset by a downward movement in prices in the next period. The former corresponds to trend extrapolators or positive feedback traders, whereas the latter corresponds to so-called contrarians. The lower right panel of Figure 2 shows the evolution of the realized price if everybody in the experiments uses AR(2) expectations \( p_{h,t+1}^e = 30 + \frac{3}{2} p_{t-1} - p_{t-2}. \)

3. Aggregate Behavior of Asset Prices

Figure 3 shows the realized asset prices in the experiment for the 10 groups. In the first seven groups the fundamental value equals 60, whereas in the last three groups the fundamental value equals 40. The horizontal line in the graphs corresponds to the fundamental price for that group.

We can classify the different groups in three different categories:

1. Monotonic convergence: the price in groups 2 and 5 seems to converge monotonically to the fundamental price from below.
2. Converging oscillations: the price in groups 4, 7, and 10 oscillates around the fundamental price but the amplitude of the oscillations decreases over time indicating convergence to the fundamental price.

3. Persistent oscillations: the price in groups 1, 6, 8, and 9 oscillates and the amplitude of these oscillations seems to be constant or even increasing. In these groups there does not seem to be convergence to the fundamental price.

Group 3 is more difficult to classify, it starts out with oscillations, but from a certain period on there seems to be monotonic convergence to the fundamental price.\(^5\)

Comparing the experimental results in Figure 3 with the simulated benchmarks in Figure 2 one observes that realized prices under the

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\(^5\)The sudden fall of the asset price in group 3 from 55.10 in period 40 to 46.93 in period 41 is due to the fact that one of the participants predicts 5.25 for period 42. It is likely that this corresponds to a typing error (may be his intention was to type 55.25), since this participant’s 5 previous predictions all were between 55.00 and 55.40, giving him the very high average earnings of 1292 out of 1300 points in these periods.
naive expectations benchmark resemble realized prices in groups 2 and 5 of the experiment remarkably well. On the other hand, the oscillatory behavior of the realized price in groups 1, 4, 6–10 in the experiment is qualitatively similar to the asset price behavior when participants use AR(2) prediction strategies. Clearly, naive and AR(2) prediction strategies give a qualitatively much better description of aggregate asset price fluctuations in the experiment than does the benchmark case of rational expectations. Recall from Section 2 that an AR(2) rule can be interpreted as a trend following forecasting strategy.

Figure 4 shows the sample mean and sample variance of realized prices for the 10 groups. The figure also represents sample means and sample variances of the three important benchmark expectational rules RE, naive expectations and an AR(2) rule, discussed in Section 2. All of these benchmarks are computed once for the case with fundamental value

![Sample mean and sample variance](image-url)

**Figure 4**

*Sample mean and sample variance*

Sample mean and sample variance (with the latter on a logarithmic scale) of realized asset prices in the 10 different groups (indicated by a ◆ and the number of the group), and of the simulated benchmark cases rational expectations ($p_{h,t+1}^R = p^f$ for all $h$), naive expectations ($p_{h,t+1}^N = p_{t-1}$ for all $h$) and AR(2) expectations ($p_{h,t+1}^{AR(2)} = \frac{3}{2}p^f + \frac{1}{2}p_{t-1} - p_{t-2}$ for all $h$). These benchmark cases are indicated by a ■ and computed for the case of $p^f = 60$ and $p^f = 40$, respectively.
\( p^f = 60 \) and once for the case with \( p^f = 40 \). Inspection of Figure 4 confirms our earlier conclusion: naive expectations or AR(2) expectations gives a much better description of aggregate price behavior than does rational expectations. There is clear evidence for excess volatility in these markets, that is, the sample variance is much larger than the RE sample variance. Moreover, in the seven experimental groups with \( p^f = 60 \), the sample average is below the fundamental value and in the three groups with \( p^f = 40 \) the sample average is above the fundamental value. This undervaluation and overvaluation can be explained as follows. We have restricted prices to lie between 0 and 100. Since agents have no prior information about the price generating process, many initial guesses lie around 50. Most of the initial guesses will therefore be smaller (higher) than the fundamental price of 60 (40), leading to undervaluation (overvaluation). The under and overvaluation is decreasing over time, but only slowly.

As a final remark on the realized asset prices we note that the influence of the fundamentalist traders on the asset pricing dynamics seems to be limited. The maximum weight of the fundamentalist traders is attained in group 4 in period 13 and equals 0.191 reducing the weight of an individual participant to 0.135 for that period. In all other periods and groups the weight of the fundamentalist traders is much smaller.

4. Individual Prediction Strategies

We now turn to the individual prediction strategies of the participants in our asset pricing experiment. In Section 4.1 we show that participants tend to coordinate on a common prediction strategy. Section 4.2 discusses earnings per group. Section 4.3 investigates whether participants use the available information efficiently. In Section 4.4 we present results on characterizing and estimating the individual prediction strategies. Section 4.5 presents four additional groups without fundamentalist traders.

4.1 Coordination

Figure 5 shows, for each of the 10 groups, the 6 individual predictions of the participants. A striking feature of Figure 5 is that different participants within one group seem to coordinate on some common prediction strategy. This coordination of expectations is obtained in all 10 groups.

In order to quantify this coordination on a common prediction strategy we consider, for each group, the average individual quadratic forecast error

\[
\frac{1}{6 \times 4!} \sum_{h=1}^{6} \sum_{t=11}^{4!} (p^e_{ht} - p_t)^2,
\]

which corresponds to the individual quadratic forecast error averaged over time and over participants within a group. Note that the first 10
observations are neglected in order to allow participants to learn how to predict prices accurately. Defining $\bar{p}_t = \frac{1}{6} \sum_{h=1}^{6} p_{ht}$ as the average prediction for period $t$ in a group (averaged over individuals in that group) we find that the average individual quadratic forecast error can be broken up into two separate terms, as follows

$$\frac{1}{6 \times 41} \sum_{h=1}^{6} \sum_{t=11}^{51} (p_{ht} - p_t)^2 = \frac{1}{6 \times 41} \sum_{h=1}^{6} \sum_{t=11}^{51} (p_{ht} - \bar{p}_t)^2 + \frac{1}{41} \sum_{t=11}^{51} (\bar{p}_t - p_t)^2.$$  \hspace{1cm} (4)

The first term on the right-hand side of Equation (4) measures the dispersion between individual predictions. It gives the distance between the individual prediction and the average prediction $\bar{p}_t$ within the group, averaged over time and participants. Note that it equals 0 if and only if all participants, in one group, use exactly the same prediction strategy. Hence, this term measures deviation from coordination on a common prediction strategy. The second term on the right-hand side of Equation (4) measures the
The average distance between the mean prediction $\hat{p}_t$ and the realized price $p_t$. If individual expectations can be described as “rational expectations with error,” where the error has mean zero and is serially uncorrelated and uncorrelated with the errors of the other participants, then we should expect that individual forecast errors cancel each other out in the aggregate. This is consistent with Muth (1961, p. 316) who gives the following formulation of the rational expectations hypothesis:

$$\text{The hypothesis can be rephrased a little more precisely as follows: that expectations of firms (or, more generally, the subjective probability distribution of outcomes) tend to be distributed, for the same information set, about the prediction of the theory (or the “objective” probability distributions of outcomes).}$$

In other words, individual expectations may be wrong, but in the aggregate expectations should be approximately correct. If this is the case then this second term should be relatively small.

Table 1 shows, for each of the 10 groups, how the average quadratic forecast error can be broken up in these two terms, the average dispersion error and the average common error.$^6$

From inspection of Table 1 it is clear that only a relatively small part (ranging from 12% in group 9 to 38% in group 3) of the average quadratic

\[ \text{avg. individual error} = \frac{1}{T} \sum_{t=1}^{T} (\hat{p}_t - p_t)^2 \]

\[ \text{avg. dispersion error} = \frac{1}{T} \sum_{t=1}^{T} (\hat{p}_t - \hat{p})^2 \]

\[ \text{avg. common error} = \frac{1}{T} \sum_{t=1}^{T} (p_t - \hat{p})^2 \]

The average (represented as avg.) quadratic forecast error of the individual prediction strategies, $\frac{1}{T} \sum_{t=1}^{T} (\hat{p}_t - p_t)^2$, averaged, for each group, over participants and time periods, starting in period 11), which is broken up, for every group, into the average dispersion error between individual predictions and the mean prediction, $\frac{1}{T} \sum_{t=1}^{T} (\hat{p}_t - \hat{p})^2$, and the average common error, $\frac{1}{T} \sum_{t=1}^{T} (p_t - \hat{p})^2$, which is the average distance between the mean prediction in a group and the realized price. For all groups, the average dispersion error is relatively low, implying strong coordination on a common prediction strategy.

For group 3, we have excluded the observation at time $t = 42$, where one of the participants appeared to make a typing error (see note 5), which has a big impact on these measures. If we include this observation we get 15.70, 11.10, and 4.60, respectively.
forecasting error (first column) can be explained by the dispersion in expectations (second column). In fact, on average 75% of the average quadratic forecast error can be attributed to the average common error. This confirms our conjecture that there is coordination on a common prediction strategy. The observation that a relatively large part of the average quadratic forecast error is due to the difference between the average expectation and the realized price (third column) implies that “rational expectations with error” is not a good description of participants’ expectation formation. In fact, it suggests that participants’ mistakes are correlated. We therefore conclude that participants make significant forecasting errors, but they are alike in the way that they make these forecasting errors.

4.2 Earnings
Comparing individual forecasts in Figure 5 with realized prices in Figure 3 suggests that the participants are performing quite well. Indeed, earnings from predicting can be substantial. The total number of points they receive when always making the correct prediction is 66,300 and under rational expectations earnings would be 65,975. In the experiment participants earn on average 46,939 points. Participants in groups 4 and 10 earn a relatively small amount (20,683 and 24,470, respectively, on average), whereas participants in groups 2 and 5 on average make substantial earnings, close to the maximum (64,168 and 63,739, respectively). The other groups are somewhere in between. The prices in groups 2 and 5 slowly converge to the fundamental price (the only rational expectations price) and the earnings in these groups are almost as high as earnings of rational forecasters. In this sense the behavior of these subjects can be considered as “close” to rational. Earnings in the other groups (with the exception of groups 4 and 10) are also relatively high, since individual forecasting errors are not too large. The groups 4 and 10 with the smallest earnings show a relatively high price volatility.

4.3 Informational efficiency
The analysis of Table 1 suggests that participants make structural forecast errors. However, if participants are rational their forecast error should be unbiased and uncorrelated with available information. To test whether participants are rational in this sense we considered the time series of the forecast errors $p_t - p^*_t$, using the last 41 observations. The sample average of these individual forecast errors is significantly different from 0 at the 5% level, for only 8 of the 60 participants. This means that for more than 85% of the individuals forecast errors are unbiased. Furthermore, we computed, for each participant, the first 10 lags of the autocorrelation function of the time series of forecast errors. The autocorrelation function of the forecast errors turns out to be significant at the first lag for many participants. However, participants do not have $p_t$ in their
information set, when predicting $p_{t+1}$. Hence, they are not able to exploit the first-order autocorrelation structure in the forecast errors to improve their predictions. Therefore one should ignore the significant first-order lags and focus on higher-order lags of the autocorrelation function. We find that for about one-fourth of the participants there is no exploitable (linear) structure in the forecast errors at all. Ignoring the first lag, the second lag is only significant for 13 out of the 60 participants. Stated differently, the most easily detected linear structure has been exploited efficiently by 47 participants. In this sense individual forecasts of about 80% of all participants may be viewed as fairly close to optimal. We also note that most structure in the forecast errors can be found in the groups where the realized price oscillates around the fundamental price. Furthermore, there is much similarity between the autocorrelation structure of participants within a group, again indicating that participants in the same group seem to coordinate on a common prediction strategy.

4.4 Characterizing individual prediction strategies

In this section we try to characterize and estimate the individual prediction strategies. Some participants try to extrapolate certain trends and by doing so overreact and predict too high or too low. Other participants are more cautious when submitting predictions. When prices are rising (declining) they usually predict a price lower (higher) than the realized price. The individual degree of overreaction can be quantified as follows. Figure 6 shows, for each group, the average absolute (one-period) change in predictions of participant $h$,

$$\Delta^e_h = \frac{1}{41} \sum_{t=11}^{51} |p_{ht}^e - p_{h,t-1}^e|.$$

The average absolute change in the price, $\Delta = \frac{1}{41} \sum_{t=11}^{51} |p_t - p_{t-1}|$ represented by the straight line. We will say that individual $h$ overreacts if $\Delta^e_h > \Delta$ and we will say that individual $h$ is cautious if $\Delta^e_h \leq \Delta$.

Figure 6 measures the degree of overreaction. For a vast majority of participants in groups 1, 3, 4, and 6–10 the individual degrees of overreaction are higher than the average changes in realized prices. Oscillatory behavior is thus caused by overreaction of a majority of agents. In groups 2 and 5 the changes in predictions are similar to the changes in prices. Convergence to the fundamental price occurs when a majority of traders is “cautious.”

The final step in our analysis of the individual prediction strategies is to try to estimate simple forecasting rules. The prediction strategies of all 60 participants can be described by the following general simple linear model

$$p_{h,t+1}^e = \alpha_h + \sum_{i=1}^{4} \beta_{hi}p_{t-i} + \sum_{j=0}^{3} \gamma_{hj}p_{h,t-j}^e + v_t,$$  \( (5) \)
where \( v_t \) is an i.i.d. noise term. Notice that this general structure includes several interesting special cases: (i) naive expectations (\( b_{h1} = 1, \) all other coefficients equal to 0); (ii) adaptive expectations (\( b_{h1} + \gamma_{h0} = 1, \) all other coefficients equal to 0), and (iii) AR(\( L \)) processes (all coefficients equal to 0, except \( a_h, b_{h1}, \ldots, b_{hL} \)). We estimated Equation (5) for all 60 participants, using observations from \( t = 11 \) to \( t = 51 \). The estimation results are qualitatively summarized in Table 2. We find 9 participants with AR(1) beliefs (of which 3 participants use naive expectations), 29 participants with AR(2) beliefs, 3 participants with AR(3) beliefs, and 3 participants with adaptive beliefs.\(^7\) Hence, for 44 participants, that is close to 75% a very simple linear forecasting rule explains the forecasting strategy quite well. The remaining 16 participants use more complicated linear prediction rules, but their prediction strategies can still be captured by linear rules with up to 4 lags. Notice that the AR(1) and adaptive rules are all found in groups 2, 3, and 5, and the AR(2) and AR(3) rules are all found in the other groups. This is consistent with the finding that in groups 2 and 5 the price seems to converge monotonically and that in groups 1, 4, and 6–10 the price oscillates around some steady state. Group 3 takes a somewhat special position, starting our with oscillations and ending with monotonic convergence to the fundamental price. Prediction strategies within groups are more similar than strategies between groups, which is

\(^7\) We arrive at the naive and adaptive expectations strategies in the following way. For the AR(1) processes we tested the joint hypothesis \( a_h = 0 \) and \( b_{h1} = 1 \) (naive expectations). For processes where only the coefficients on \( p_{t-1} \) and \( p'_{h0} \) are significant we tested the joint hypothesis \( a_h = 0 \) and \( b_{h1} + \gamma_{h0} = 1 \) (adaptive expectations).
consistent with the finding that participants within one group seem to coordinate on a common prediction strategy.

The AR(2) prediction strategy can be rewritten as a trend following rule

$$p^e_{h,t+1} = \alpha_h + \beta_{h1}p_{t-1} + \delta_h(p_{t-1} - p_{t-2}),$$

where $\beta_h \equiv \beta_{h1} + \beta_{h2}$ and $\delta_h \equiv -\beta_{h2}$. For all of the 26 AR(2) prediction strategies in the “oscillating” groups (1, 4, 6–10) we have $\beta_{h1} > 0$ and $\beta_{h2} < 0$. The latter inequality is equivalent with $\delta_h > 0$, which implies that all these participants try to extrapolate a trend: they expect that a recent upward (or downward) movement in prices will continue in the near future. These participants therefore correspond to so-called positive feedback or momentum traders.

One final remark is in order. From the estimation results we should not draw the conclusion that these prediction strategies are typical for the different individuals, in the sense that these individuals will use the same rule in another context as well. Actually, participants coordinate on some kind of behavior within their group and this behavior becomes self-fulfilling: The estimated relationships are consistent with that behavior.

4.5 The impact of the fundamentalist traders

In this section we discuss the influence of the fundamentalist traders. We ran four additional groups, where the only difference with the other sessions is that there are no fundamentalist traders [$n_t = 0$, for all $t$, in Equation (1)]. Figure 7 shows the realized asset prices and individual predictions per group.

<table>
<thead>
<tr>
<th>Group 1</th>
<th>0</th>
<th>5</th>
<th>0</th>
<th>0</th>
<th>B (4,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 2</td>
<td>4(3)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>B (1,2)</td>
</tr>
<tr>
<td>Group 3</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>—</td>
</tr>
<tr>
<td>Group 4</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>B (3,1), B (4,3)</td>
</tr>
<tr>
<td>Group 5</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>B (2,1)</td>
</tr>
<tr>
<td>Group 6</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>B (2,2)</td>
</tr>
<tr>
<td>Group 7</td>
<td>0</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>B (1,2)</td>
</tr>
<tr>
<td>Group 8</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>B (1,1), B (4,3)</td>
</tr>
<tr>
<td>Group 9</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>B (1,1), B (2,2), B (2,3), B (4,1)</td>
</tr>
<tr>
<td>Group 10</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>2×B (1,1), B (3,0)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>9</td>
<td>29</td>
<td>3</td>
<td>3</td>
<td>16</td>
</tr>
</tbody>
</table>

For each participant we estimated a linear forecasting rule $p^e_{h,t+1} = \alpha_h + \sum_{i=1}^{4} \beta_{hi}p_{t-i} + \sum_{j=0}^{3} \gamma_j p^e_{h,t-j} + \nu_t$, from $t = 11$ to $t = 51$. AR(1) means that only $\alpha$ and $\beta_1$ are significant at the 5% level. Naive for three of the participants in group 2 refers to the fact that the null hypothesis $\beta_1 = 1$ and all other coefficients equal to 0 cannot be rejected at the 5% significance level. For AR(2) only $\alpha$, $\beta_1$, and $\beta_2$ are significant and for AR(3) only $\alpha$, $\beta_1$, $\beta_2$, and $\beta_3$ are significant. Adaptive refers to the fact that the null hypothesis $\beta_1 + \gamma_0 = 1$, and all other coefficients equal to 0 cannot be rejected at the 5% significance level. B(k, l) refers to a prediction strategy where $k$ is the highest significant lag of the price and $l$ is the highest significant lag of the prediction (which does not necessarily mean that all smaller lags are also significant) in the regression.
This figure shows that also in the case without fundamentalist traders coordination of individual forecasting strategies on a common prediction strategy occurs.

A total of 24 subjects participated in this session and their average earnings were 32,664 points (17.56 Euro), which is below the average earnings of the 10 other groups. For the four additional groups, the sample mean of realized prices was 56.48, so that also in these groups on average the market is undervalued. The sample variance of realized prices is quite large, especially in groups 11–13 (647 on average). Hence, in accordance with what one would expect, without computerized fundamentalist traders market volatility is higher than in the presence of fundamentalist traders.

Figure 7 also shows that in three of the four groups temporary bubbles and crashes occur. The fourth group shows a steady oscillation around the fundamental value of $p^f = 60$. These results are similar to those from Figure 7.
a related asset pricing experiment without fundamentalist traders, recently obtained in Hommes et al. (2002).\(^8\)

From Figure 7 it is clear that also in the additional groups without fundamentalist traders, participants coordinate on a common prediction strategy. Computations similar to those in Section 4.1 show that 75% of the average individual quadratic forecast error can be attributed to the common error. Estimating individual forecasting strategies, as in Section 4.4, shows that the majority of the individual prediction strategies can be classified as AR(2), AR(3), or AR(4) strategies. These results are similar to the results obtained for the oscillatory groups 1, 4, 6–10 with fundamentalist trader.

In summary, also in the absence of the fundamentalist traders our key finding remains that there is coordination on a common prediction strategy. This coordination of expectations therefore seems to be a robust result in these asset pricing experiments.

5. Concluding Remarks

In this article, we investigated expectation formation in a simple experimental asset pricing model. Ten markets are populated by six participants and a certain fraction of computerized fundamentalist traders; four additional markets without computerized fundamentalist traders have also been investigated. We observe slow and monotonic convergence to the fundamental price, as well as regular oscillations around the fundamental price. In most groups the asset is undervalued and exhibits excess volatility. Simple expectation schemes — or popular models [Shiller (1990)] — such as naive or autoregressive expectations give a much better description of aggregate market behavior than do rational expectations. From the analysis of the individual prediction strategies we find that participants within a group coordinate on a common prediction strategy. Moreover, these popular models can be estimated rather accurately, and this reveals that participants indeed tend to use simple (linear) forecasting models. In the stable markets, a majority of participants is cautious and uses naive, adaptive, or AR(1) forecasting strategies. In the oscillatory groups, a majority of participants exhibits overreaction and uses trend following strategies. Although the participants are not completely rational like standard economic theories assume, they perform very well. For a large majority of individuals, forecasting errors are unbiased and without autocorrelation in the smallest exploitable lag (lag 2) and

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\(^8\) The main difference in these experiments is that the participants have no a priori information about an upperbound on their prediction. The most striking feature of these experiments is that bubbles increasing up to a value of 1000 (i.e., more than 15 times the fundamental price) occur. Therefore, in these related experiments without fundamentalist traders and without an a priori given upperbound, participants coordinate on a common prediction strategy, predicting (exponentially) growing asset prices. See also van de Velden (2001) for further details.
their earnings are high. Our experimental outcomes thus support the common hypothesis in behavioral finance that individuals use simple, but reasonably successful, rules of thumb.

One may ask whether our experimental results can also be explained by a rational theory. In fact, it has been pointed out recently, for example, in Brav and Heaton (2002, p. 575), that it is difficult to distinguish between “behavioral theories built on investor irrationality and rational structural uncertainty theories built on incomplete information about the structure of the economic environment.” In particular, Brav and Heaton (2002) consider a model with a one-period risky asset paying an uncertain dividend at the end of the period. They compare the model with a rational agent who does not know the true underlying generating process for dividends, but behaves rationally given his incomplete information about economic fundamentals, to the model with an irrational, behavioral investor, who knows the true underlying dividend process, but behaves according to a representativeness heuristic or conservatism. They then show that both the rational agent model and the behavioral model can generate a form of overreaction and underreaction in asset prices. In other recent work, rational explanations of momentum trading have been proposed, for example, by Johnson (2002) in a rational model with time-varying expected dividend growth rates and by Chordia and Shivakumar (2002) in a rational model with time-varying expected returns due to macroeconomic effects. Conrad and Kaul (1998) argue that the profitability of momentum strategies could be entirely due to cross-sectional variations in expected returns; see also Jegadeesh and Titman (2001) for a discussion.

Laboratory experiments are well suited to distinguish rational and behavioral theories, because the experimenter can control the “economic fundamentals” as well as the information about these fundamentals. In our experiments economic fundamentals are stationary, and participants know the mean of the dividend process and the risk free interest rate, and can use these to compute the constant fundamental price. Clearly, this is not what participants did in the experiments. In an unknown stationary environment a rational agent would use the sample average as his price forecast, and this would lead to slow convergence to the fundamental price. Again, this is not what happened in the experiment. The slowly, monotonically converging groups 2 and 5 may perhaps be explained by rational Bayesian learning with appropriate weight given to some prior beliefs, but this can not explain the remaining oscillatory groups. A rational explanation of the oscillatory groups could perhaps be that individuals (wrongly) believe that economic fundamentals (dividends) are time varying and act rationally given their belief. Although in theory such a “rational” explanation is possible, it seems unlikely that six individuals in a group coordinate on the same (wrong) belief about market expectations in asset pricing experiments.
fundamentals, not supported by any observations of dividends during the experiment, and act rationally on it. In contrast, the behavioral theories of naive expectations, low-order linear forecasting rules and trend following rules, have been estimated from observable quantities, and these parsimonious rules fit our experimental data surprisingly well. We therefore view our experimental results as evidence for behavioral theories.

Let us finally try to develop some intuition for the emergence of expectational coordination. Participants in these experiments have an incentive to coordinate their prediction strategies, since the market clearing price is close to the average prediction. Participants who succeed in predicting the average prediction well, perform well in the experiment. This feature of the asset pricing experiment may be similar to real asset markets, and is consistent with the ideas of Keynes (1936) in his famous beauty contests. From our experiments we find that participants are rather successful in “anticipating what average opinion expects the average opinion to be.”

If there are forces toward coordination of individual expectations, the question then is what kind of “average” equilibrium outcome will individuals coordinate on? One possibility would be coordination on the fundamental price, but in our experiments (slow) convergence to the fundamental price only happens in a minority of cases. From a theoretical perspective another possibility for coordination is a (rational) self-fulfilling bubble solution growing at the risk free interest rate. In the absence of a robot trader and in the absence of upper and lower bounds, these bubble solutions are rational expectations (perfect foresight) equilibria. The presence of a robot trader, who acts as a stabilizer in the direction of the fundamental price, makes coordination on these bubble solutions less likely. In the experiment however, coordination on temporary bubbles, triggered by simple trend following strategies, does occur even in the presence of computerized fundamentalist traders. These trends are triggered by overreaction of a majority of participants, and once triggered become self-fulfilling and lead to momentum persisting. However, the trends cannot continue forever and are reversed, due to the lower and upper bounds 0 and 100, and/or the presence of robot traders. The upward trend reverses and once reversed, trend extrapolating forecasting rules reinforce the downward trend. The result is then coordination of individual expectations on damped or permanent oscillatory price fluctuations with upward and downward trends, as observed in most of our groups. Our experiments thus provide evidence for a number of behavioral modes popular in behavioral finance, in particular correlated imperfect rational forecasting due to trend extrapolation, overreaction, and momentum trading. Our experiments suggest that estimating a behavioral model, with agents using simple strategies, on real financial data is an important challenge for future work.
References


