M.Sc. in Economics
Advanced Microeconomics
Module on General Equilibrium Theory

Prof. Giorgio Fagiolo
Sant'Anna School of Advanced Studies
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Waiver Examination

Instructions: This is a closed-book exam. You have **90 minutes**. Please try to answer ALL questions. You must write in English. Please be concise but logically and mathematically coherent. If you are asked to give a definition or prove a proposition, please explain verbally any single step of your argument.

1. **Pareto Efficient Allocations**. Consider an economy

$$E = \left(\{X_i, u_i\}_{i=1}^I, \{Y_j\}_{j=1}^J; \{\omega_i, \theta_i\}_{i=1}^I \right),$$

   where I is the number of consumers, J is the number of firms, $X_i \in \mathbb{R}_+^L$ are consumption sets (L=number of commodities), $u_i : X_i \to \mathbb{R}$ are consumers' utility functions, $Y_j \subseteq \mathbb{R}^L$ are firms' technology sets, $\omega_i \in \mathbb{R}_+^L$ are consumers' initial-endowment vectors, and $\theta_{ij} \geq 0$ is the share of firm j owned by consumer i, where $\theta_i \in [0,1]^J$ and $\sum_i \theta_{ij} = 1$, all j.

   (a) Define the concept of "feasible allocation".

   (b) Define the concepts of Strongly Pareto Optimal (WPO) allocation and Weakly Pareto Optimal (SPO) allocation.

   (c) Prove that if an allocation is SPO then it is also WPO.

2. **Equilibrium and Welfare in a Pure-Exchange Economy**. Consider a pure-exchange economy composed of I consumers and L commodities. Let $X^i \in \mathbb{R}_+^L$ be consumption sets, $u^i : X^i \to \mathbb{R}$ consumers' utility functions, $e^i \in \mathbb{R}_+^L$ consumers' initial endowments, and $p \in \mathbb{R}_{++}^L$ the vector of commodity prices.

   (a) Define a Walrasian Equilibrium (WE) for this economy.

   (b) **First Welfare Theorem**: Prove that if $(x^i)_{i \in I}$ and p is a WE, and u^i are all strictly increasing (i.e. $u^i(z^i) > u^i(x^i)$ for any $z^i \gg x^i$), then $(x^i)_{i \in I}$ is strongly Pareto Optimal.