Advanced Microeconomics

Partial and General Equilibrium

Giorgio Fagiolo
giorgio.fagiolo@sssup.it
http://www.lem.sssup.it/fagiolo/Welcome.html

LEM, Sant'Anna School of Advanced Studies, Pisa (Italy)

Part 5

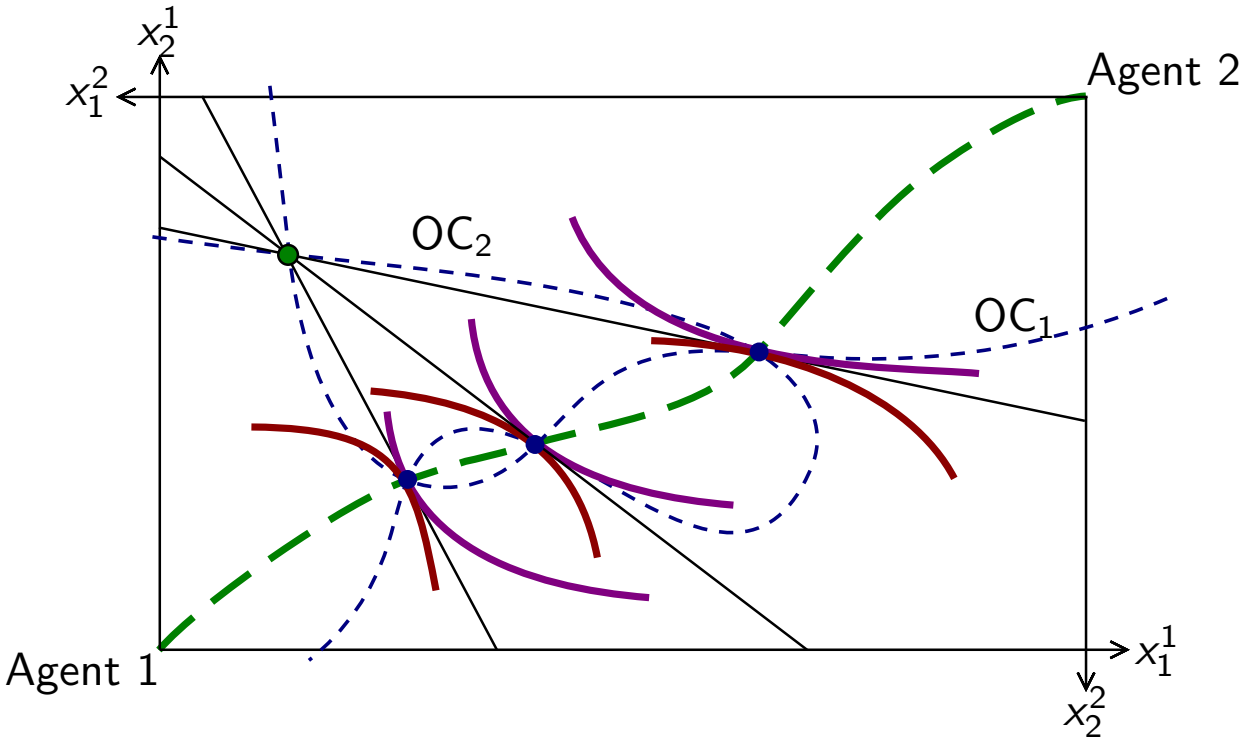
This and Next Lectures

So far: We have established that if the economy satisfies certain (very stringent) properties, then at least one WE exists. We are left with other crucial questions to answer...

1. Uniqueness: When a WE exists, is it unique? How many WE are there?
2. Stability: Suppose that a unique WE does exist. How does the economy (as distinct from the modeler) "find" the equilibrium, i.e. how does it go there? And if the economy starts far from the equilibrium, is it able to actually go back to the WE?
3. Empirical validity: Can we take the GET to the data and test whether its implications are true? What implications can we test?

Uniqueness

Question: Is there a unique WE up to price normalization? If not, how many WE are there?



In general, there may be many!

Uniqueness

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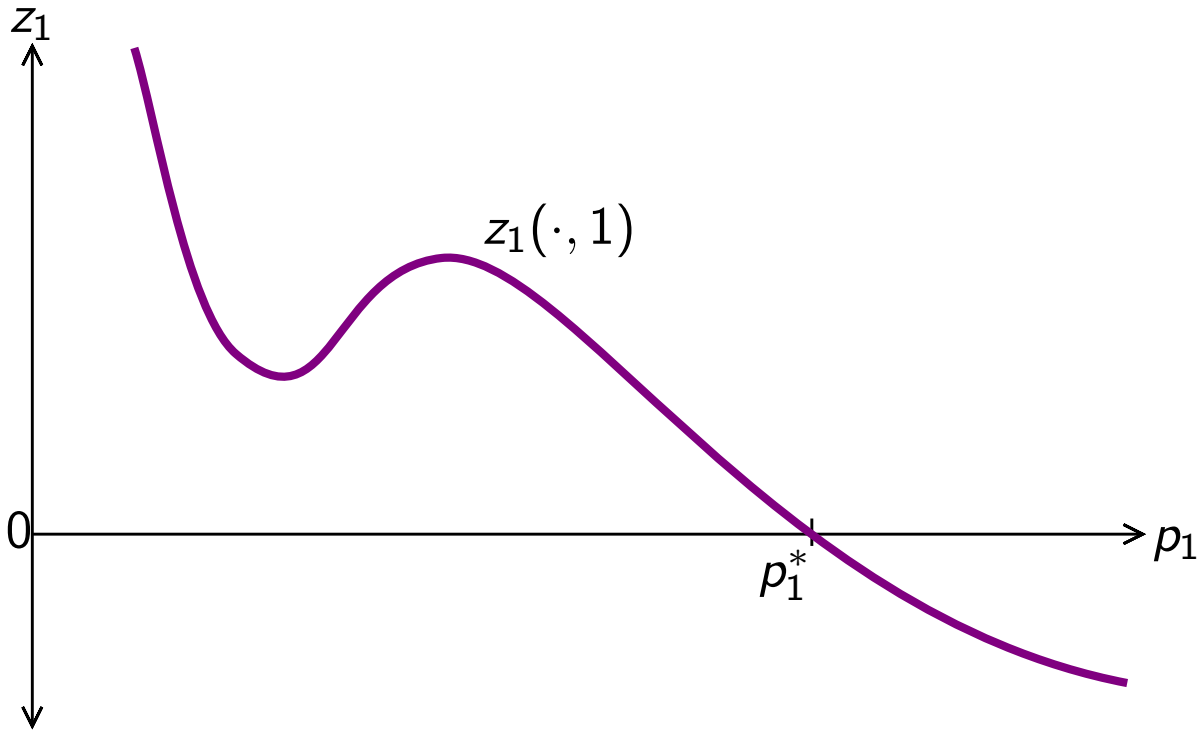
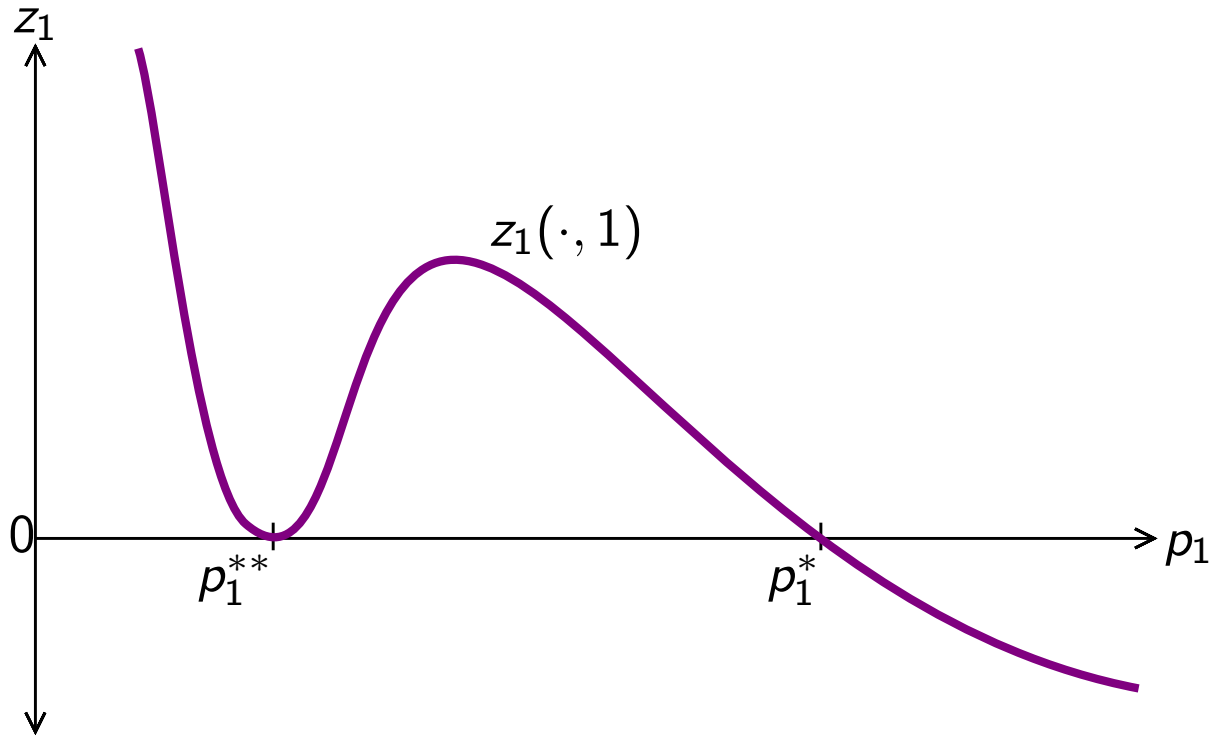


There could be a unique WE

Uniqueness

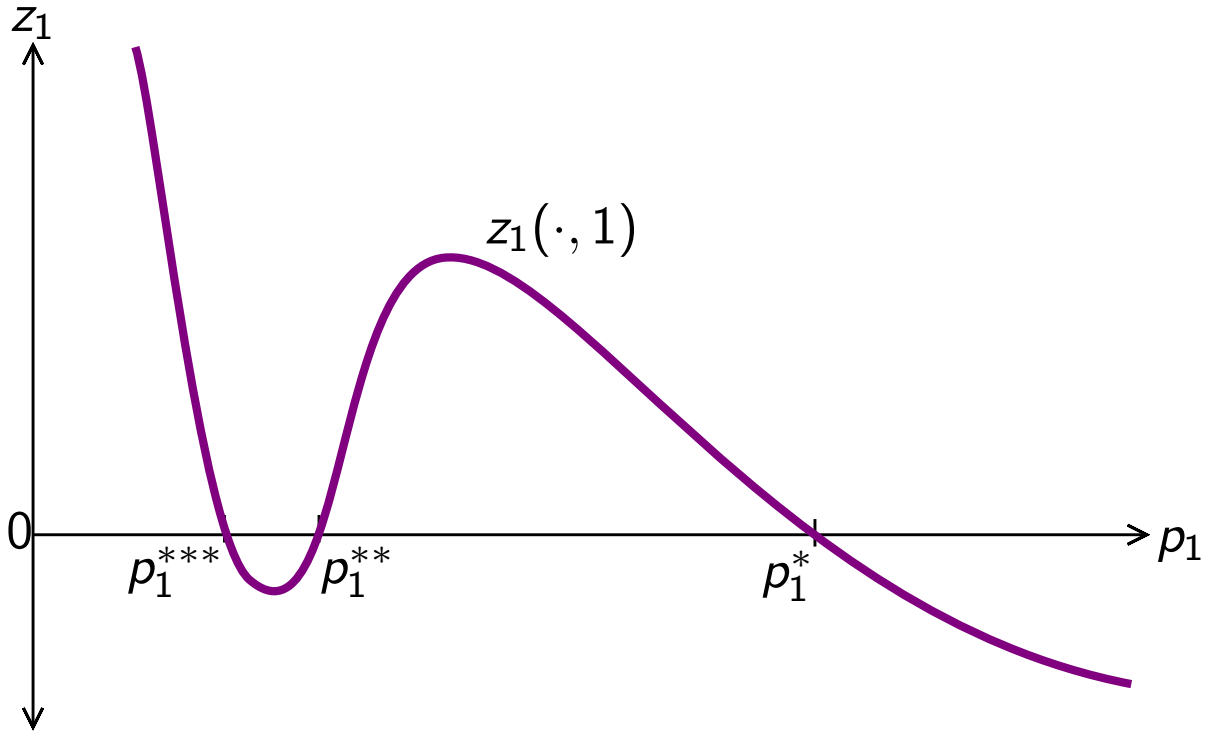
Question: Is there a unique WE up to price normalization? If not, how many WE are there?



There could be two WEs

Uniqueness

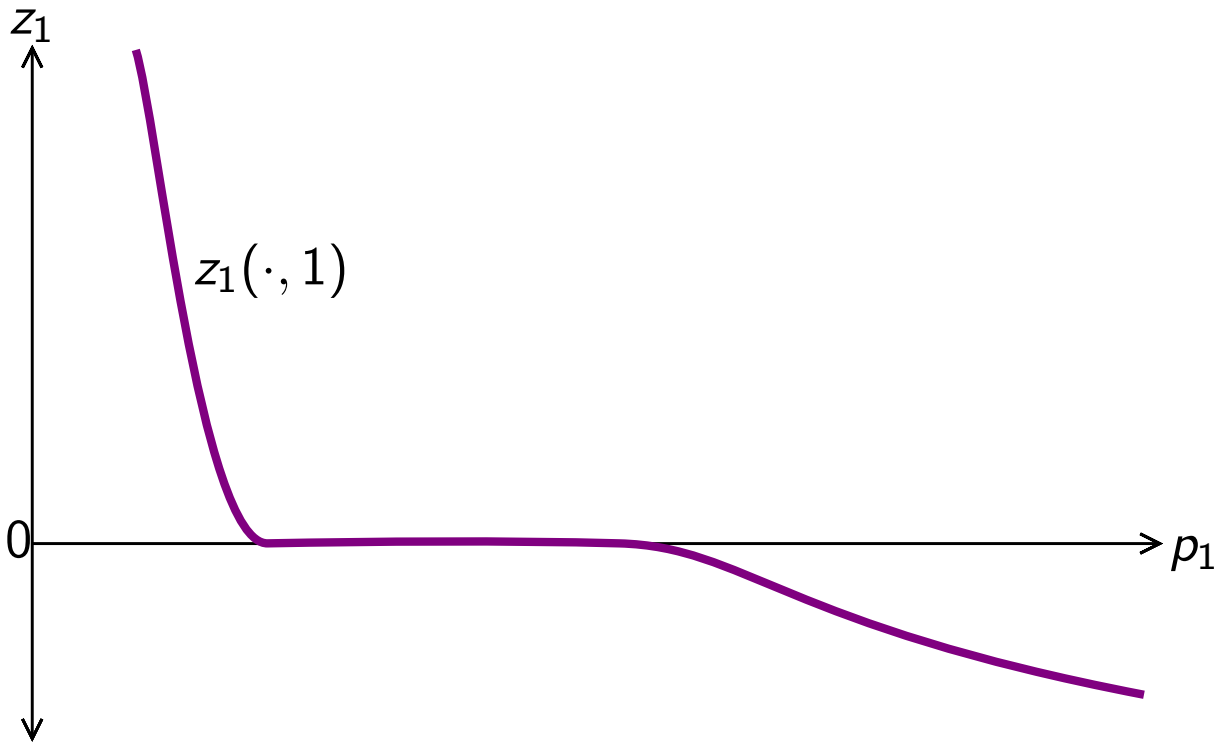
Question: Is there a unique WE up to price normalization? If not, how many WE are there?



There could be three WEs

Uniqueness

Question: Is there a unique WE up to price normalization? If not, how many WE are there?



There could be infinitely many WEs

Uniqueness

- **Where does multiplicity come from?**
  - Wealth effects generate the possibility of multiple equilibria.
  - When a price of a particular good changes there are two effects
    1. The relative attractiveness of various commodities changes
    2. The wealth distribution of individual agents is altered
  - These two effects can offset or reinforce each other in ways that make it possible for more than one set of prices to constitute an equilibrium

- **Is multiplicity of equilibria pervasive?**
  - In general, YES
  - Two related concepts: global uniqueness (only one equilibrium) vs. local uniqueness (a number of equilibria "sufficiently far apart" in the price space)
  - To ensure global uniqueness — or at the very least local uniqueness — we need to be prepared to add further restrictions to preferences and technologies

The Sonnenschein-Mantel-Debreu (SMD) Theorem

- **An "anything goes" result?**
  - The aggregate (excess) demand function inherits only certain properties of individual's demand functions
  - These are: continuity, homogeneity of degree zero, Walras' law and boundary behavior when prices are near zero
  - These properties are not sufficient to restrict the admissible aggregate excess demand function in a way which would ensure uniqueness of equilibrium
  - Therefore a GE model can have an arbitrary number of equilibria in general

Theorem (Sonnenschein-Mantel-Debreu)

Consider a continuous function $f : B \to \mathbb{R}^L$ *on an open and bounded set* $B \subseteq \mathbb{R}^L_{++}$ *such that*

- $f(\cdot)$ *is homogeneous of degree zero, and*
- $p \cdot f(p) = 0$ *for all* $p \in B$.

Then there exists an economy (goods, agents, preferences, and endowments) with aggregate excess demand function $z(\cdot)$ *satisfying* $z(p) = f(p)$ *for all* $p \in B$.

Local Uniqueness and Regular Economies

- **Local uniqueness**
  - Given that multiplicity cannot be in general avoided, under what conditions equilibria are finite and hence locally unique?
  - Local uniqueness implies that comparative statics can be applied as long as the shocks to the system are not too large
- **Regular economies**
  - A regular economy is an economy characterized by an excess demand function which has the property that its slope at any equilibrium price vector is non-zero
  - If we graph the excess demand function of a regular economy against prices, then the excess demand function cuts the price-axis in such a way to ensure that each equilibrium is locally unique
- **Regular economies and local uniqueness**
  - Debreu (1970): almost any economy, defined by an initial distribution of consumer's endowments, is regular (i.e. non regular economies have a zero measure). Therefore local uniqueness is almost always the case
  - Moreover, it can be shown that in regular economies the number of equilibria is finite (≥ 1) and odd

Gross Substitutability and Global Uniqueness

Question: Is it possible to further restrict excess demand functions so as to get a globally-unique equilibrium?

Recall that

Definition (Gross substitute—partial equilibrium)

Good ℓ is a (strict) gross substitute for good m iff $x_\ell(p, w)$ is (strictly) increasing in p_m.

In our G.E. framework, wealth depends on prices ($w = e \cdot p$) so

Definition (Gross substitute—general equilibrium)

Good ℓ is a (strict) gross substitute for good m iff $x_\ell(p, e \cdot p)$ is (strictly) increasing in p_m.

Gross Substitutability and Global Uniqueness

Definition (Gross substitutes property)

Marshallian demand function $x(p) \equiv x(p, e \cdot p)$ has the (strict) gross substitutes property if every good is a (strict) gross substitute for every other good.

More generally...

Definition (Gross substitutes property)

A function $f(\cdot)$ has the (strict) gross substitutes property if $f_\ell(p)$ is (strictly) increasing in p_m for all $\ell \neq m$.

Gross Substitutability and Global Uniqueness

Suppose each individual's Marshallian demand satisfies the gross substitutes property; i.e., $x_\ell^i(p)$ is increasing in p_m for all $\ell \neq m$

Then

- Individual excess demands also satisfy it: $z_\ell^i(p) \equiv x_\ell^i(p) - e_\ell^i$ is increasing in p_m
- Aggregate excess demand also satisfies it: $z_\ell(p) \equiv \sum_i z_\ell^i(p)$ is increasing in p_m

Gross Substitutability and Global Uniqueness

Theorem

If aggregate excess demand $z(\cdot)$ satisfies the strict gross substitutes property, then the economy has at most one Walrasian equilibrium (up to price normalization).

Proof.

Suppose in contradiction that there are two non-collinear Walrasian equilibrium prices p and p'; i.e., $z(p) = z(p') = \mathbf{0}$. Define $\lambda_\ell \equiv p'_\ell / p_\ell$, and consider $\tilde{\ell} \equiv \text{argmax}_\ell \, \lambda_\ell$. Finally, define $\tilde{p} \equiv \lambda_{\tilde{\ell}} p$. This normalization ensures that $\tilde{p}_{\tilde{\ell}} = p'_{\tilde{\ell}}$, and

$$\tilde{p}_\ell = \lambda_{\tilde{\ell}} p_\ell \geq \lambda_\ell p_\ell = p'_\ell,$$

with strict inequality for some ℓ (since otherwise $p' = \lambda_{\tilde{\ell}} p$).

Gross Substitutability and Global Uniqueness

Proof (continued).

Consider moving from p' to $\tilde{p}$ by increasing the price of each good one at a time. By gross substitutes,

$$\begin{aligned} 0 = z_{\tilde{\ell}}(p') &\leq z_{\tilde{\ell}}(\tilde{p}_1, p'_2, \ldots, p'_L) \\ &\leq z_{\tilde{\ell}}(\tilde{p}_1, \tilde{p}_2, p'_3, \ldots, p'_L) \\ &\vdots \\ &< z_{\tilde{\ell}}(\tilde{p}) \end{aligned}$$

where strict inequality obtains since $\tilde{p}_\ell > p'_\ell$ for some ℓ.

By homogeneity of degree zero of $z(\cdot)$, we have $z_{\tilde{\ell}}(\tilde{p}) = z_{\tilde{\ell}}(\lambda_{\tilde{\ell}} p) = z_{\tilde{\ell}}(p) = 0$, a contradiction. □

How Does the Economy Find Prices?

- **Out-of-Equilibrium Dynamics and Convergence**
  - Global stability: Suppose the economy has only one equilibrium and we start from any given price vector different from the equilibrium one. Does the economy converge to the unique equilibrium?
  - Local stability: Suppose there is a finite number of equilibria. Start from one equilibrium and assume the economy is hit by a shock that moves it a little bit away from the equilibrium. Does the economy go back to it?

- **Problems**
  - The GE model only says what happens in equilibrium: no hints on any disequilibrium behavior
  - The GE model is not dynamic: everything happens simultaneously and we do not know anything about what could happen through time
  - We must endow the model with an external set of assumptions that allow for some price adjustment when prices are not equilibria ones

Walrasian Tatonnement

- **Introduce a central auctioneer**
  1. Auctioneer suggests prices
  2. Agents report demand at these prices
  3. If excess demand is non-zero, return to step 1

- **How does the auctioneer suggest/adjust prices?**

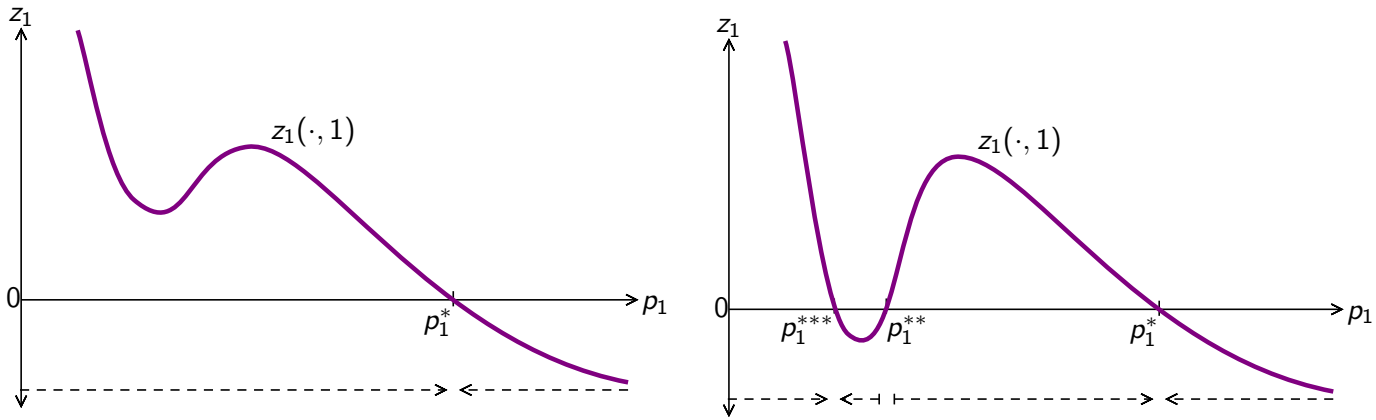
$$p_{t+1} = p_t + \alpha(t)z(p_t)$$

where $\alpha(t) > 0$ controls for the speed of adjustment

- **Problems**
  - Where does the auctioneer come from?
  - What does time really mean?
  - Any disequilibrium price is not feasible, therefore not admissible within the model!
  - Decentralized economy or centrally-planned economy?

Does the tatonnement process lead to stability?

- Suppose the economy adjusts prices using a tatonnement process. What happens when we start from a non-equilibrium price? Do we converge to an equilibrium?
- Not necessarily!! Everything depends on the slope of excess demand at equilibrium prices



- In general, the MSD theorem tells us that excess demand shapes cannot be restricted ex-ante: therefore we cannot say anything general about the stability of equilibria
- However, when one does further restrict preferences and technologies, stability can be achieved
- For example: if aggregate excess demand satisfies the strict gross substitutes property, then the unique is also globally stable under a tatonnement adjustment

Testability of GET Implications

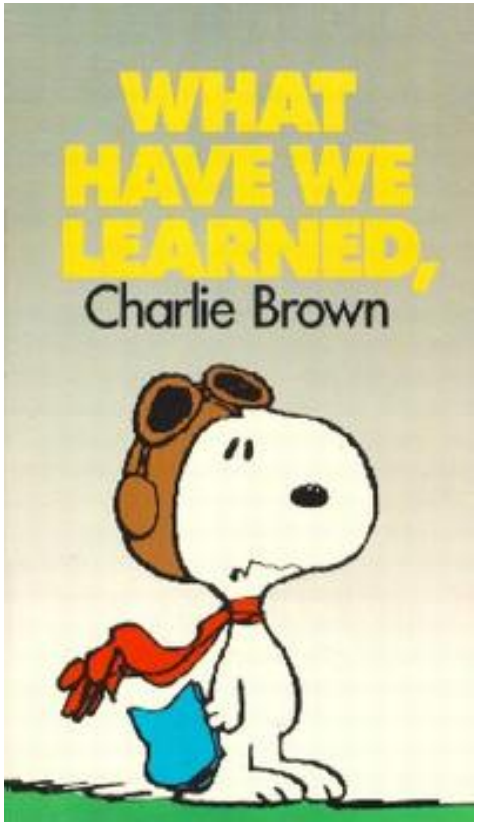
- **Does Walrasian equilibrium impose meaningful restrictions on observable data?**
  - Friedman: Models should be judged according to their predictive ability and explanatory power
  - What kind of empirically-testable implications does GET allow for?

- **Testable implications of excess demand function**
  1. Continuity
  2. Homogeneity of degree zero
  3. Walras' Law ($pz(p) = 0$ for all p)
  4. Limit properties when prices go to zero or to infinity

- **Actually, this is all we get!**
  - The MDS reloaded: the above properties are the only ones we can expect to hold in general for $z(p)$
  - Anything goes: apart from these testable implications, the predictive power of GET is almost irrelevant (e.g., no implications on the slope of excess demand)
  - A rather depressing viewpoint: GET is unable to generate falsifiable predictions (and thus is unscientific from a Popperian viewpoint)

What have we learned in this module?

- **The good side of GET**
  - Modeling price emergence in decentralized markets
  - A formal model of Smith's "Invisible Hand" idea
  - A very elegant set of strong statements based on very stringent and minimalistic assumptions
  - A perfect world: equilibrium, efficiency, social welfare
  - A micro model that can be used to micro-found all macroeconomic models
- **The bad side of GET**
  - An exercise in pure mathematics? An utterly unrealistic model completely disjoint from real-world intuitions of how markets work
  - Perfect worlds may exist only in the mind of (some) economists: when do normative statements become descriptive statements? Ex: welfare theorems
  - GET and anything goes implications (MSD theorem): do we really need so much mathematics to end up with almost irrelevant testable implications?



Beyond the Basic GET Framework

- **Extensions of basic GET**
  - Time
  - Imperfect information
  - Out-of-equilibrium trade
  - Money
  - Computable large GE models

- **An alternative approach to modeling decentralized economies**
  - The "economy as a complex system" approach
  - Agent-based computational economics (ACE)
  - Agent-based models (ABMs)
    - Old assumptions: Rationality, equilibrium, no interactions
    - What do experiments and empirical evidence tell us?
    - New assumptions: Bounded rationality, disequilibrium, interactions
  - ABMs: analytical tractability and simulations
  - See my course on ACE at:

    `http://www.lem.sssup.it/fagiolo/Teaching.html`