An Introduction to the Statistical Analysis of Agent-Based Models

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Outline of the Talk

- Agent-Based Models (ABM)
  - Why?
  - A Common Framework
- Statistical Analysis of ABM
- An Example
- Conclusions

Which Class of Models Are We Talking About?

- Micro-Macro Relationship
  - Studying macro properties of socio-economic dynamic systems changing over time due to behavior of interacting micro-entities (firms, consumers, etc.)
  - Adam Smith “Invisible Hand” Metaphor

- “Mainstream” Reference: Micro-founded macro models
  - Often based on over-simplifying assumptions
    - Interactions: Game Theory vs. General Equilibrium
    - Heterogeneity: Representative Individual
    - Behaviors: Hyper-rationality
    - Macro Properties as Equilibria

Why evolutionary/ACE approach in modeling decentralized economies?

- ABM as a complementary approach
  - Exploring dimensions difficult to address jointly with standard models
    - Bounded Rationality, Endogenous Novelty, Disequilibrium, etc.
  - Grounding behavioral assumptions into empirical / experimental evidence
    - Individual behavior, markets, industries…

- ABM as an alternative approach
  - Providing robustly an alternative view of how decentralized economies work
  - ABM replicating reality, generating fresh implications, allow for policy implications and predictions

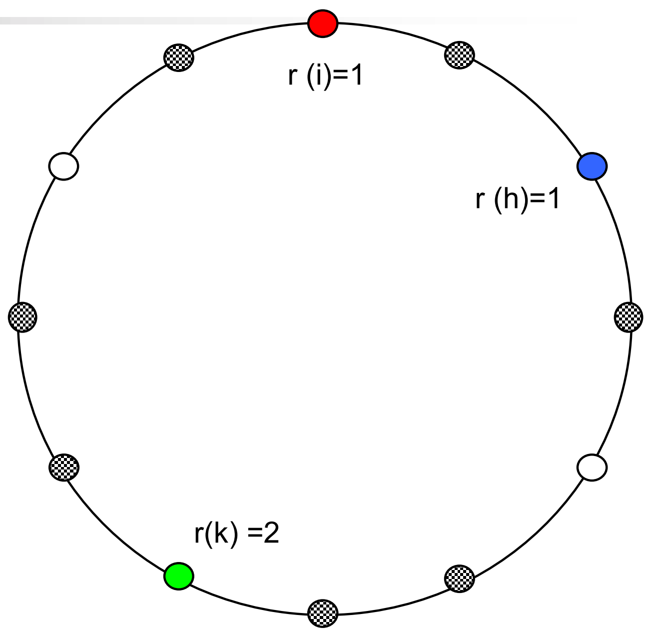
Cooking an ABM Model: Ingredients for a Recipe

- **Bottom-up** (agent-based) Philosophy (Tesfatsion, 1997)
- Agents live in **complex systems** evolving through time (Kirman, 1998)
- Agents are (or might be) **heterogeneous** in almost all their characteristics
- Environment is too complex: **“hyper-rationality” not viable** (Dosi et al., 1996)
- Agents behave as **boundedly rational** entities with adaptive expectations
- **“True” dynamics**: Systems are typically non-reversible
- Agents **interact directly**, interaction networks may change over time (Fagiolo, 1998)
- Endogenous and persistent **novelty** (technological change): open-ended spaces
- **Selection**-based market mechanisms (Nelson & Winter, 1982)

The Structure of ACE/EV Models

• Time	$t = 0, 1, 2, \ldots$	… Quarters, Years
• Set of Agents	$I = \{1, 2, \ldots, N\}$	… Firms
• Vectors of Micro-States	$i \rightarrow \underline{x}_{i,t}$	… Firms' output
• Vectors of Micro-Parameters	$i \rightarrow \underline{\theta}_i$	… R&D Propensity
• Vector of Macro-Parameters	$\Theta \in \Re^m$	… Opportunities
• Interaction Structures	$G_t \in \wp(I_t)$	… Networks
• Micro Decision Rules	$R_{i,t} (\bullet \mid \bullet)$	… Output Rule
• Aggregate variables	$\underline{X}_t = f (\underline{x}_{1,t}, \ldots, \underline{x}_{N_t,t})$	… GNP

Example: Dynamic Games



• Time	$t = 0, 1, 2, \ldots$	
• Sets of Agents	$I = \{1, 2, \ldots, N\}$	
• Sets of Micro States	$i \rightarrow \{-1,+1\}$	
• Interaction Structures	G_t = 1-Dim Lattice	
• Micro-Parameters	r(i)	Interaction Radius
• Vector of Macro-Parameters	$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$	Stage-Game Payoffs
• Micro Decision Rules	BR Rule	Strategy Updating
• Aggregate variables	Average Action	Coordination Level

Flexibility of ACE/EV Paradigm (1/5)

- **Micro Decision rules**

Deterministic

Myopic BR:	At each t, agents only know the state of the system they are allowed to observe at t-1 and choose among the available actions the one that maximizes current payoffs (assuming that tomorrow e.t. will remain the same
Routines:	Pay-Back Rule: Adjust current variables using a deterministic rule (function)

Stochastic

BR w/ mistakes:	Apply Deterministic BR and flips it
Linear Prob:	Choose actions prop to payoffs
Logistic Prob:	Choose actions prop to logistics of payoffs

Algorithmic

… Updating rules cannot be summarized by some function

Flexibility of ACE/EV Paradigm (2/5)

- **Dynamics of Micro Decision Rules**

Fixed	Agents always employ the same rules (with constant parameters)
Exogenously Changing	Micro decision rules change due to exogenous shocks that change e.g. the parameters (payback parameter, willingness to explore,…) – mutations, technological innovation, demand shocks,…
Endogenously Adapting	Micro decision rules endogenously change because agents are able to select (or are selected against) among a pool of different rules: learning over the space of rules…

Flexibility of ACE/EV Paradigm (3/5)

- **Expectations**

Myopic/Adaptive/Econometric

Agents employ the past to form expectations about the future in a naïve way: tomorrow expectations is a simple (linear) function of past observations

$x(t+1) = x(t)$

$x(t+1) = f\,(\, x(t), x(t-1)\,), \ldots, x(t-k)\,)$

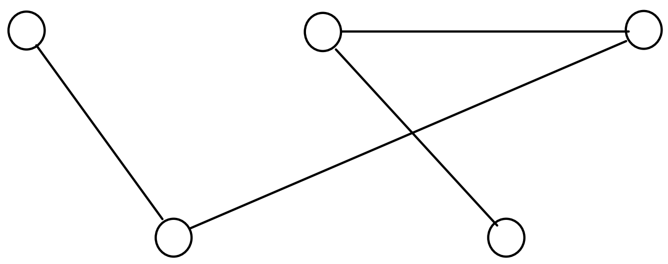
$x(t+1) \leftarrow x(t), x(t-1), \ldots, x(t-k)$

AI-Based

Neural Networks

Flexibility of ACE/EV Paradigm (4/5)

- **Interactions**
  - What does it mean “direct interactions” ?
  - Interaction structure: who interacts with whom at each point in time
  - Interaction structure described by a graph
    - Nodes and Edges



Flexibility of ACE/EV Paradigm (4/5)

- **Interactions**
  - What does it mean “direct interactions” ?
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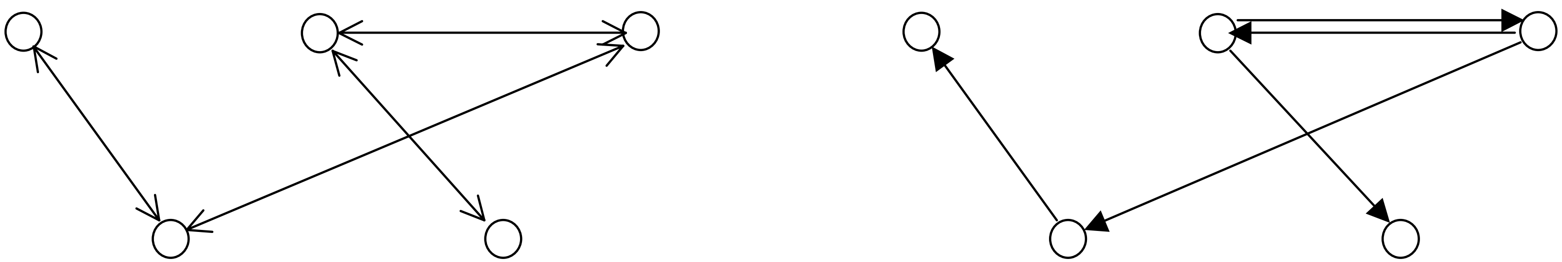
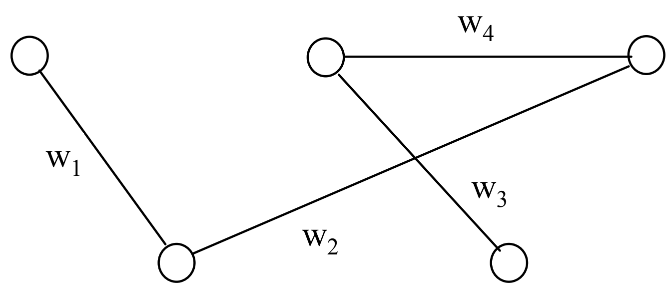
  - Interaction structure described by a graph
    - Undirected vs. Directed



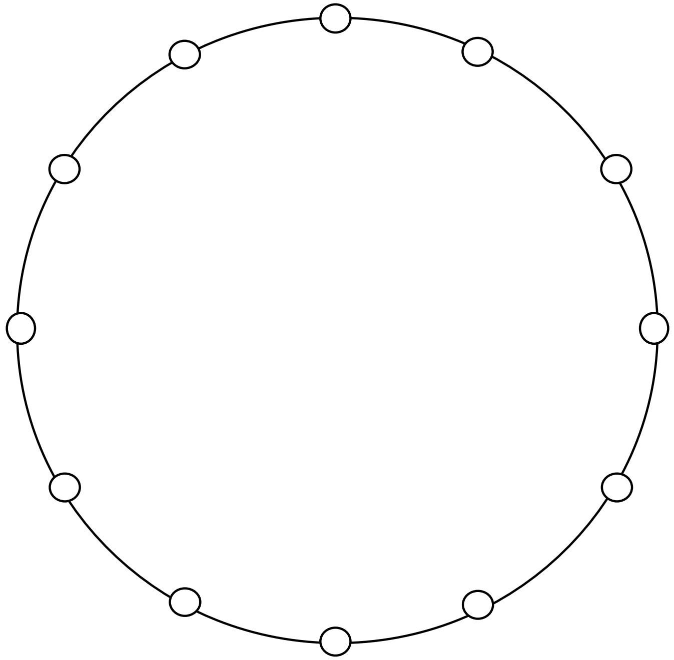
Flexibility of ACE/EV Paradigm (4/5)

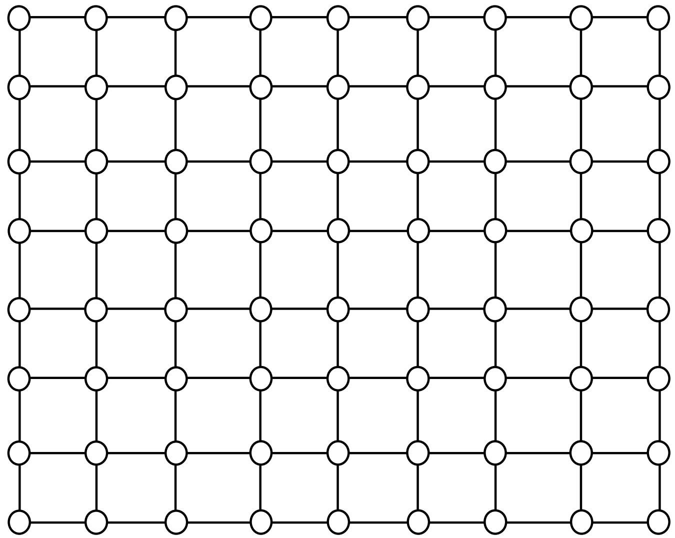
- **Interactions**
  - What does it mean “direct interactions” ?
  - Interaction structure: who interacts with whom at each point in time
  - Interaction structure described by a graph
    - ❑Unweighted vs. Weighted



Flexibility of ACE/EV Paradigm (4/5)

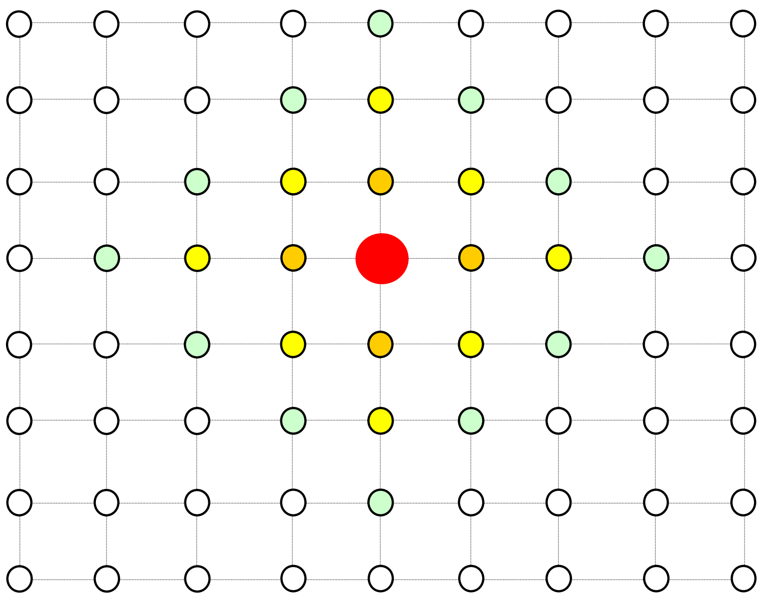
- **Interactions**
  - Interaction structure described by a graph
    - ❑ Lattices

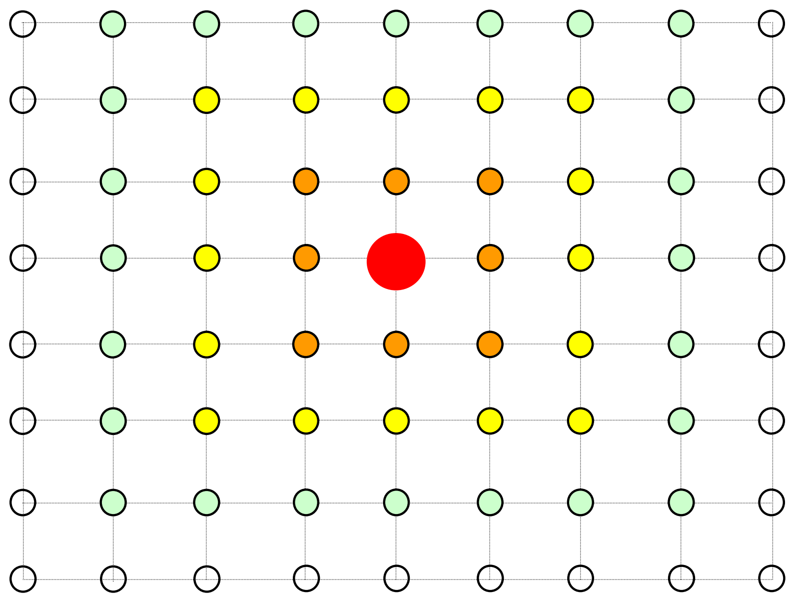




Flexibility of ACE/EV Paradigm (4/5)

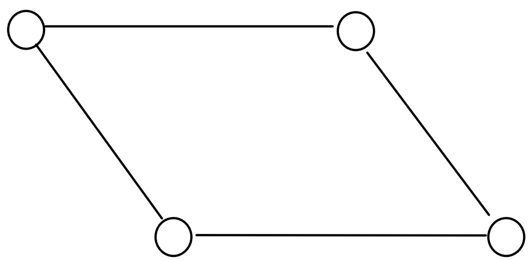
- **Interactions**
  - Lattice useful to describe local (spatial) interactions
  - Playing with different neighborhood structures

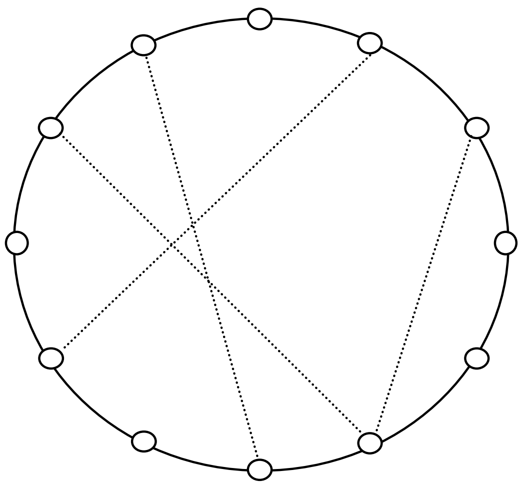


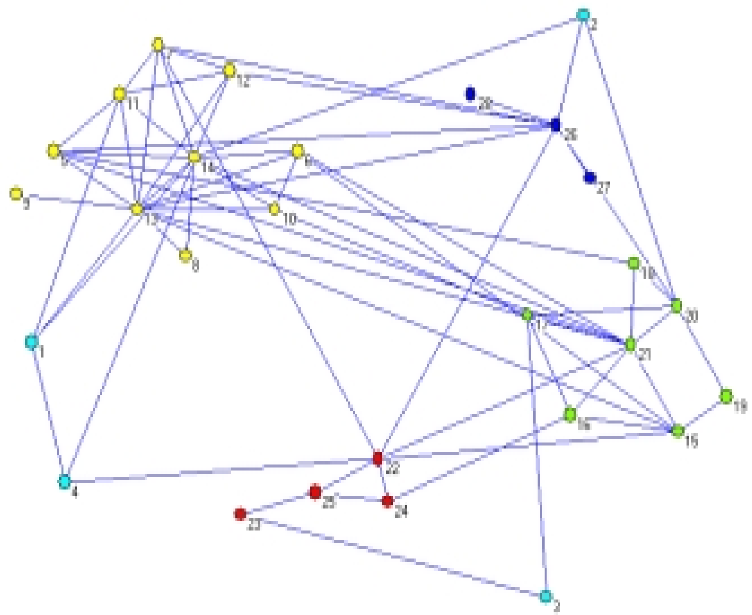


Flexibility of ACE/EV Paradigm (4/5)

- **Interactions**
  - Interaction structure described by a graph
    - Regular Graphs, Small-World Nets, Generic Graphs







Flexibility of ACE/EV Paradigm (5/5)

- **Dynamics of Interaction Structures**

Static	Interaction Structures are fixed across time
Exogenously Evolving	Interaction Structures change due to exogenous shocks (e.g. after the system has converged to some stable state)
Endogenously Evolving	Agents are able to choose whom to interact with (interaction structures become micro-variables and are updated in a strategic way thanks to properly defined decision rules)

A Large Set of Models…

- Evolutionary-Games (P. Young, Kandori et al., Blume, Ellison…)
- (Local) Interaction Models (Kirman, Weisbuch, Lux, Topol, IPD Models…)
- Endogenous Network Formation Models (Vega-Redondo, Goyal, Jackson-Watts…)
- Polya-Urn Schemes (Arthur, Dosi, Kaniovski, Lane, …)
- ….
- ….
- Industry-Dynamics Models (Nelson + Winter tradition, History-Friendly Models)
- Evolutionary Growth Models (Silverberg, Verspagen, Dosi et al., …)
- ACE Models of Market Dynamics (Axtell, Epstein, Tesfatsion, Vriend, …)

The Outcomes of ACE/EV Models (1/2)

Micro-Dynamics
(induced by decision rules, interactions and expectations)

Macro-Dynamics
(obtained as aggregation of individual behaviors)

- Stochastic components in decision rules, expectations, interactions imply that the dynamics of micro and macro variables can be described by some (Markovian) stochastic process parametrized by $(\underline{\theta}_i)$, Θ :

$$(\underline{x}_{i,t}) \mid (\underline{x}_{i,t-1}), (\underline{x}_{i,t-2}), \ldots ; (\underline{\theta}_i), \Theta$$

$$\underline{X}_t \mid (\underline{X}_{t-1}, \underline{X}_{t-2}, \ldots ; (\underline{\theta}_i), \Theta)$$

- Non-linearities in decision rules, expectations, interactions **may** imply that it is **hard** to analytically derive laws of motion, kernel distributions, time-*t* probability distributions, etc.

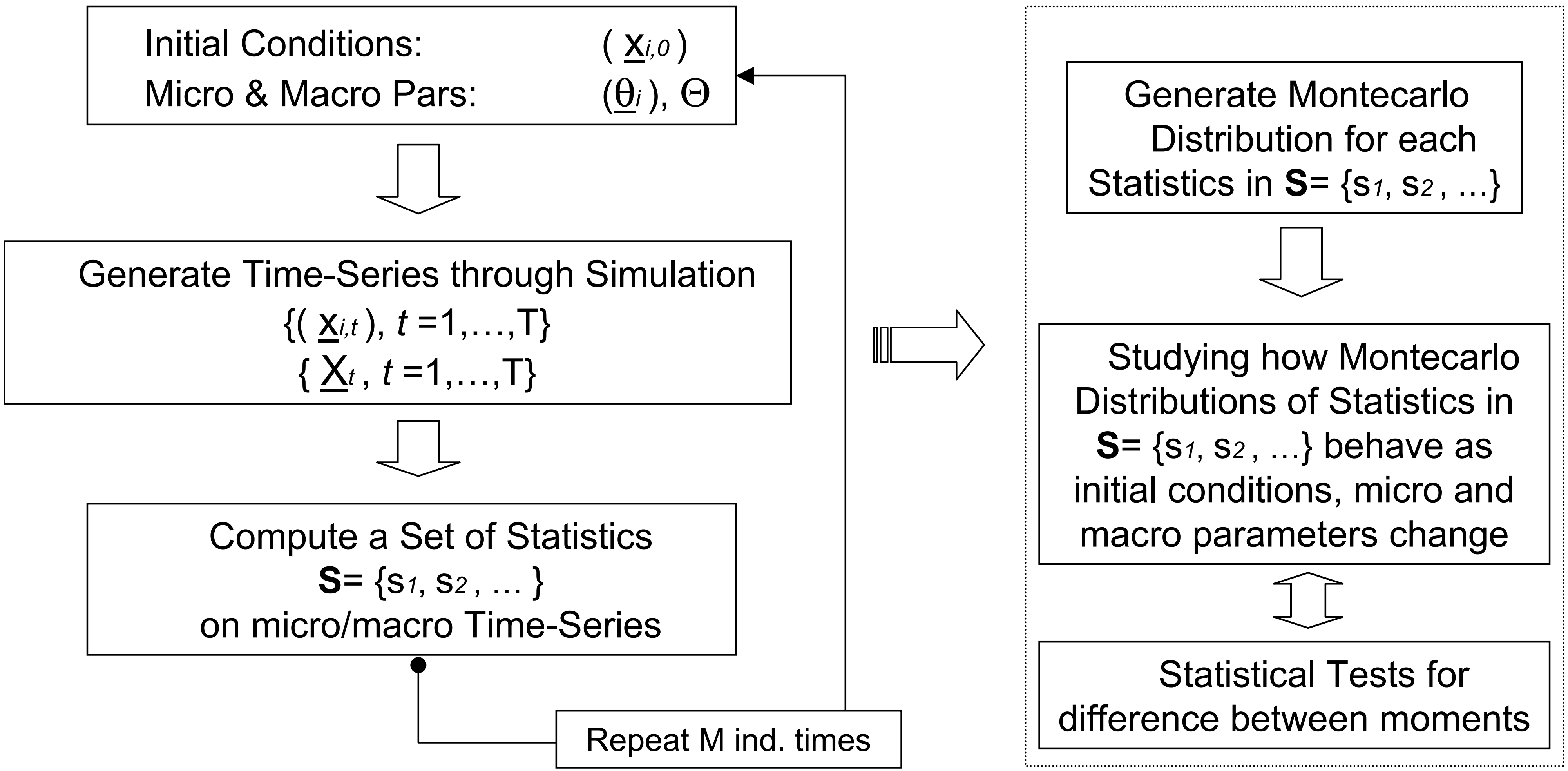
The Outcomes of ACE/EV Models (2/2)

- Often: Need to resort to **computer simulation** as tool of analysis to study the properties of (stochastic) processes describing $\underline{x}_{i,t}$ and $\underline{X}_t$

- Simple Models:
  - Steady-states, stationary distributions, etc.

- Complicated Models:
  - The economy is by definition out-of-equilibrium: Focus on disequilibria paths
  - Looking for emergent or transient (statistical) properties in aggregate dynamics

Analyzing ACE/EV Models



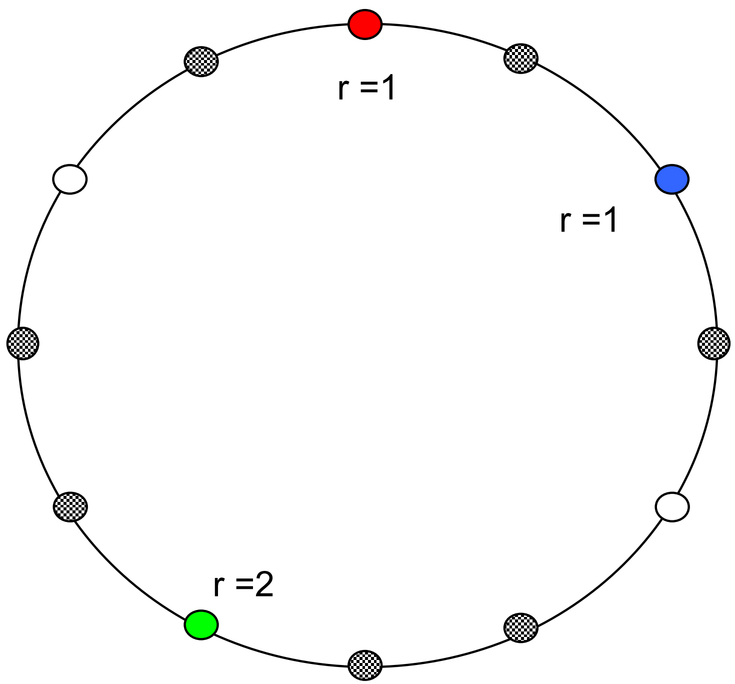
Example: Dynamic Games (1/5)

- Time — $t = 0, 1, 2, \ldots$
- Sets of Agents — $I = \{1, 2, \ldots, N\}$ — Players
- Sets of Micro States — $i \rightarrow s(i) \in \{-1,+1\}$ — Pure strategies
- Strategic Problem: Overall Coordination out of 2-person games (a > 1.5)

		+1	-1	
Pareto-Efficient Strategy →	+1	2a	0	$EU(+1)=2a \cdot ½+0 \cdot ½ = a$
Risk-Efficient Strategy if a<2.5 →	-1	3	2	$EU(-1)=3 \cdot ½+2 \cdot ½ = 2.5$

Example: Dynamic Games (2/5)

- Interaction Structures G_t = 1-Dim Lattice Circle
- Each agent i interacts with neighbors closer than r(i)



$$V(i) = \{ j : | i - j | \le r(i) \}$$

Example: Dynamic Games (3/5)

- Micro-Parameters $r(i)$ Interaction Radius
- Macro-Parameter a Stage-Game payoff of (+1,+1)
- Micro Decision Rules and Dynamics
  - At t=0 random draw of strategies
  - At each t>0 one agent is chosen at random
  - Chooses $s_t(i)$ s.t. max total payoffs given neighbors choices at t-1

$$s_t^*(i) = \arg\max_{s \in \{-1,+1\}} \sum_{j \in V(i)} u(s; s_{t-1}(j))$$

Example: Dynamic Games (4/5)

- Aggregate Variable: LR Coordination Level

$$c = \frac{1}{N}\sum_{i=1}^{N} s_T(i) \in [-1,+1]$$

- Choosing T large enough (stability/convergence of moments)

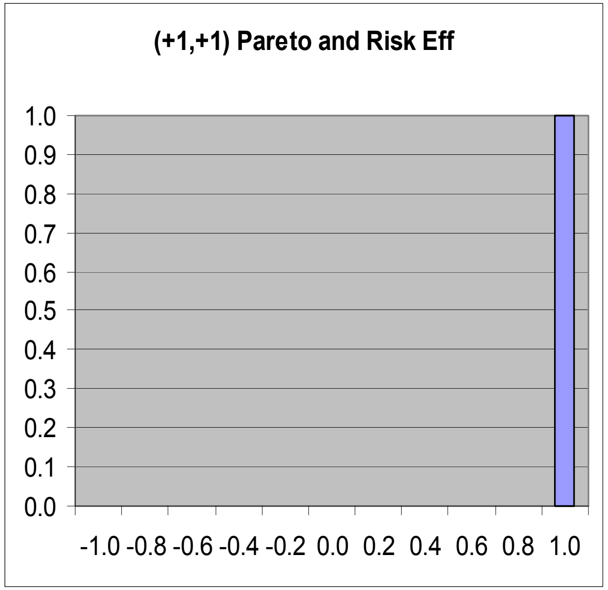
- Goal: Studying MC distributions of LR coordination levels as a function of

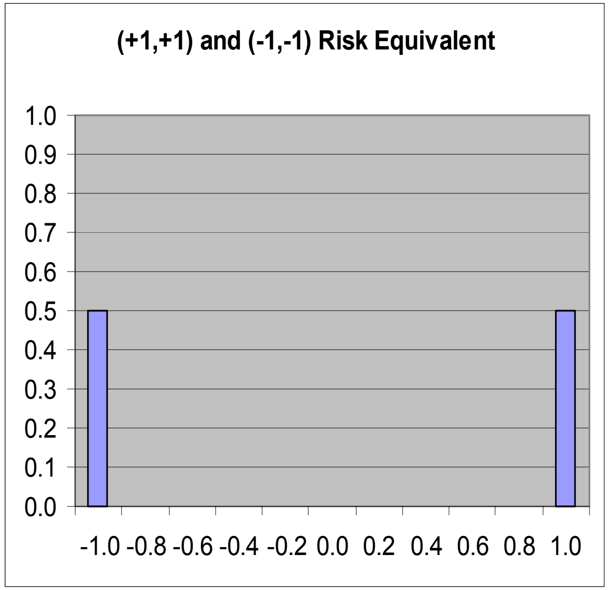
    1) Aggregate Parameter (a)
    2) Micro Parameters (e.g. average radius)

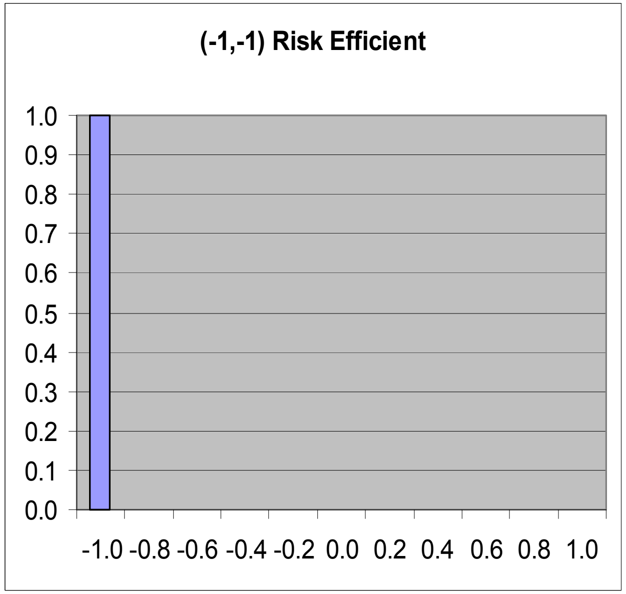
Example: Dynamic Games (5/5)

- Results with r(i)=1 all i:

1) (+1,+1) Pareto-Efficient and Risk Efficient (a > 2.5)
2) (+1,+1) and (-1,-1) Risk Equivalent (a = 2.5)
3) (-1,-1) Risk Efficient (a < 2.5)







Concluding Remarks

- From Stylized to Micro-Founded Models
  - No interpretative commitment to equilibrium
  - Allowing endogenously for structural change
  - Avoiding assumptions which are “too far away” from empirical evidence on individual behavior and the microeconomics of the system we want to model

- ABM and Real-World Data
  - Replicating Statistical Properties of Real-World Data (e.g. GNP)
  - Three hot topics:
    - Strategies for model building
    - Tools for Statistical Analysis of ABM and Empirical validation
    - Policy implications