On the Distributional Properties of Household Consumption Expenditures

Empirical Results and Modeling Perspectives

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Presentation Prepared for the Workshop
"The Role of Consumption for Structural Change in the Economy"
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Outline of the talk

1. **Introduction: Motivations and Literature**

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   - **Data (Italy)**
   - **Methodology**
   - **Results: Household Consumption Expenditures**

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   - **Statistical equilibrium**
   - **Diffusion processes**
   - **Micro-founded agent-based model**

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4. **Conclusions and Agenda**

Background Papers

- Fagiolo, G., Alessi, L., Barigozzi, M. and Capasso, M. (2007), "On the distributional properties of household consumption expenditures. The case of Italy", LEM Working Paper, 2007/24. Under revision for *Empirical Economics*.

- Alessi, L., Barigozzi, M., Capasso, M. and Fagiolo, G. (2007), “A Simple Characterizion of the Distribution of Household Consumption Budget Shares", LEM Working Paper, 2008, forthcoming.

- Capasso, M., Alessi, L., Barigozzi, M. and Fagiolo, G. (2007), "On approximating the distributions of goodness-of-fit test statistics based on the empirical distribution function: The case of unknown parameters.", LEM Working Paper, 2007/23. Under revision for *Advances in Complex Systems*.

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Motivations (1/2)

- **Characterizing Distributional Properties of Economic Variables**
  - Rem: *Characterization* here means "parametric statistical description"
  - Wealth and income distributions (Chatterjee et al., 2005)
  - Firm sizes and growth rates (Axtell, 2001; Bottazzi and Secchi, 2006)
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  - Despite existing turbulence at micro level (e.g. firm entry/exit; shocks to personal income; etc.) there exist a lot of regularity in distributional shapes of micro- and macro-economic variables and indicators

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- **Examples**
  - Income: Log-normal body + Pareto tail
  - Output growth rates: Exponential-power
    - Firms, industries, countries; cross-section and time-series

Motivations (2/2)

- **What about Household Consumption Expenditures (HCE)?**
  - A lot of cross-section micro data available
  - Understanding demand side is crucial in both micro and macro
  - Consumption: 2/3 of GDP
  - Importance of consumption for social welfare and normative aspects

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  - Attanasio (1999): Lot of knowledge on properties of aggregate consumption time series and microeconomic life-cycle profiles
  - Almost no empirical studies on the statistical properties of HCEDs
- **Why is it so?**
  - Most of consumption-related literature is (neoclassical) theory-driven
  - Representative-individual hypothesis almost always requires only knowledge of first and second moment of HCEDs (Caselli and Ventura, 2000, AER)

Why Characterizing HCEDs May Be Important? I

- **Characterizing HCEDs means understanding heterogeneity**
  - Heterogeneity crucial for aggregation issues (Forni and Lippi, 1997)
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- **Consumption and Heterogeneity**
  - Caselli and Ventura (2000): Representative individual hypotheses impose almost no restrictions on HCEDs ("anything goes" result)
  - Hildenbrand (1994): Law of demand and Wald's axiom can be obtained as the outcome of aggregation of not-necessarily-rational households, provided that some constraints on household heterogeneity hold
  - Kirman, Gallegati and others (90's): Aggregate well-behaved demand schedules can be obtained as the outcome of aggregation of heterogeneous, badly-behaved, individual demand schedules

Why Characterizing HCEDs May Be Important? II

- **A Phenomenological (Data-Driven) Approach to Economics**
  - Most of applied economics in theory-driven: first theory, then data
  - (Almost) Theory-free approaches to economics: first data, then theory
    - Kaldor's stylized facts in macroeconomics
    - Cambridge's data-driven (encompassing) approach to econometrics (Hendry)
    - Recent applications to financial markets and industrial dynamics (Econophysics)

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- **Two almost-alternative paradigms (oversimplifying…)**
  - Theory-driven: Use data to test theories often based on metaphysical entities and constructions (utility, rationality, RA hypotheses, etc.)
  - Data-driven:
    - Explore data from a theory-free perspective
    - Single-out robust stylized facts or statistical regularities valid at different disaggregation levels and dimensions
    - Look for theories or data-generating processes (DGPs) able to explain/reproduce stylized facts based on causal relations and/or simple probabilistic constraints
    - Problems: explanation vs. reproduction; existence of theory-free stylized facts

Why Characterizing HCEDs May Be Important? III

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  - Better understanding of causal mechanisms and forces at work?

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- **Brock (1999): Unconditional-Object Argument**
  - HCEDs are unconditional objects: therefore little information on the DGP that has generated them can be inferred from the data
  - Observed stylized facts can be the outcome of many different DGPs
  - However:
    - A statistically-sound parametric description of HCEDs can help in discriminating among “general classes” of stylized facts or rule out others
    - This can give us a rough idea on the “minimal” causal mechanisms at work, or on those NOT at work

Battistin, Blundell and Lewbel (2007)

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  - Cohort and age-conditioned HCE data from UK and US surveys
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  - Assuming perfectly-rational households maximizing the expectation of time-separable utility functions under expected intertemporal budget constraint
  - If shocks to consumption resulting from new information satisfy CLT and marginal utility is linear in log consumption, then HCEDs are log-normally distributed

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- **Implications**
  - HCED can be explained by a simple multiplicative growth model where consumption results from cumulation of random shocks
  - Only first two moments are needed to characterize HCEDs

Our Research Project: Research Questions

- **Is that true also for Italian HCEDs?**
  - Studying dynamics of Italian HCEDs: Stability? Structural changes?
  - Checking departures from log-normality using more sophisticated statistical methods
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  - Three-levels of modeling: from less to more micro-founded
- **Fitting consumption-expenditure budget shares distributions**
  - From HCE for commodity classes to consumption-expenditure BS
  - Taxonomize BS distributions according to moments

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- **Disaggregation into 8 Age Classes (Head of Household)**
  - Following Battistin et al. (2007)
  - Age classes: {≤30; 31-35; 36-40; 41-45; 46-50; 51-55; 56-60; ≥61}
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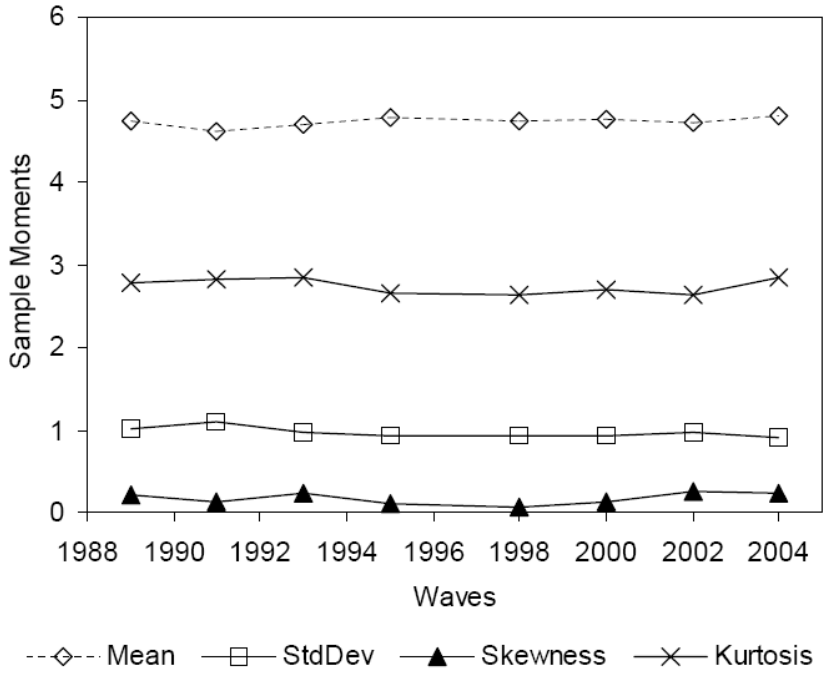
- **For BS Analysis Only: Expenditure Commodity Classes**
  - Nondurable (food), durables, insurance, [others]
  - Available classes make up for 80% of total consumption expenditure

Evolution of Time of HCEDs Moments

- **A First Check: Structural Breaks?**
  - First four moments of **aggregate** HCEDs are stable over time



  - NB: Kurtosis slightly below 3; skewness slightly above zero

Evolution of Time of HCEDs Moments

- **A First Check: Structural Breaks?**
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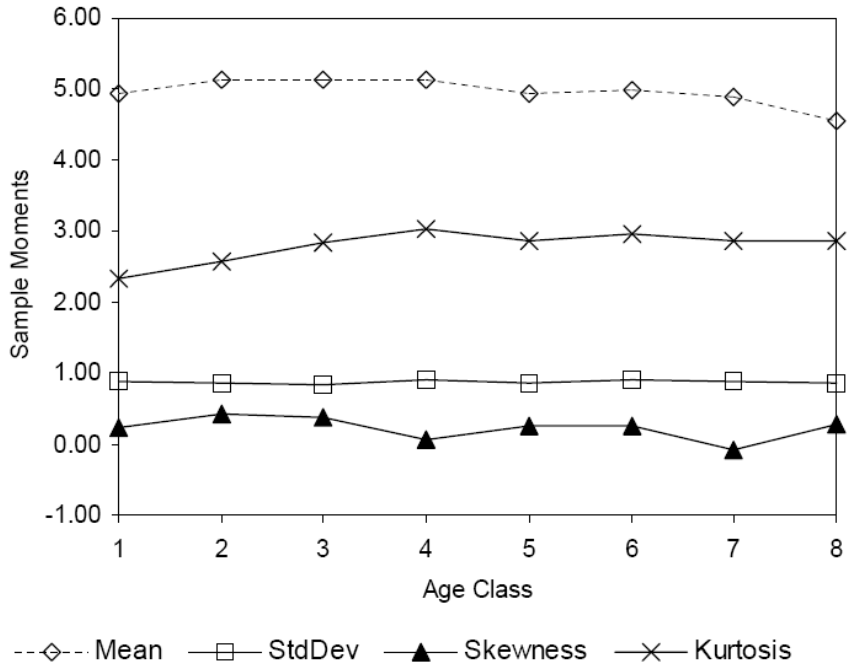
  - First four moments of **age-conditioned** HCEDs stable across age classes



  - NB: Income-conditioned data reveal similar patterns

Are HCEDs Log-Normal?

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  - Aggregate HCEDs: All tests reject log-normality (in every wave)
  - Age-conditioned HCEDs: In 55/64 HCEDs log-normality is rejected
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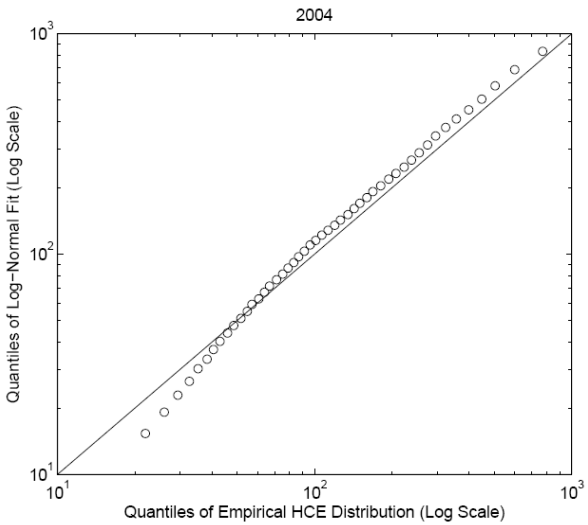
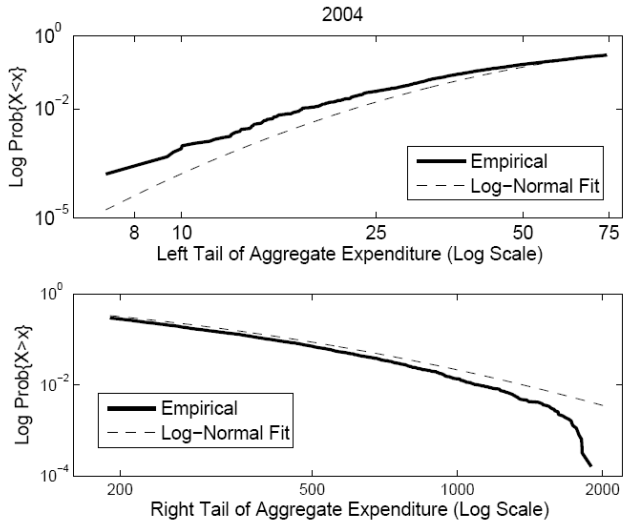
- **What are the causes of departure from log-normality?**
  - Two levels of heterogeneity
  - Across HCEDs: A lot of variation in first four moments (as expected)
  - **Within HCEDs: Asymmetric tail-behavior (left vs. right)**

Why Are HCEDs *Not* Log-Normal?

- **Tail behavior: Thick left tail and thin right tail**
  - Large (small) HCE less (more) likely than expected
  - Tail asymmetries also imply variability asymmetries (wrt modal value)





- Graphical evidence confirmed by estimation of Hill's Alpha Statistics

The Asymmetric Exponential-Power (AEP) Density

- **Five-parameter density**

$$g(x; a_l, a_r, b_l, b_r, m) = \begin{cases} K^{-1} e^{-\frac{1}{b_l}\left|\frac{x-m}{a_l}\right|^{b_l}}, & x < m \\ K^{-1} e^{-\frac{1}{b_r}\left|\frac{x-m}{a_r}\right|^{b_r}}, & x \geq m \end{cases}$$

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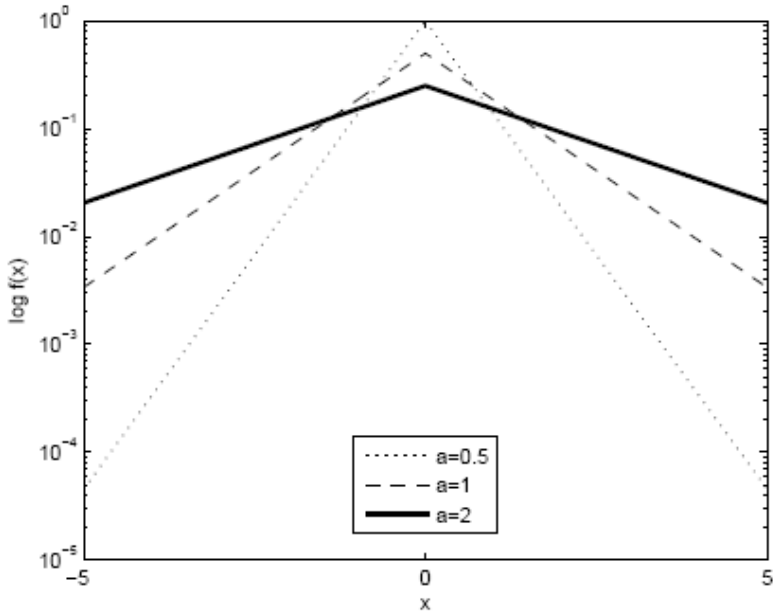
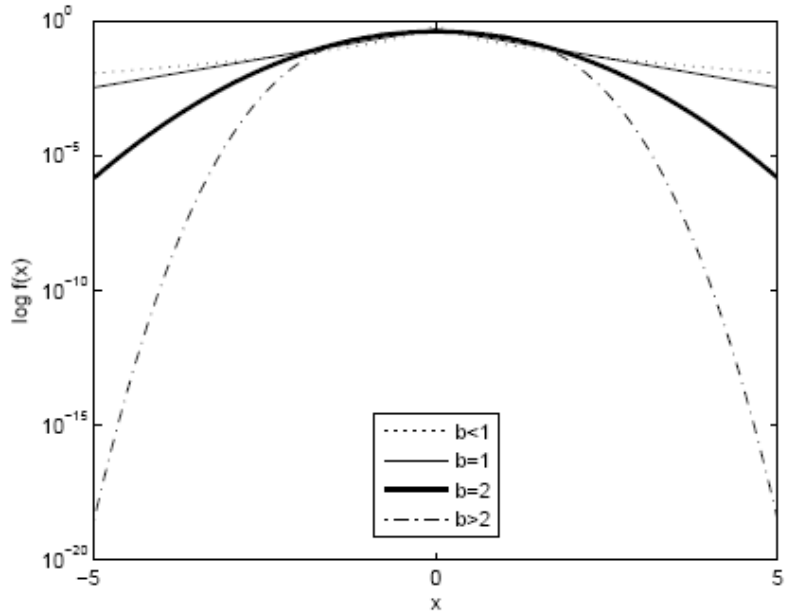
- **Some features of the AEP**
  - Modal parameter (m), shape parameters (a), tail parameters (b)
  - Recovers symmetric EP (Subbotin) for $a_l{=}a_r$ and $b_l{=}b_r$
  - Recovers Gaussian density for $b_l{=}b_r{=}2$ ($a_l{=}a_r$)
  - Recovers Laplace density for $b_l{=}b_r{=}1$ ($a_l{=}a_r$)
  - Allows for thinner-than-normal tails ($b{>}2$)
  - Allows for thicker-than-normal tails ($b{<}2$)
  - All the moments of the AEP exist finite (not a heavy-tailed distribution)

AEP: Symmetric Case (Logs on y-axis)

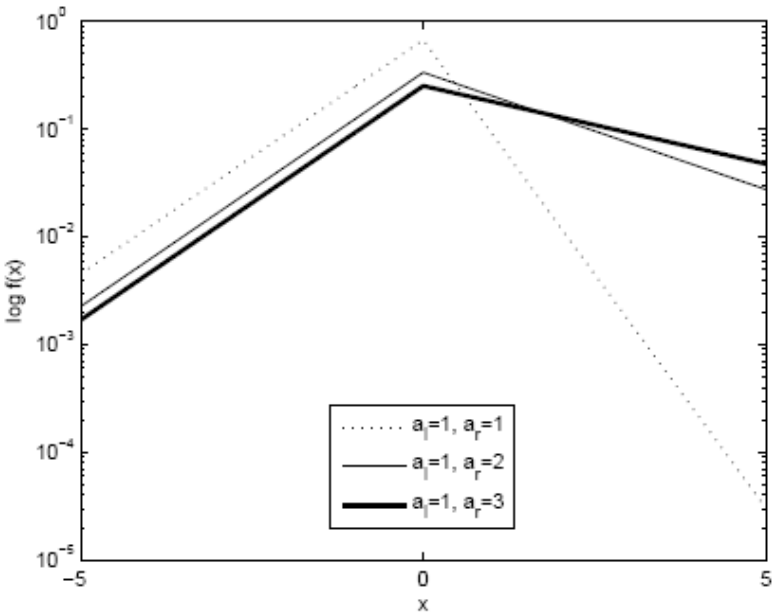


b=1: Laplace for increasing a's

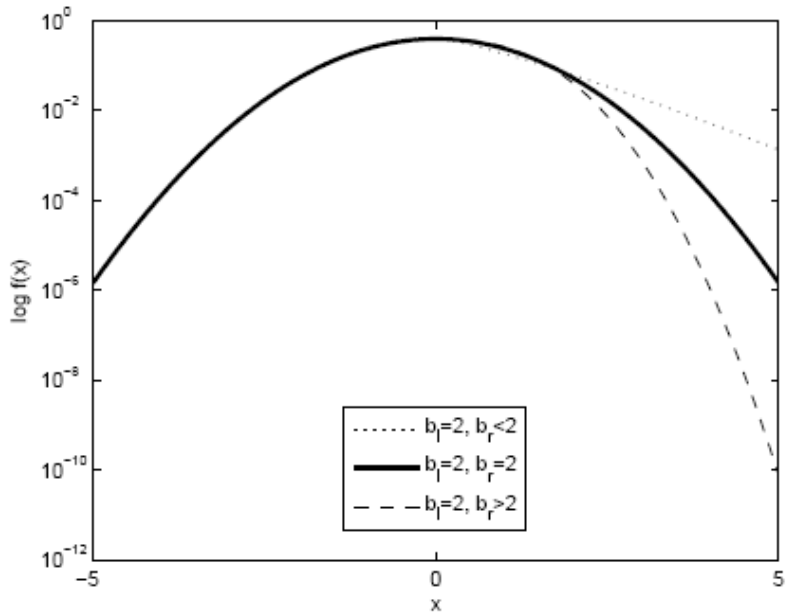


a=1: Tail thickness for decreasing b's

AEP: Asymmetric Case (Logs on y-axis)



$b_l = b_r = 1$: Increasing a_r



$b_l = 2$: Varying b_r

MLE Fits of AEP Density to Logs of HCEDs

- **Goodness-of-Fit (Monte-Carlo) Tests**
  - Kuiper, Cramér-Von Mises, Quadratic Anderson-Darling Tests
  - Null hypothesis (H_0): HCEDs are AEP-distributed

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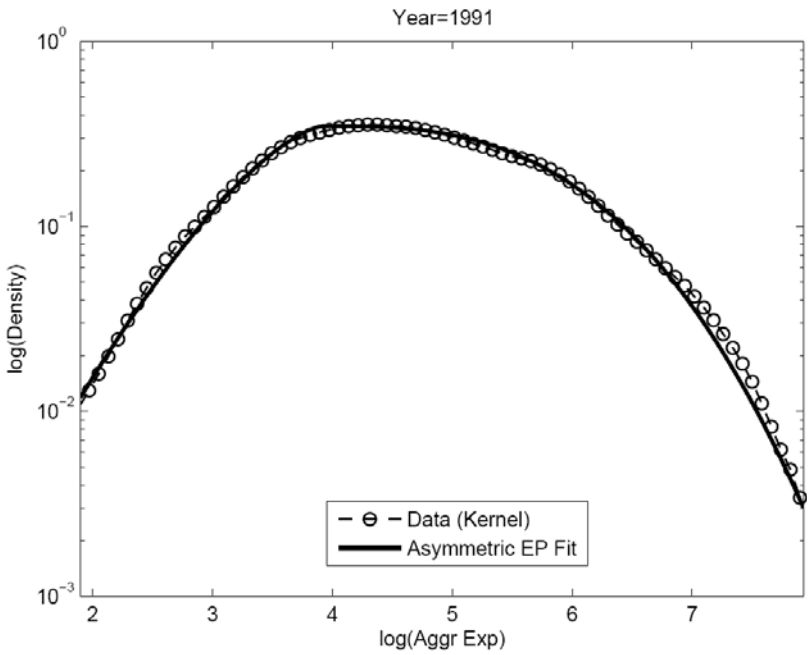
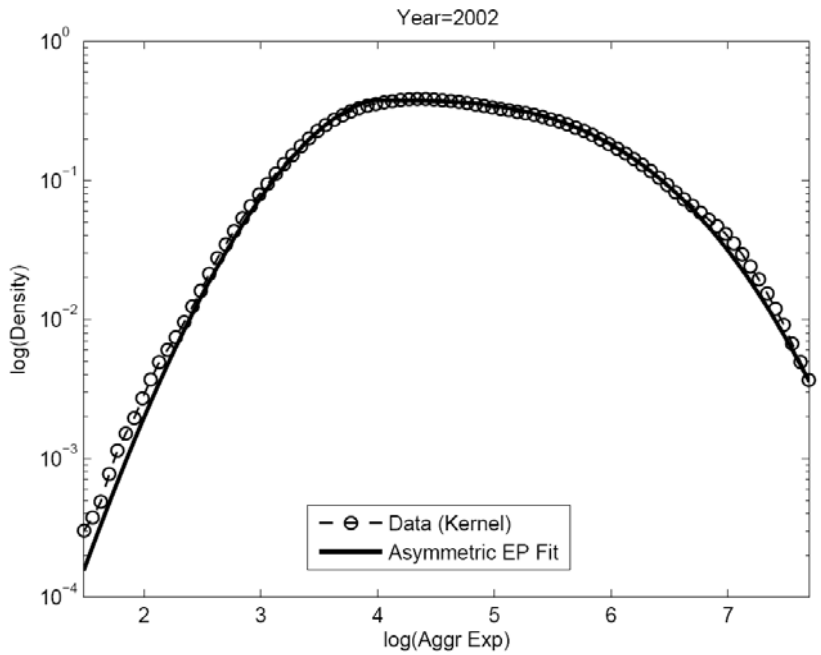
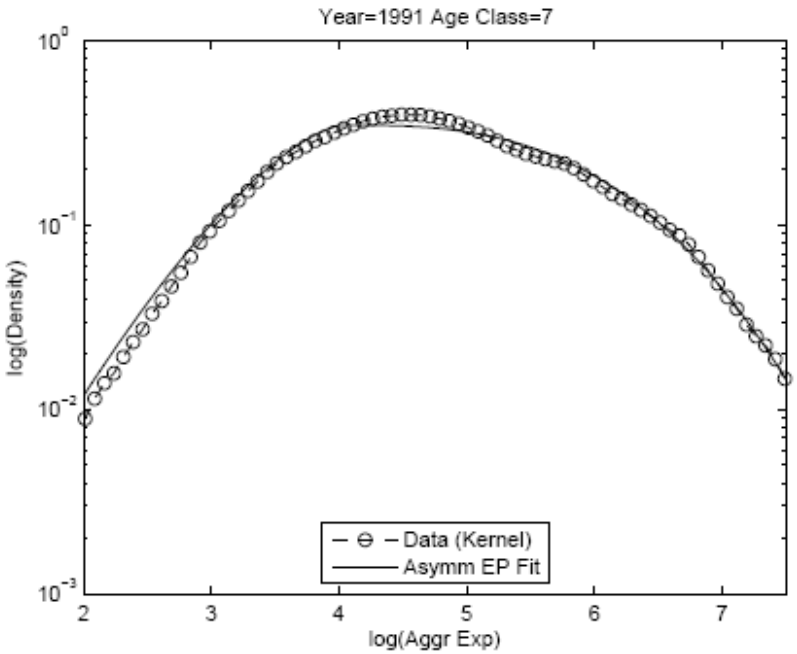
- **Why Does AEP Perform So Well? Asymmetry in Tail Thickness**
  - Right tail
    - 79% of cases: thinner than left tail
    - 89% of cases: thinner than normal
  - Left tail
    - 47% of cases: thicker than normal

AEP Fits: Aggregate HECDs

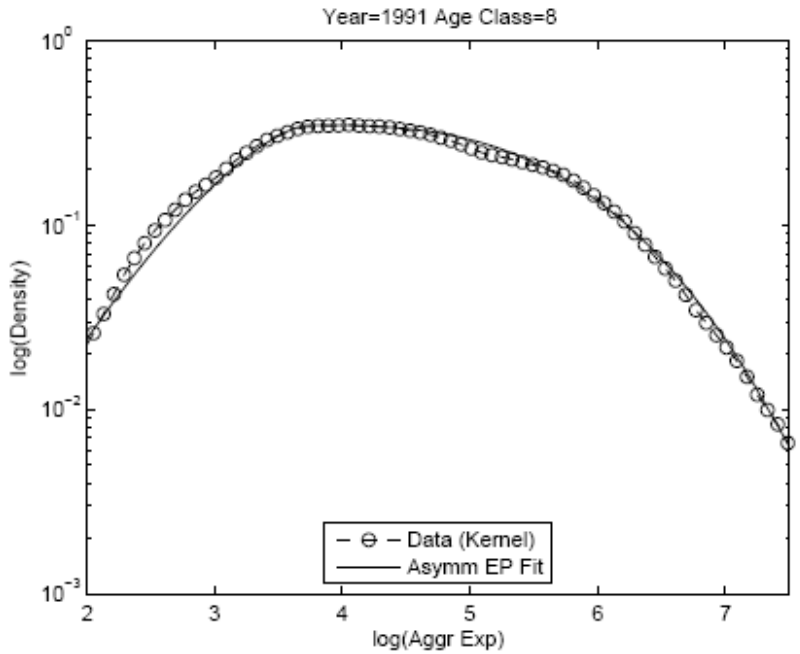




AEP Fits: Age-Conditioned HECDs (Wave=1991)



56-60



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Implications and Current Empirical Research

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  - HCEDs are not lognormal
  - Within-distribution asymmetries call for parsimonious but more flexible parametric characterizations
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- **Current Empirical Research**
  - Apply similar methods to levels of HCEDs in other countries (UK)
  - Disaggregation of HCEDs wrt commodity classes and budget-shares
    - Finding: Disaggregated HCEDs seem more lognormal than aggregated ones (!)
    - Are our finding only the effect of aggregation? Why?
    - It is possible to find a closed-form density for consumption budget-shares stemming from log-normal commodity-class HCEDs
    - This density performs very well in characterizing observed BS distributions
    - It allows to taxonomize commodity classes in economically-meaningful ways

Modeling: Some Speculative Remarks

- **Rationalizing emergence of AEP-Distributed Logs of HCEs**
  - If simple multiplicative process cannot be the cause, what is it?
  - Problem: Is it possible to explain emergence of AEPs for HCEDs on the basis of simple-enough causal mechanisms?
  - If not, what is the minimal set of assumptions able to reproduce such stylized fact?

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- **Brock (1999) Argument (Cont'd)**
  - Explanations of a single (unconditional) stylized fact (SF) may lead to a plethora of competing simple models able to reproduce it
  - Risk of coming out with very simple (and maybe economically-meaningless) processes and causal mechanisms
  - Our emerging SF should be explained jointly with other SFs (e.g., BS taxonomy, etc.)
  - Strategy: Starting to look at plausible ways to interpret AEP-distributed HCEs and then proposing more micro-founded models

Level-I Modeling: Statistical Equilibrium (1/3)

- **AEP Distributions as the Solution of a Max-Ent Problem**
  - Consumption choices as inherently stochastic phenomena
  - Consumers live in complex-evolving systems (Kirman, 1997)
  - Expenditure choices modeled as ongoing “exploration” of new, feasible alternatives given **constraints**
  - **Constraints** embody tastes, income, and equalization tendency (imitation); may be formalized as dispersion=constant

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- **Statistical Equilibrium (Jaynes, 1978; Foley, 1994; Aoki, 2000)**
  - Observed cross-section distribution of (logs of) HCE can be formalized as the one that maximizes **entropy** of the system subject to finite-dispersion constraints
  - Applications:
    - Foley (1994, JET): Exchange-economy
    - Krebs (1996, JET): OLG models
    - Gallegati et al. (2008): Industry-dynamics and financial markets

Level-I Modeling: Statistical Equilibrium (2/3)

- **Symmetric Case: If Density $f(c)$ Solves**

$$\max_{f(c)} H = \max_{f(c)} [-\int_{-\infty}^{\infty} f(c) log f(c) dc]$$

$$s.t. \int_{-\infty}^{\infty} f(c) dc = 1$$

$$\int_{-\infty}^{+\infty} f(c) |c - m|^{\alpha} dc < \infty$$

Finite Standardized Alpha-Moment

Then $f(c)$ is EP distributed with tail-parameter α (see Alfarano et al., 2008)

Level-I Modeling: Statistical Equilibrium (3/3)

- **From Symmetric to Asymmetric EP Densities**
  - Imposing constraints on the direction of deviations from modal value

Level-I Modeling: Statistical Equilibrium (3/3)

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- **Economic Interpretation of Max-Ent Principle (Foley, 1994)**
  - Entropy H measures the number of micro-states (consumer choices) consistent with a macro-state (distribution)
  - Number of ways a macro-state can be realized (entropy landscape) is shaped by income dynamics, tastes, imitation, etc., which are formally summarized through moments constraints
  - Economy constantly explores neighboring micro-states going (probabilistically) from a lower to a higher entropy state
  - Statistical equilibrium is achieved when entropy is maximized, i.e. at the combinatorially most-likely distribution

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- **Calibrating the Model?**
  - Using empirical tail parameter to interpret moments constraints (α)

Level-II Modeling: Diffusion Process

- **AEP as Limit Distribution of a Diffusion Process (Aoki, 2000)**
  - Consumers in interacting-particle systems playing “against the field”
  - Changes of HCE in continuous time depend on
    - Interactive term: Imitation tendency (consumers try to be like their neighbors)
    - Innovation term: Innovation tendency (shocks to income + exploration)

$$dc_t = -\frac{D}{2s}\left|\frac{c_t - m}{s}\right|^{\alpha-1} sign(c_t - m)dt + \sqrt{D}dW_t$$

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- **Interesting Property**
  - Both interactive and innovative term are jointly needed!

Level-III Modeling: ACE Models (1/3)

- **Statistical Equilibrium and Diffusion Processes**
  - Plausible but very stylized DGPs
  - Common underlying mechanisms (imitation/exploration)
  - Brock (1999) critique prompts for models able jointly to explain many empirical regularities at once: need for more micro-foundation

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- **Aversi et al (1999, ICC): ACE Model of Consumption Dynamics**
  - Building blocks: ABM philosophy (Pyka and Fagiolo, 2005)
- **Main Ingredients**
  - N consumers
  - Finite initial number of commodities, possibly expanding in time
  - Goods ordered from basic to luxury
  - Consumers with lexicographically-ordered preferences encoded in binary strings, e.g. {111100100101…}: 0=not desired; 1=desired
  - Individual incomes and prices vary stochastically

Level-III Modeling: ACE Models (2/3)

- **Dynamics I: Consumption**
  - Given income, prices, and preferences, agents decide how to allocate expenditures across existing desired goods
  - Build up “frustrated memory” (desired but not affordable expenditures)

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- **Dynamics III: Imitation and Innovation**
  - Given new income, prices and goods, agents can
    - Leave unchanged their consumption basket
    - Use new income to satisfy (part of) frustrated memory
    - Try one on the newly-introduced goods (innovation)
    - Imitate consumers in same/neighboring income class (GA-dynamics)

Level-III Modeling: ACE Models (3/3)

- **Main Results:**
  - Homogeneous initial conditions evolve into heterogeneous, stable, consumption patterns in path-dependent and socially-embedded ways
  - S-shaped diffusion curves (good adoption/consumption) and cross-section Engel curves with empirically-plausible shapes do emerge
  - Income elasticity of basic goods tend to decline over time
  - BSs of basic goods tend to decrease, BSs of luxury goods tend to increase in the long run
  - “Law of Demand” and “Wald Axiom” emerge -- checking Hildenbrand’s (1994) sufficient conditions

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- **Further Research**
  - Is the model able also to generate empirically-plausible cross-section distributions for HCEs (AEP) and BS?

Conclusions: Agenda

- **First Part: Empirical Properties of HCEDs in Italy**
  - Extend the analysis to BS (current work)
  - Apply same analysis to other Countries (next step)
  - Explore properties of consumption-income-age joint distributions

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- **Second Part: Perspectives on Modeling**
  - Extend the diffusion-process approach to asymmetric case
  - Aversi et al. (1999): promising point of departure due to interaction between imitation-innovation forces already found in statistical-equilibrium and diffusion-process approaches
    - Simplify the model so as to retain its explanative ability
    - Complicate the model so as to account for more stylized facts

Thanks!

https://mail.sssup.it/~fagiolo/welcome.html