# Agent-Based Models in Economics

**Giorgio Fagiolo**

**Sant'Anna School of Advanced Studies, Pisa (Italy)**
giorgio.fagiolo@sssup.it
https://mail.sssup.it/~fagiolo

DIMETIC 2008
Maastricht, October 2008

# Research Areas

- Agent-Based Computational Economics (ACE)
  - Methodology: Empirical validation in ACE models
  - Applications: ACE models and policy
- Networks
  - Game-theoretic models of strategic network formation
  - Empirical properties of economic networks
- Industrial dynamics: models/empirical evidence
  - Geography of industrial agglomeration
  - Firm size and growth dynamics: the role of financial constraints
- Statistical properties of micro/macro dynamics
  - Statistical properties of household consumption patterns
  - Statistical properties of country-output growth

# Homepage

***Giorgio Fagiolo***
Associate Professor of Economics
Laboratory of Economics and Management
Sant'Anna School of Advanced Studies
Piazza Martiri della Libertà, 33 I-56127 Pisa (Italy)
E-Mail

Curriculum Vitae (pdf)
Research
Seminars & Lectures
Publications
Teaching
Software
Links

Last Update: Apr 2007

https://mail.sssup.it/~fagiolo/welcome.html

# Sneak-in Preview

- What is an ABM? Standard definition:
  - Computational model of an economy consisting in populations of heterogeneous, interacting agents with a limited knowledge of the system
  - Aggregate dynamics not-necessarily characterized by equilibria in the standard sense; micro dynamics may allow for meta-stable patterns and emergent phenomena

- This (and following) lecture
  - A statistical perspective on ABMs in terms of stochastic processes
  - How ABMs can be analyzed and taken to the data

# Outline

- Why Agent-Based Models in Economics?
  - Problems with neoclassical models
  - Empirical and experimental findings
  - Philosophical underpinnings
- Building Blocks of Agent-Based Models
  - Classes of assumptions
  - The structure of an agent-based model
  - Analysis of an agent-based model
- Open Issues in Agent-Based Models
  - Interactions with mainstream community
  - Policy Implications
  - Empirical validation (next lecture)

# Background Papers


- Pyka, A. and **Fagiolo, G.** (2005), "Agent-Based Modelling: A Methodology for Neo-Schumpeterian Economics". In: Hanusch, H. and Pyka, A. (Eds.), *The Elgar Companion to Neo-Schumpeterian Economics*, Edward Elgar, Cheltenham.
- Dosi, G., **Fagiolo, G.** and Roventini, A. (2006), "An Evolutionary Model of Endogenous Business Cycles", Computational Economics, 27, 1: 3-34 (published version of LEM Working Paper 2005/04).
- Axelrod, R. (1997) Advancing the Art of Simulation in the Social Sciences, Complexity, 3: 16-22.

# Additional Material

- Miller and Page (2007), *Complex Adaptive Systems*, Princeton University Press.
- Tesfatsion and Judd (Eds), Handbook of Computational Economics, Agent-Based Computational Economics, Volume 2, North-Holland, 2006
- Flake (1998), *The computational beauty of nature*, The MIT press.
- Leigh Tesfatsion's web site: www.econ.iastate.edu/tesfatsi
- My own course on ABMs: see https://mail.sssup.it/~fagiolo/teaching.html

# Agent-Based vs. Neoclassical Models

- Benchmark: micro-macro (neoclassical) models
  - Endogenous or exogenous micro-founded growth models

- Based on over-simplifying assumptions
  - Heterogeneity irrelevant: the “representative individual” hypothesis and its consequences for aggregation
  - Fully-rational agents without computational bounds
  - Equilibrium analysis: empirical observations as equilibria
  - No interactions among agents (other than price-related ones)

- Why such a set of assumptions?
  - Need for a sharp relation between assumptions and implications
  - Analytical solutions strongly required
  - Instrumentalist approach *à la* Friedman

# Empirical and Experimental Findings (1/2)

- Persistence of heterogeneity among agents
  - Examples: Firms and industry characteristics
  - Aggregation in theory: average of behaviors different from behavior of the average (Kirman, Lippi)
  - Aggregation in practice: aggregate properties may have nothing do to with individual characteristics (ex: law of demand, see Gallegati, Kirman, etc.)

- Equilibrium analysis?
  - Economy as a complex evolving system
  - Economic observations as equilibria of some kind?
  - Ex: Turbulence in the patterns of industrial dynamics
  - State of the economy as meta-stable states: statistical features that last sufficiently long to be observed

# Empirical and Experimental Findings (2/2)

- Real-World Economic Agents are not Rational
  - Majority of rationality axioms persistently violated in reality
  - Departures from axioms are systematic (“predictably irrational”)
  - Examples: framing, probabilistic judgment and intuition

- Relevance of Interaction Networks
  - Agents form interaction networks to exchange commodities, information, knowledge
  - Real networks have peculiar and persistent properties (small-worlds, scale-free, etc.)
  - Network structure does influence aggregate results (ex: market design)

# Evidence vs. Models (1/2)

- Relevance of standard neoclassical models
  - Dick Day: "*Can one do good science based on models whose assumptions are clearly at odds with empirical evidence?*"
- An old (but still open) philosophical problem
  - Models as abstractions of reality
  - What does "realistic assumptions" really mean?
  - Models as solutions of the trade off between simplicity and usefulness
- Empirical validity of an economic model
  - To what extent is a model able to explain and replicate existing reality (and possibly predict future trends)?
  - Are neoclassical models really good at explaining and replicating stylized facts?

# Evidence vs. Models (2/2)

- Empirical validity of neoclassical models?
  - Industrial dynamics and organization
  - Micro-founded models of growth
  - Macroeconomic models of investment and output dynamics
  - Micro-founded models of labor-market dynamics
  - … and so on

- Difficulties
  - Dynamics and distributions
  - Joint replications of SFs

- Exploiting “instrumentalism” at its best
  - If the model is not able to replicate stylized facts, assumptions can be freely replaced
  - Why not using assumptions “more in line” with empirical evidence?

# Agent-Based Models

- A tool to model economies where agents
  - are boundedly rational entities
  - directly interact in non trivial networks
  - might be persistently heterogeneous
- … and
  - State of the economy is not necessary an “equilibrium”
- A bottom-up approach
  - Modeling agents behaviors and their interactions first
  - Statistical analysis of models output
  - Matching with empirical data

# ACE/Evolutionary Approaches

- Two competing brands?
  - Sharing almost same ingredients and philosophical underpinnings
- Evolutionary Models
  - Stress on selection-based market mechanisms…
  - … less on tools used
- ACE Models
  - Stress on tool used (OOP)…
  - … focus on open-ended systems where behavioral rules endogenously evolve as well

# The Structure of Agent-Based Models

- Main ingredients (to cook an ABM)
  - Bottom-up (agent-based) Philosophy (Tesfatsion, 1997)
  - Agents live in complex systems evolving through time (Kirman, 1998)
  - Agents might be heterogeneous in almost all their characteristics
  - “Hyper-rationality” not viable (Dosi et al., 1996)
  - Agents as boundedly rational entities with adaptive expectations
  - “True” dynamics: Systems are typically non-reversible
  - Agents interact directly, networks change over time (Fagiolo, 1997)
  - Endogenous and persistent novelty: open-ended spaces
  - Selection-based market mechanisms (Nelson & Winter, 1982)

# The Structure of Agent-Based Models

| | | |
|---|---|---|
| • Time | $t = 0, 1, 2, \ldots, (T)$ | … Discrete |
| • Sets of Agents | $I_t = \{1, 2, \ldots, N_t\}$ | … Often $N_t = N$ |
| • Sets of Micro States | $i \rightarrow \underline{x}_{i,t}$ | … Firm's output |
| • Vectors of Micro-Parameters | $i \rightarrow \underline{\theta}_i$ | … Res. Wage |
| • Vector of Macro-Parameters | $\Theta \in \Re^m$ | … Opportunities |
| • Interaction Structures | $G_t \in \wp(I_t)$ | … Networks |
| • Micro Decision Rules | $R_{i,t}(\bullet \mid \bullet)$ | … Innovation rule |
| • Aggregate variables | $\underline{X}_t = f(\underline{x}_{1,t}, \ldots, \underline{x}_{N_t,t})$ | … GNP |

# Example: Schelling Segregation Model

| | | |
|---|---|---|
| • Time | $t = 0, 1, 2, \ldots, (T)$ | … Discrete |
| • Sets of Agents | $I = \{1, 2, \ldots, N\}$ | … Households |
| • Sets of Micro States | $i \rightarrow \underline{x}_{i,t}$ | … Location, Neighbors |
| • Vectors of Micro-Parameters | $i \rightarrow \underline{\theta}_i$ | … Type, Utility cutoff |
| • Vector of Macro-Parameters | $\Theta \in \Re^m$ | … Empty nodes, Size |
| • Interaction Structures | $G_t \in \wp(I_t)$ | … Lattice |
| • Micro Decision Rules | $R_{i,t}(\bullet \mid \bullet)$ | … If and where to move |
| • Aggregate variables | $\underline{X}_t = f(\underline{x}_{1,t}, \ldots, \underline{x}_{N_t,t})$ | … Segregation index |

# Exercise: Map objects in Dosi et al. 2006

| | | |
|---|---|---|
| • Time | $t = 0, 1, 2, \ldots, (T)$ | ??? |
| • Sets of Agents | $I = \{1, 2, \ldots, N\}$ | ??? |
| • Sets of Micro States | $i \rightarrow \underline{x}_{i,t}$ | ??? |
| • Vectors of Micro-Parameters | $i \rightarrow \underline{\theta}_i$ | ??? |
| • Vector of Macro-Parameters | $\Theta \in \Re^m$ | ??? |
| • Interaction Structures | $G_t \in \wp(I_t)$ | ??? |
| • Micro Decision Rules | $R_{i,t}(\bullet \mid \bullet)$ | ??? |
| • Aggregate variables | $\underline{X}_t = f(\underline{x}_{1,t}, \ldots, \underline{x}_{N_t,t})$ | ??? |

# Flexibility of ACE/EV Paradigm

- Micro Decision rules
  - deterministic (best-replies, routines) → stochastic → algorithmic
- Dynamics of Micro Decision Rules
  - fixed → exogenously changing → endogenously adapting
- Expectations
  - myopic/adaptive → econometric → AI-based (neural networks)
- Interactions
  - global → local
  - symmetric, bilateral → asymmetric, unilateral
- Dynamics of Interaction Structures
  - static → exogenously evolving → endogenously evolving

# A Large Set of Models…

- Evolutionary-Games (P. Young, Kandori et al., Blume, Ellison…)
- (Local) Interaction Models (Kirman, Weisbuch, Lux, Topol, IPD Models…)
- Endogenous Network Formation (Vega-Redondo, Goyal, Jackson-Watts…)
- Polya-Urn Schemes (Arthur, Dosi, Kaniovski, Lane, …)
- ….
- ….
- Industry-Dynamics Models (Nelson + Winter tradition)
- Evolutionary Growth Models (Silverberg, Verspagen, Dosi et al., …)
- ACE Models of Market Dynamics (Axtell, Epstein, Tesfatsion, Vriend, …)

# The Outcomes of ACE/EV Models

**Micro-Dynamics**
(induced by decision rules, interactions and expectations)

⇨

**Macro-Dynamics**
(obtained as aggregation of individual behaviors)

- Stochastic components in decision rules, expectations, interactions imply that the dynamics of micro and macro variables can be described by some (Markovian) stochastic process parametrized by $(\underline{\theta}_i)$, $\Theta$ :

$$(\underline{x}_{i,t}) \mid (\underline{x}_{i,t-1}), (\underline{x}_{i,t-2}), \ldots ; (\underline{\theta}_i), \Theta$$

$$\underline{X}_t \mid (\underline{X}_{t-1}, \underline{X}_{t-2}, \ldots ; (\underline{\theta}_i), \Theta)$$

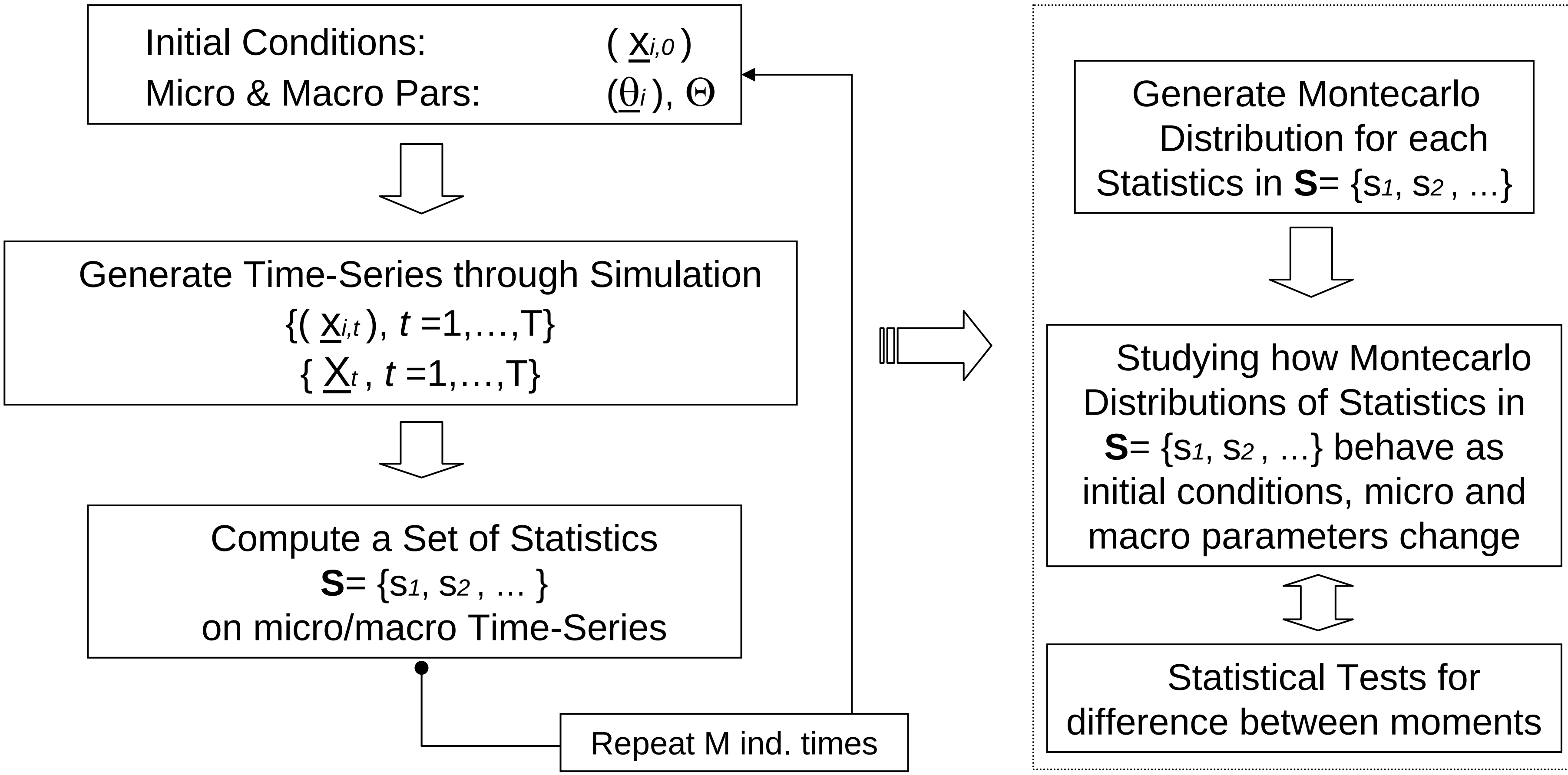
- Non-linearities in decision rules, expectations, interactions **may** imply that it is **hard** to analytically derive laws of motion, kernel distributions, time-*t* probability distributions, etc.

# Analysis of Agent-Based Models

- Analytical tractability?
  - Analytical solutions only for particular cases
  - Models must often be built and simulated (via computer)
  - Object-oriented programming languages (C++) as natural tools for agent-based models

- Analyzing the output of agent-based models
  - Initial conditions for all micro and macro variables of interest
  - Parameterization of the model
  - Model as a "data generation process" for the underlying unknown mechanisms
  - Run of the model: set of time-series (and statistics thereof)
  - Stochastic elements and need for Monte-Carlo analysis
  - Sensitivity analysis vs. parameters and initial conditions

# Analysis of Agent-Based Models



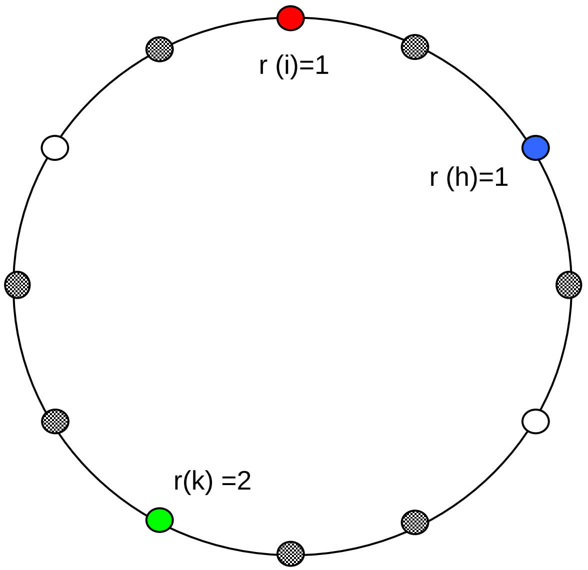
# Example: Dynamic Games (1/7)

- Studying equilibrium selection in coordination games
  - Old problem: selection among multiple equilibria
  - Coordination game: Two pure-strategy equilibria, possibly Pareto ranked; Inefficiency may arise
  - What happens in players do not play games with anyone else?

- Model
  - N players arranged on 1-dimensional lattice
  - Play games with nearest neighbors, care about total payoff from plays
  - At each t, one is drawn at random and plays with nearest neighbors

- Goal
  - Studying long-run coordination in the aggregate
  - Likelihood of Pareto efficient equilibria?

# Example: Dynamic Games (2/7)




| | | |
|---|---|---|
| • Time | $t = 0, 1, 2, \ldots$ | |
| • Sets of Agents | $I = \{1, 2, \ldots, N\}$ | |
| • Sets of Micro States | $i \rightarrow \{-1,+1\}$ | |
| • Interaction Structures | $G_t$ = 1-Dim Lattice | |
| • Micro-Parameters | $r$(i) | Interaction Radius |
| • Vector of Macro-Parameters | $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ | Stage-Game Payoffs |
| • Micro Decision Rules | BR Rule | Strategy Updating |
| • Aggregate variables | Average Action | Coordination Level |

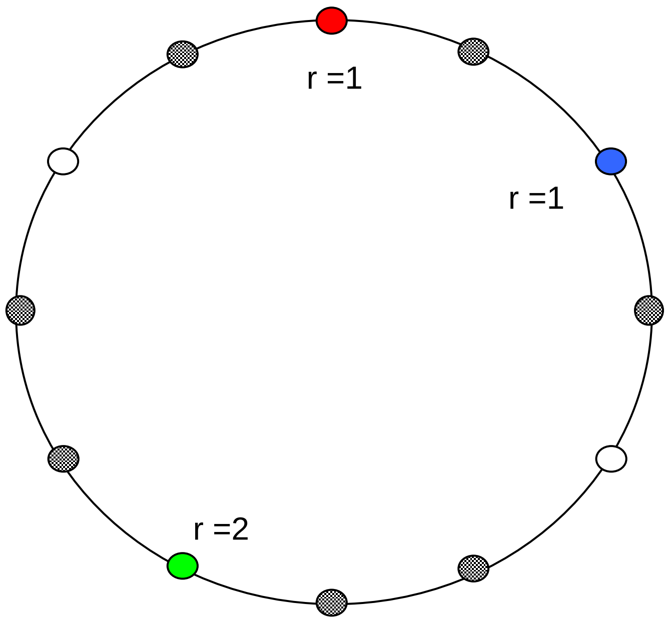
# Example: Dynamic Games (3/7)

- Time $t = 0, 1, 2, \ldots$
- Sets of Agents $I = \{1, 2, \ldots, N\}$ Players
- Sets of Micro States $i \rightarrow s(i) \in \{-1,+1\}$ Pure strategies
- Strategic Problem: Overall Coordination out of 2-person games ( a > 1)

| | | +1 | -1 | |
|---|---|---|---|---|
| Pareto-Efficient Strategy → | +1 | 2a | 0 | EU(+1)=2a•½+0 •½ = a |
| Risk-Efficient Strategy if a<2.5 → | -1 | 3 | 2 | EU(-1)=3•½+2 •½ = 2.5 |

# Example: Dynamic Games (4/7)

- Interaction Structures $G_t$ = 1-Dim Lattice Circle
- Each agent i interacts with neighbors closer than r(i)




$$V(i) = \{ j : |i - j| \leq r(i) \}$$

# Example: Dynamic Games (5/7)

| | | |
|---|---|---|
| • Micro-Parameters | *r*(i) | Interaction Radius |
| • Macro-Parameter | *a* | Stage-Game payoff of (+1,+1) |

• Micro Decision Rules and Dynamics

- At t=0 random draw of strategies
- At each t>0 one agent is chosen at random
- Chooses $s_t$(i) s.t. max total payoffs given neighbors choices at t-1

$$s_t^*(i) = \arg\max_{s \in \{-1,+1\}} \sum_{j \in V(i)} u(s; s_{t-1}(j))$$

# Example: Dynamic Games (6/7)

- Aggregate Variable: LR Coordination Level

$$c = \frac{1}{N}\sum_{i=1}^{N} s_T(i) \in [-1,+1]$$

- Choosing T large enough (stability/convergence of moments)

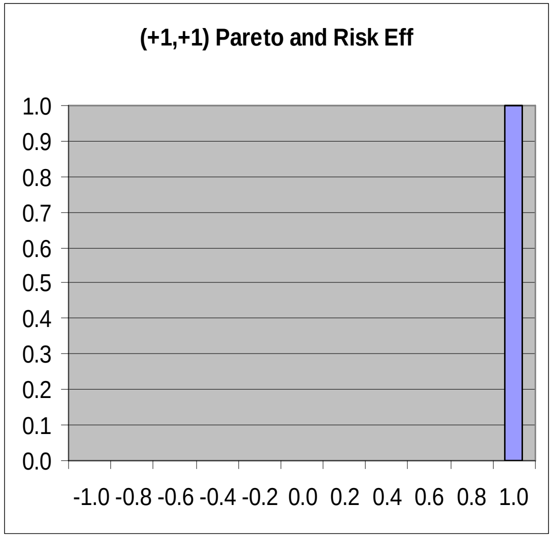
- Goal: Studying MC distributions of LR coordination levels as a function of

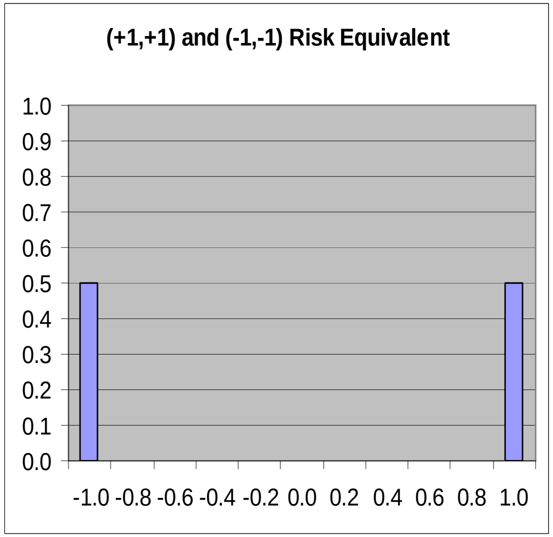
  1) Aggregate Parameter (a)
  2) Micro Parameters (e.g. average radius)

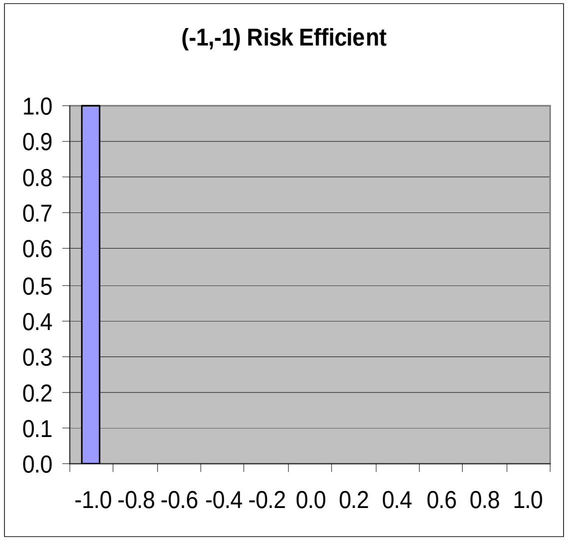
# Example: Dynamic Games (7/7)

- Results with r(i)=1 all i:

    1) (+1,+1) Pareto-Efficient and Risk Efficient ( a > 2.5)
    2) (+1,+1) and (-1,-1) Risk Equivalent ( a = 2.5)
    3) (-1,-1) Risk Efficient ( a < 2.5)



# Remarks (1/2)

- A new way of doing economics?
  - Large community: Ph.D. programs, journals, conferences
  - Still a minority vs. neoclassical economics
  - Two ways of seeing agent-based modeling approach
- ABM as a complementary approach
  - Exploring dimensions difficult to address jointly
  - Grounding behavioral assumptions into empirical/experimental evidence
- ABM as an alternative approach
  - Providing robustly an alternative view of how decentralized economies work
  - ABM replicating reality, generating fresh implications, allow for policy implications and predictions

# Remarks (2/2)

- Crucial, open issues
  - Pushing policy and design exercises
  - Fostering empirical validation techniques
- Policy implications and market design
  - Agent-based models as very flexible “laboratory” tools
  - Experimenting with alternative policy designs
  - Testing different market designs: the U.S. experience
- Empirical validation of ABMs
  - Allow for a better and deeper replication of stylized-facts
  - Over-parameterization of agent-based models
  - Developing more powerful calibration techniques
  - A new econometrics of ABMs? Causality and graphical models