INSTITUTE
OF ECONOMICS

Scuola Superiore
Sant'Anna

LEM | Laboratory of Economics and Management

Institute of Economics
Scuola Superiore Sant'Anna

Piazza Martiri della Libertà, 33 - 56127 Pisa, Italy
ph. +39 050 88.33.43
institute.economics@sssup.it

LEM

WORKING PAPER SERIES

Stock Recommendations from Stochastic Discounted Cash Flows

Giulio Bottazzi [a]
Francesco Cordoni [b]
Giulia Livieri [b]
Stefano Marmi [b]

[a] Institute of Economics and Department EMbeDS, Scuola Superiore Sant'Anna, Pisa, Italy.
[b] Scuola Normale Superiore, Pisa, Italy.

2020/17 June 2020
ISSN(ONLINE) 2284-0400

Stock Recommendations from Stochastic Discounted Cash Flows*

Giulio Bottazzi[†a], Francesco Cordoni[‡b], Giulia Livieri[§b], and Stefano Marmi[¶b]

[a]*Scuola Superiore Sant'Anna,*
Piazza Martiri della Libertà, 33, 56127 Pisa Italy.
[b]*Scuola Normale Superiore,*
Piazza dei Cavalieri 7, 56123, Pisa, Italy.

June 9, 2020

*We are indebted to Giacomo Toscano and Iacopo Raffaelli for helpful comments and fruitful discussion. We are grateful to participants at the Quantitative Finance Workshop Qfw2020 (Napoli, January 28-30, 2020). Francesco Cordoni, Giulia Livieri and Stefano Marmi acknowledges the support by Unicredit S.P.A. under the project "Dynamics and Information Research Institute-Quantum Information (Teoria dell'Informazione), Quantum Technologies". All errors are our own.

†E-mail: giulio.bottazzi@santannapisa.it.
‡E-mail: francesco.cordoni@sns.it.
§E-mail: giulia.livieri@sns.it
¶E-mail: stefano.marmi@sns.it.

Abstract

This paper presents two stocks recommendation systems based on a stochastic characterization of firm present value that extends the conventional discounted cash flow analysis. In the *Single-Stock Quantile* recommendation system, the market price of a company's stocks is compared with the estimated distribution of the company fair value to obtain an individual measure of mispricing, while in the *Cross-Sectional Quantile* system, a relative measure of mispricing is built using the fair value distribution of all firms at the same time. Both systems use mispricing information to build sell side and buy side portfolios. We provide a series of statistical exercises that show how these portfolios can consistently deliver significant excess returns, also when rebalancing costs are accounted for.

Keywords: Stochastic Discounted Cash Flow; Asset Valuation; Valuation Uncertainty; Portfolio Strategy.
JEL codes: G11,G17,G32.

1. Introduction

In this work we investigate whether investors can profit from two stocks recommendations systems constructed by using the information provided by the Stochastic Discounted Cash Flow (SDCF, henceforth) analysis, a new approach for the estimation of the shareholder value proposed by the authors of the present work in Bottazzi et al. (2020). The SDCF is a fairly general and theoretically grounded econometric methodology which allows to replace the pointwise estimate of the conventional discounted cash flow method with a random variable whose empirical distribution, the *fair value distribution*, can be used to obtain both an estimate of the expected fair value of the company and of its degree of uncertainty.

The two recommendation systems compare the fair value distributions with market prices in order to detect possible mispriced assets. Under the hypothesis that some degree of market efficiency is at work, undervalued assets should recover, at least in part, their true value and are thus expected to increase their price. As such, they constitute good candidates for buy portfolios. Conversely, overvalued assets are expected to face a decrease in their price and are good candidates for sell portfolios. The misvaluation assessment is run differently in the two system. In the *Single-Stock Quantile* system, the degree and direction of mispricing for a company is derived from the likelihood to obtain the observed market price from the fair value distributions of that company. In the *Cross-Sectional Quantile* instead, mispricing information is derived in a comparative fashion, jointly using the fair value distributions of all considered companies, through the definition of a mispricing index defined as the difference between the market price of the company and its expected fair value, divided by the fair value distribution standard deviation. The advantage of the former method is to fully use all information available from the fair value distribution, while the advantage of the latter derives from its robustness with respect to possible fair value estimation biases.

The analysis of the performance of the two systems is carried out through an intercept test of the excess returns of portfolios build following their prescription, using the Fama-French three-factor model (Fama and French, 1993) augmented with the momentum factor (Carhart, 1997). We show that equally weighted portfolios composed by the most highly recommended stocks consistently earn positive abnormal gross returns. The comparison with the much weaker results obtained using similar portfolios built from the analyst recommendations from Thomson Reuters Institutional Brokers' Estimate System (I/B/E/S) database further stresses the advantage of our methodology. Moreover, our recommendation systems remain profitable also when an high degree of turnover costs is included.

On a broader perspective, we believe that this study fits within the existing works that have highlighted the necessity of developing probabilistic and statistical tools to extend the conventional DCF approach to include some measure of uncertainty associated with the estimated value (Bradshaw, 2004; Brown et al., 2015; Baule and Wilke, 2016; Casey, 2001).

In Section 2 the SDCF methodology is briefly reviewed. In Section 3 the database and variables used in our analysis are described. In Section 4 we introduce the two stocks recommendations systems and in Section 5 we analyse their performance. Finally, we conclude in Section 6.

2. The SDCF approach

The idea of the SDCF approach is replacing the usual pointwise estimate of the preset value of a company with a *fair value distribution*, that takes into consideration the intrinsic uncertainty about the future firm performance. We provide a short review of the procedure below. For more details the reader is referred to Bottazzi et al. (2020). Given the difficulty in estimating the debt cash-flow with the available data, we adopt an *Unlevered Free Cash Flow* (UFCF) approach and derive the present value of equity V_0 from the present value of the firm $\widetilde{V}_0$ subtracting the current value of "debt"

$$V_0 = \widetilde{V}_0 - (TD - CsI + MI + PS) \ , \tag{1}$$

where TD represents the total debt, used as a proxy for the market value of debt (Damodaran, 2007, 2012), CsI the cash and short-term investments, MI the minority interest and PS denotes the preferred stocks. The firm present values does not require an estimation of new debt issues nor debt repayment and can be obtained directly from an estimate of future unlevered free cash flow CF_t defined at each time t as (Damodaran, 2007, 2012; Chang et al., 2014; Gryglewicz et al., 2019)

$$CF_t := NOPAT_t + D\&A_t - CAPEX_t - \Delta WC_t, \tag{2}$$

where $NOPAT$ denotes the net operating profit after tax, $D\&A$ the depreciation and amortization, $CAPEX$ the capital expenditure and ΔWC the changes in the working capital. In turn, $NOPAT$ can be obtained from the earning before interest, taxes, depreciation, and amortization $EBITDA$ using the relation $NOPAT_t = (EBITDA - D\&A) \cdot (1 - \tau_0)$, where τ_0 represents the marginal tax rate. In order to derive the firm present value from estimated future cash flow we make two assumptions. First, an homogeneous cost of capital k to be used to discount future cash flows. The cost of capital is computed as a weighted average of the cost of equity, the after-tax cost of debt and the cost of preferred stocks. Second, in line with a large body of literature (see for instance Damodaran (2007) and Ali et al. (2010)), we consider a two-stage model and assume that there exists a date $T > 0$ and a "perpetual growth rate" g, with $0 < g < k$, such that for any $t \geq T$ it is $CF_{t+1} = CF_t \cdot (1 + g)$. Thus, assuming that all the random quantities are defined on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t), \mathcal{P})$, for any possible future realization of cash flows $CF_t(\omega)$ with $\omega \in \Omega$ the associated firm present value reads

$$\widetilde{V}_0(\omega) = \sum_{t=1}^{T} \frac{CF_t(\omega)}{(1+k)^t} + \frac{CF_T(\omega)(1+g)}{(1+k)^T(k-g)} \ . \tag{3}$$

The model heavily depends on reliable estimates of CF_t. To estimate future cash flows, we start by expressing, through a firm-specific regression model, all the relevant accounting variables in (2) as *margins* with respect to the revenues. Then we use a battery of econometric models, including stationary models, a local level model and a local linear trend model (Harvey (1990) and Durbin and Koopman (2012)), to describe the dynamics of log-revenues. The models are calibrated at the level of the single firm. For each firm we select the best performing model and we use it in a Monte Carlo exercise to generate future revenues realizations which, substituted in (3), produce a distribution of $\widetilde{V}_0$ values (see Bottazzi et al. (2020) for further details on model selection). The distribution of firm present values is adjusted using (1) and divided by the number of company's outstanding shares to obtain the *fair value distribution*. Under the hypothesis of the model, $\mathbb{E}[V_0]$

represents the traditional point-wise present value estimate of company's shares. Figure 2 shows the logarithm of the fair value distribution for Booking Holdings Inc. (ticker BKNG) computed at different dates. The red dotted lines indicate the market log-price at the evaluation date. At the end of the first quarter in 2009, the company results heavily undervalued, while it results only mildly undervalued in 2013 Q1 and 2018 Q1.

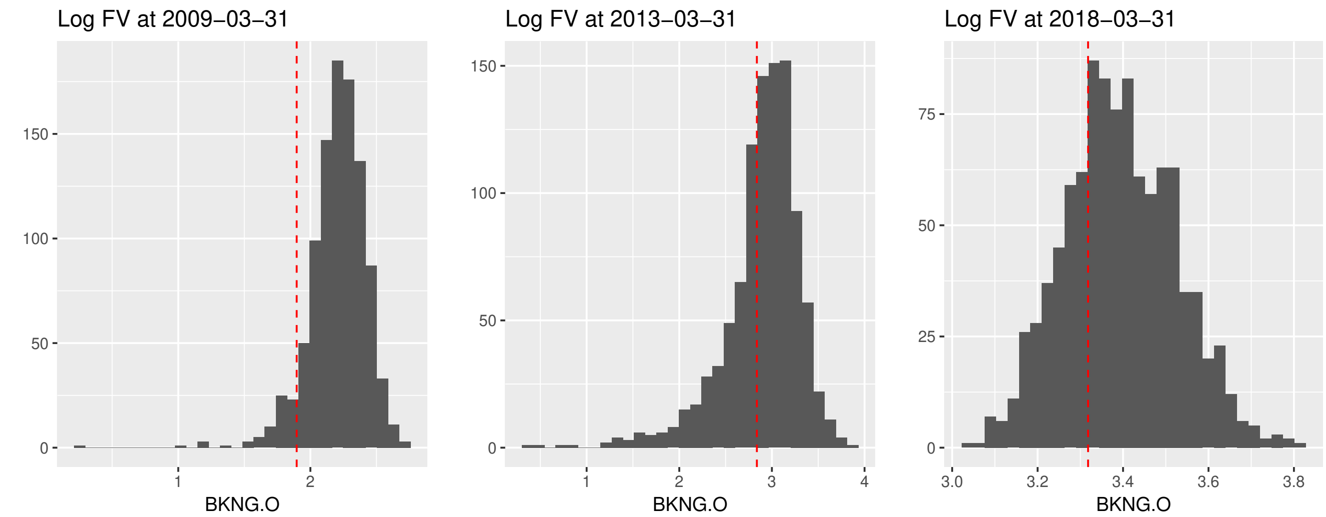

Figure 1: Distribution of the logarithm of the *fair value distribution* for Booking Holdings Inc. (ticker BKNG) computed at different dates. The red dotted lines indicate the market price at the evaluation date.

3. Data and Sample Selection Criteria

While the SDCF model relies upon general considerations, many details of its implementation and validation depend on specific company level data. In this Section we review the different data sources we use to develop and test our methodology. All the data in the subsequent description are taken at a quarterly frequency.

The required equity prices, along with the corresponding fundamental data are collected from Thomson Reuters Eikon, Datastream database. Our initial sample covers the period December 1990–December 2017 and comprises all the 505 companies currently listed in the S&P 500. We exclude companies belonging to the financial sector as they are subject to industry-specific regulations that are likely to badly affect our cash flow estimation. We also exclude firms with partial data over the period of investigations and a few firms for which the econometric models for revenues produce inconsistent estimates.[1] We are finally left with a sample of 140 firms. The sample is rather heterogeneous: we have 17 firms in both the Oil & Gas (ICB 1) and the Basic Material (ICB 1000) sector, 44 Industrial firms (ICB 2000), 22 Consumer Good firms (ICB 3000), 19 Healthcare firms (ICB 4000), 12 firms in the Consumer Service sector (ICB 5000), 3 firms in the Telecommunication sector (ICB 6000), 7 Utilities firms (ICB 7000) and 16 Technology Firms (ICB 9000). We also get from Datastream the estimate of the firm-specific cost of capital k and the company marginal tax rate τ_0. Following an industry standard, the discount rate for the cash flow terminal value is computed by considering the fixed corporate tax rate provided by KPMG instead of the individual tax rate, albeit the difference is minimal for all firms and all years considered. Since we set the terminal year T to 5, the perpetual growth rate g is set equal to the 5-year T-bond rate obtained from the Federal Reserve Economic Data (FRED) database.

Data employed in the analysis of the returns of different portfolios within standard multi-factor models, performed in Section 5, are taken from the Kenneth R. French Data Library.[2] Data on analyst recommendations, used in a benchmarking exercise in Section 5, are obtained from I/B/E/S database. We look at the Summary History-Recommendation file which compiles a monthly snapshot of each company in the database by sell-side analyst whose brokerage firm provides data to IBES. The database tracks the number of analyst following the stock, the average consensus rating level (which is a number between 1 and 5) along with its standard deviation and the number of analysts upgrading and downgrading their opinion level from the previous month. A rating of 1 reflects a strong buy recommendation, 2 a buy, 3 a hold, 4 a sell, and 5 a strong sell. On average each firm is followed by 20 analysts. Table 1 reports sector-specific summary statistics about analyst coverage.

4. Recommendations from Fair Value Distributions

The fair value distribution defined in the Section 2 can be straightforwardly used to obtain portfolio recommendations for company stocks. The basic idea is to use the valuation model to identify possibly mispriced companies. Under the hypothesis that mispriced companies will recover their correct price, undervalued firms represent prospective buys and overvalued firms prospective sells. Following a standard practice, stocks will be classified as Strong Buy (SB), Buy (B), Hold (H), Sell (S) and Strong Sell (SS). Let FV_t^i be the distribution function of the fair value of company i at time t and P_t^i its market price. The quantity $q_t^i = FV_{t_h}^i(P_{t_h}^i)$ represents the probability that the

[1]See Bottazzi et al. (2020) for further details concerning the filtering procedures.

[2]Freely available at https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Table 1: Summary statistics of analysts coverage for the 140 firms in our sample, grouped by ICB code. Two "snapshots" are reported: one referring to January 2009, and one to December 2017 (between brackets), which correspond to the beginning and the end date of the period under investigation. For each sector, the average consensus rating level and the average number of analysts following the stocks is reported.

	Oil & Gas and Basic Materials	Industrial	Consumer Goods	Healthcare	Consumer Service	Telecommunication and Utilities	Technology	All
ICB codes	1 and 1000	2000	3000	4000	5000	6000 and 7000	9000	
Number of firms	17	44	22	19	12	10	16	140
Rating level								
Mean	2.29 (2.35)	2.50 (2.53)	2.64 (2.68)	2.32 (2.24)	2.5 (2.42)	2.25 (2.80)	2.22 (2.31)	2.42 (2.48)
Number of analysts								
Mean	15.12 (22.35)	12.73 (17.91)	11.77 (18.77)	14.89 (21.05)	17 (24)	13.70 (16.80)	24.12 (26.06)	14.90 (20.39)

company fair value is lower than or equal to the observed price. In general, if q_t^i is near 0.5, the market price is near the median of the fair value distribution and we can conclude that, according to our model, the company is fairly priced. If instead, the value that our valuation model assigns to the company is higher (lower) then the market price, then the company is undervalued (overvalued) and $q_{t_h}^i$ is close to zero (one). Based on this consideration, the classification of stocks is performed in the following way: if $q_{t_h}^i < 0.125$ company i is classified SB, if $0.125 \leq q_{t_h}^i < 0.25$ is classified B, if $0.25 \leq q_{t_h}^i < 0.75$ is classified H, if $0.75 \leq q_{t_h}^i < 1$ is classified S and SS if $1 \leq q_{t_h}^i$.[3] This classification system, denoted *Single-Stock Quantile* (henceforth, SSQ), has the advantage of using all the information provided by the distribution of the company fair value. The recommendation for each firm is obtained using exclusively its own fair value distribution, without reference to the valuation of other firms.

A second possible approach is to use the fair value distribution of all firms at the same time. For this purpose, we introduce a second recommendations system based on the definition of a company-specific mispricing indicator. Let μ_t^i and σ_t^i be the empirical mean and standard deviation of the distribution of the logarithm of the fair value of stock i at some time t, computed using the SDCF method, and let p_t^i be the closing log-price at day t of the same company. The mispricing indicator z_t^i of company i at time t is defined as the difference of the company's expected log-fair value and its log-price divided by the standard deviation of the log-fair value distribution,

$$z_t^i := \frac{p_t^i - \mu_t^i}{\sigma_t^i} \quad i \in \{1, \dots, N\} \ .$$

[3]The class assignment is broadly in line with the values adopted by the *Morningstar*® equity research methodology, see MorningstarEquityResearchMethodology.pdf for further details.

The uncertainty of the evaluation procedure, captured by the standard deviation σ_t^i, is used to modify the observed difference (in log) between market price and expected present value. If the uncertain is high, the indicator is reduced as the observed difference is considered less significant. If instead the uncertain is low, the observed difference becomes more relevant and the indicator takes a larger value. Consider now the empirical distribution function of all mispricing indicators z_t^i, $\forall i = 1, \ldots, N$ and let $\rho_t(\alpha)$ be its α-quantile. The sock of company i is classified SB if $z_t^i < \rho_t(0.1)$, B if $\rho_t(0.1) \leq z_t^i < \rho_t(0.4)$, H if $\rho_t(0.4) \leq z_t^i < \rho_t(0.6)$, S if $\rho_t(0.6) \leq z_t^i < \rho_t(0.9)$ and SS if $\rho_t(0.9) \leq z_t^i$. Firms having a misvaluation indicator near to the median of the empirical distribution of all indicators are assigned to the hold class. Firms with a high mispricing are assigned to the sell class, that becomes strong sell if they are in the top decile. Conversely firms with low, respect to the median, misvaluation are buy, and strong buy if they are in the bottom decile. We denote this system as *Cross-Sectional Quantile* (henceforth, CSQ). The advantage of this system is that a common shift of the market prices, having no effect on the relative rankings of the different companies, has no effect on their classification. The system is also insensitive to the presence of a common bias affecting the valuation procedure of the different companies.

To test the performance of the SSQ and CSQ systems we run an in sample analysis. We consider 19 non-overlapping periods of six months, from FQ1 2009 to FQ1 2018.[4] At the beginning of each period, we classify the firms using both systems. We use the first available closing price for the computation of the mispricing indicator. In the case of SSQ, both the number of firms in each class and the associated market capitalization, with respect to our universe of stocks, can vary from period to period, while for CSQ, the numbers of stocks in each class is constant in all periods. On average, for the SSQ system, SB class has 31 stocks (28% market capitalization), B has 25 (18%), H has 62 (40%), S has 18 (11%) and SS has 3 (3%), while for the CSQ system, SB class has 14 stocks (13% market capitalization), B has 42 (37%), H has 28 (16%), S has 42 (25%) and SS has 14 (9%).

For each recommendation system, we build equally weighted portfolios with all companies in a given rating class at the beginning of each semester and we compute the daily return of these portfolios R_t^p, with p taking values SS, S, H, B, SB, in each day t in the semester.[5]

[4]The period of six months has been chosen because long enough for the calibration of the cash flow model to be reliable but short enough to give us a good number of data points to analyze. In any case it is broadly consistent with several portfolio strategies discussed in (Li et al., 2019).

[5]In Barber et al. (2001) market-weighted rather then equally-weighted portfolios are considered. Their choice is consistent with the use of daily rebalancing and with the size of their sample. However, the same authors warn about the possibility that using market-weighted returns may bias against finding evidence of abnormal returns, so we opt for a more conservative choice.

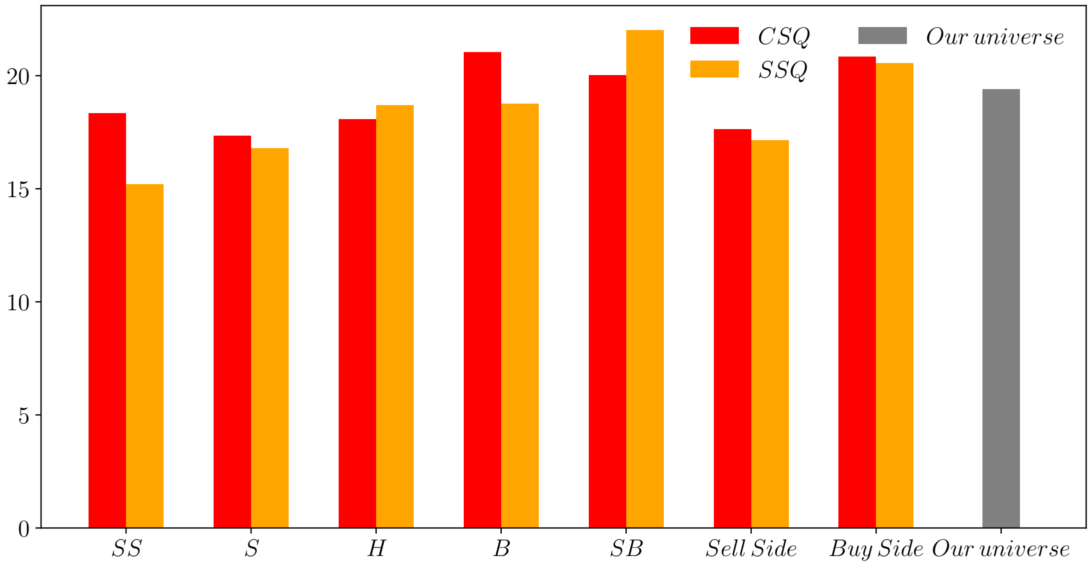

Figure 2: Annualized log-returns in percentage for each of the constructed portfolio according to *CSQ* and to the *SSQ* recommendation. *Our-universe* is the return of an equally-weighted portfolio that goes long in all the stocks of our universe. The sample period is April 1, 2009 to September 28, 2018.

5. Performance Evaluation

We begin with a simple calculation, over the entire time-period considered, of the annualized log-returns (in percentage) for each of our constructed portfolios. In Figure 2 they are compared with the annualized log-returns of a benchmark equally-weighted portfolio that goes long in all the stocks of our universe, labeled *Our universe*. As can be seen, undervalued assets tend to growth significantly faster than overvalued ones. For instance, the annualized log-return of the *SB* portfolio built following the *SSQ* system is 22.00% while that of the *SS* portfolio is 15.20%. This remains true also if we consider a more coarse-grained classification merging portfolios in the buy and sell side. The use of just two broad classes seems to enhance the performances of the *CSQ* system. The goodness of the portfolio obtained with the two recommendation systems is confirmed also when a measure of risk is included. The Sharpe Ratio (Sharpe, 1994) of the *Buy Side* portfolios is 1.43 for the *CSQ* system and 1.40 for the *SSQ* system. They are both significantly larger, according to the Ledoit and Wolf (2008) and Ardia and Boudt (2018) test (p-value around 0.003), then the Sharpe Ratio of the *Our universe* portfolio, which is 1.24. In turn, the two *Sell Side* portfolios have value which are significantly lower then the *Our universe* portfolio (1.05 for the *CSQ* system and 0.97 for

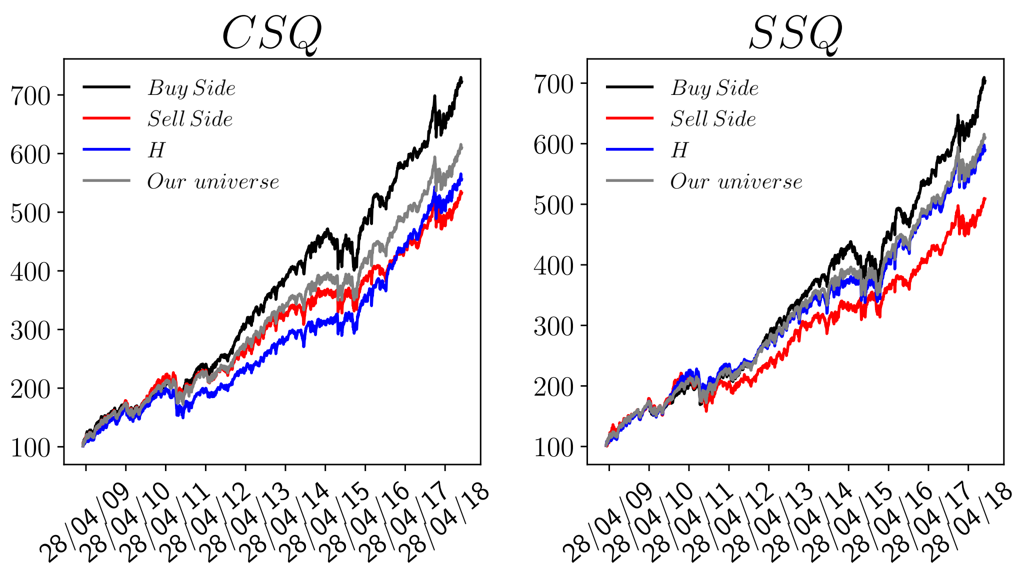

Figure 3: Cumulative sum of daily returns when investing 100$ in the *Buy Side*, *H* and *Sell Side* portfolio constructed by employing the *CSQ* (left panel) and *SSQ* (right panel) methodology. The grey line is the cumulative sum of daily returns of *Our universe*. The sample period is April 1, 2009 to September 28, 2018.

the *SSQ* system).[6]. Figure 3 represents the cumulative sum of daily returns when investing 100$ in the *Buy Side*, *H* and *Sell Side* portfolio constructed by employing the *CSQ* (left panel) and *SSQ* (right panel) methodology, compared with the *Our universe* portfolio.

In order to obtain a more precise estimate of portfolio performances we employ an intercept test using the Fama-French three-factor model (Fama and French, 1993) augmented with the momentum factor (Carhart, 1997). We estimate the following daily time-series regression

$$R_{p,t} - R_{F,t} = \alpha_p + \beta_p(R_{M,t} - R_{F,t}) + s_p, SMB_t + h_p, HML_t + m_p MOM_t + e_{p,t}\,, \tag{4}$$

where $R_{F,t}$ is the risk-free return for period t, $R_{M,t}$ is the return of the value-weighted market portfolio, SMB_t is the difference between the daily returns of a value-weighted portfolio of small stocks and one of large stocks, HML_t is the difference between daily returns of a value-weighted portfolio of high book-to-market stocks and one of low book-to-market stocks, MOM_t is the momentum factor and e_{pt} is the error term. The regression yields parameter estimates of α_p, β_p, s_p, h_p and m_p but the relevant parameter here is the intercept α_p, as it captures the presence of abnormal returns. Results

[6]The Sharpe Ratio has been computed setting to zero the benchmark return. We compared the portfolios performances also using the Sortino ratio (Sortino and Price, 1994), obtaining identical results.

Table 2: Estimated gross annual abnormal returns earned by portfolios constructed with our *CSQ* and *SSQ* systems, and using the analysts recommendation from the IBES database. Coefficients significant at 10%, 5%, 1% and 0.1% level are marked with '.', '$*$', '$**$' and '$***$' respectively. The *SS* portfolios in the case of *SSQ* system and both the *SS* and *BB* portfolios in the case of analysts are empty for a few rebalancing date. In this case, the corresponding return is set to zero.

α_p	*SS*	*S*	*H*	*B*	*SB*	*Sell Side*	*Buy Side*
CSQ	1.5733	0.8190	2.4977	5.7706	7.0838	1.0076	6.0989
$p-value$	(4.24e-01)	(5.57e-01)	(8.72e-02)	(5.00e-06***)	(6.00e-04***)	(4.39e-01)	(3.90e-07***)
SSQ	1.4665	1.2052	2.0189	3.3869	7.9444	1.4182	5.8442
$p-value$	(7.08e-01)	(6.57e-01)	(1.20e-01)	(4.65e-02*)	(6.55e-06***)	(5.94e-01)	(2.60e-05***)
Analysts	11.0480	5.0222	2.4576	2.1187	1.9982	5.795	2.0579
$p-value$	(3.70e-03**)	(1.30e-03**)	(0.0234*)	(0.1053)	(0.6117)	(2.00e-04***)	(0.1111)

are reported in Table 2. The most highly recommended stocks (*B*, *SB* and *Buy Side*) earn positive abnormal gross returns, whereas the least favourably recommended ones do not. In addition, the abnormal gross excess returns of these portfolios are greater than that of *Our universe*, which has a gross annual excess return of 3.34% with a p-value of $3.16e-04$. These results suggest that investors following our SDCF-based recommendations and building concentrated portfolios could be able to obtain market-beating returns.

As a further check, we repeat the same analysis using expert recommendations from the IBES database, described in Section 3. Let $\bar{A}_t^i$ be the average analysts' rating for firm i on date t. We follow Barber et al. (2001) and if $1 \leq \bar{A}_t^i \leq 1.5$ we classify company i as *SB*, if $1.5 < \bar{A}_t^i \leq 2$ as *B*, if $2 < \bar{A}_t^i \leq 2.5$ as *H*, if $2.5 < \bar{A}_t^i \leq 3$ as *S* and a *SS* whenever $\bar{A}_t^i > 3$. The downward shift accounts for the observed over-optimistic recommendation scores provided by experts (see Barber et al. (2001) and reference therein). Using the expert recommendation system we build six-month rebalanced portfolios in exactly the same way we did for our systems and we perform the regression in (4). The results reported in Table 2 show that the experts' *Buy Side* does not provide significant abnormal returns. Awkwardly, abnormal returns are observed for the experts' *Sell Side* portfolio.

The previous calculated returns are gross of any trading costs. To assess the size of these costs we calculate a measure of annual turnover. Let t_h with $h = 1, \ldots, 19$ be the rebalancing dates, N_h^p the number of companies in portfolio p at date t_h and δ_{i,t_h}^p equal to 1 if company i is in portfolio p at date t_h and zero otherwise. Turnover at date $t_{h+1}$is calculated as

$$\mathrm{TO}_{t_{h+1}}^p := \sum_i \left| \frac{\delta_{i,t_h}^p}{N_h^p} - \frac{\delta_{i,t_{h+1}}^p}{N_{h+1}^p} \right|$$

Table 3: Annualized percentage turnover of the *Cross-Sectional Quantile* and *Single-Stock Quantile* portfolios.

	SS	*S*	*H*	*B*	*SB*	*Sell-Side*	*Buy-Side*
CSQ	136.51	158.20	245.24	148.15	142.86	102.38	98.016
SSQ	164.51	236.37	149.70	241.56	167.48	200.81	136.16

where the sum is over all companies composing our universe. Annualized total turnover TO^p is two times the average of the previous quantity across the entire period. Values are reported in Table 3.

The transaction cost of portfolio p is computed as the product between the annualized turnover and the round-trip cost (henceforth, RTC) (see Baule and Wilke, 2016, and references therein) and the "critical" round-trip cost, RTC^p_{crit}, which is the rebalancing cost that makes the net abnormal return of the portfolio equal to that of the *Our universe* benchmark

$$\alpha^p - \mathrm{RTC}^p_{crit} \cdot \mathrm{TO}^p = \alpha^{\text{Our universe}}.$$

The critical round-trip cost is equal to 2.81% for the *Buy Side CSQ* and 1.84% for the *Buy Side SSQ* portfolios, the only ones with an abnormal return above that of the benchmark. Table 4 displays the adjusted (for transaction cost) annualized abnormal returns and the adjusted Sharpe Ratio (i.e. the Sharpe Ratio computed by using the adjusted returns defined as the difference between the actual return and the transaction costs) for different levels of round-trip costs. As can be seen, the considered portfolios remain profitable also for pretty high transaction costs. The *Buy Side CSQ*, having the lowest turnover is less sensitive to transaction costs.

6. Conclusions

We propose two recommendation systems based on the comparison of observed market prices with the fair value distributions obtained through the SDCF method introduced in Bottazzi et al. (2020). The *Single-Stock Quantile* system derives recommendations for each company by computing the probability of the observed market price given the fair value distribution of the company. The *Cross-Sectional Quantile* system builds a mispricing indicator for each company and then derive recommendations by comparing the indicators across all companies. While the former method fully uses all information available from the fair value distribution, the latter turns out to be more robust with respect to possible fair value estimation biases.

Table 4: The Table displays the annualized abnormal returns and adjusted annualized percentage Sharpe ratio for the *Buy Side* portfolios as a function of $RTC^p(\%)$ for both *CSQ* and *SSQ* systems. Sharpe ratios which result to be significantly different from our universe, according to the Ledoit and Wolf (2008) and Ardia and Boudt (2018) test, at 10%, 5%, 1% and 0.1% level are marked with '.', '∗', '∗∗' and '∗ ∗ ∗' respectively.

	CSQ		*SSQ*	
$RTC^p(\%)$	Ann. Abn. Return (%)	Ann. Sharpe Ratio (%)	Ann. Abn. Return (%)	Ann. Sharpe Ratio (%)
0.00	6.10	1.43 **	5.84	1.40 *
0.31	5.79	1.41 **	5.42	1.37 .
0.63	5.49	1.39 *	4.99	1.34
0.94	5.18	1.37 *	4.57	1.31
1.25	4.87	1.35 .	4.14	1.28
1.31	4.81	1.34	4.06	1.28
1.38	4.75	1.34	3.97	1.27
1.56	4.57	1.32	3.72	1.25
1.88	4.26	1.30	3.29	1.22
2.19	3.95	1.28	2.87	1.20
2.50	3.65	1.26	2.44	1.17
2.81	3.34	1.24	2.01	1.14

For each recommendation system we build buy and sell side portfolios and we estimate the abnormal returns, both gross and net of trading costs, earned on diverse investment strategies. The *Buy Sides* provide a significant average annual abnormal gross return of about 6% percent, after controlling for market risk, size, book-to-market and price momentum effects, which doubles the market abnormal gross return (of our universe), which is about 3%. Contrary to what happens with portfolios based on analysts' stock recommendations (i.e. the I/B/E/S recommendation system), our investment strategies (portfolios) are always consistent, as buying the stocks with the more favorable consensus recommendation earns an annualized log-return greater than buying the stocks with the less favourable consensus recommendation.

References

Ali, M., R. El-Haddadeh, T. Eldabi, and E. Mansour (2010). Simulation discounted cash flow valuation for internet companies. *International Journal of Business Information Systems 6*(1), 18–33.

Ardia, D. and K. Boudt (2018). The Peer Ratios Performance of Hedge Funds. *Journal of Banking and Finance 87*, 351.

Barber, B., R. Lehavy, M. McNichols, and B. Trueman (2001). Can Investors Profit from the Prophets? Security Analyst Recommendations and Stock Returns. *The Journal of Finance 56*(2), 531–563.

Baule, R. and H. Wilke (2016). On the profitability of portfolio strategies based on analyst consensus eps forecasts. In *28th Australasian Finance and Banking Conference Paper.*

Bottazzi, G., F. Cordoni, L. Giulia, and S. Marmi (2020). Uncertainty in firm valuation and a cross-sectional misvaluation measure. *Available at SSRN: 3614988*.

Bradshaw, M. T. (2004). How do analysts use their earnings forecasts in generating stock recommendations? *The Accounting Review 79*(1), 25–50.

Brown, L. D., A. C. Call, M. B. Clement, and N. Y. Sharp (2015). Inside the Black Box of sell-side financial analysts. *Journal of Accounting Research 53*(1), 1–47.

Carhart, M. M. (1997). On Persistence in Mutual Fund Performance. *The Journal of finance 52*(1), 57–82.

Casey, C. (2001). Corporate valuation, capital structure and risk management: A stochastic DCF approach. *European Journal of Operational Research 135*(2), 311–325.

Chang, X., S. Dasgupta, G. Wong, and J. Yao (2014). Cash-flow sensitivities and the allocation of internal cash flow. *The Review of Financial Studies 27*(12), 3628–3657.

Damodaran, A. (2007). Valuation Approaches and Metrics: A Survey of the Theory and Evidence. *Foundations and Trends® in Finance 1*(8), 693–784.

Damodaran, A. (2012). *Investment valuation: Tools and techniques for determining the value of any asset.* John Wiley & Sons.

Durbin, J. and S. J. Koopman (2012). *Time series analysis by state space methods*, Volume 38. Oxford University Press.

Fama, E. F. and K. R. French (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics 33*(1), 3–56.

Gryglewicz, S., L. Mancini, E. Morellec, E. Schroth, and P. Valta (2019). Cash flow shocks and corporate liquidity.

Harvey, A. C. (1990). *Forecasting, structural time series models and the Kalman filter*. Cambridge university press.

Ledoit, O. and M. Wolf (2008). Robust performance hypothesis testing with the Sharpe ratio. *Journal of Empirical Finance 15*(5), 850–859.

Li, F., T.-M. Chow, A. Pickard, and Y. Garg (2019). Transaction Costs of Factor-Investing Strategies. *Financial Analysts Journal 75*(2), 62–78.

Sharpe, W. F. (1994). The Sharpe Ratio. *Journal of Portfolio Management 21*(1), 49–58.

Sortino, F. A. and L. N. Price (1994). Performance Measurement in a Downside Risk Framework. *The Journal of Investing 3*(3), 59–64.