Synchronization Patterns in the European Union

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Abstract

We propose a novel approach to investigate the synchronization of business cycles and we apply it to a Eurostat database of manufacturing industrial production time-series in the European Union (EU) over the 2000-2017 period. Our approach exploits Random Matrix Theory and extracts the latent information contained in a balanced panel data by cleaning it from possible spurious correlation. We employ this method to study the synchronization among different countries over time. Our empirical exercise tracks the evolution of the European synchronization patterns and identifies the emergence of synchronization clusters among different EU economies. We find that synchronization in the Euro Area increased during the first decade of the century and that it reached a peak during the Great Recession period. It then decreased in the aftermath of the crisis, reverting to the levels observable at the beginning of the 21\textsuperscript{st} century. Second, we show that the asynchronous business cycle dynamics at the beginning of the century was structured along a East-West axis, with eastern European countries having a diverging business cycle dynamics with respect to their western partners. The recession brought about a structural transformation of business cycles co-movements in Europe. Nowadays the divide can be identified along the North vs. South axis. This recent surge in asynchronization might be harmful for the European Union because it implies countries’ heterogeneous responses to common policies.

Keywords: Business Cycle Synchronization, Random Matrix Theory, European Union.

JEL Codes: E32, F44, F45.
1 Introduction

Since the financial and economic crisis hit in 2008, fierce political pressures concerning the legitimacy of the European Union (EU) have emerged. Parties and movements from various EU countries have started to attack some of the very fundamental pillars of the Union. Technically speaking, these movements have been questioning the EU ability to provide coherent policy replies for countries that were asymmetrically affected by common macroeconomic shocks (Eichengreen, 2014). For a group of economies composing an economic union, to have a common policy is not a problem per se (Frankel and Rose, 1998). As long as the members have similar business cycle and development phases, the same policy receipts might have similar effects in all countries. One of the key conditions that allows economic unions (and currency areas) to function, is thus the synchronization of the business cycles among member states. The synchronization indeed, helps reducing the asymmetries arising from common shocks. In contrast, common policies might become problematic whenever countries display heterogeneous business cycle phases (Burriel and Galesi, 2018). In such a situation, a common policy would clearly not fit all of the union’s members alike, possibly generating additional distress in some of them.

Questions about business cycles synchronization across different economies have already occupied a central role in the EU debate (Artis et al., 1997; Wynne and Koo, 2000). Before the formation of the common currency, economists and policy-makers were aware of the fact that a lack of synchronization would have downsized the benefits of the common currency, as documented by Feldstein (1997), Mundell (1997) and Mongelli (2002) among the others. Given the state of the political economy debate in Europe however, new contributions and perspectives enriching this discussion are needed. First, to evaluate ex-post the extent to which the introduction of the common currency has affected the process of synchronization in the European economies. Second, to learn from past mistakes and to find possible remedies for the future.

In this paper we contribute to the above debate by means of a novel econometric procedure that allows one to identify and easily quantify synchronization patterns across European economies. More precisely, we employ the methods of Random Matrix Theory (RMT), a technique introduced by Wigner (1955) that it has recently gained success in the finance literature for performing empirical analyses upon assets cross-correlations and portfolios overlaps (Wang et al., 2011; Bun et al., 2017; Lux et al., 2019). The RMT technique has commonalities with Principal Component Analysis (PCA) and with Dynamic Factor (DF) models (see e.g. Forni et al., 2000; Stock and Watson, 2011). Its main advantage lies in the possibility of providing an intuitive and simple selection criteria for the number of factors to be employed. We apply the RMT to monthly EU industrial production data in order to extract a number of significant factors that accounts for the common dynamics of industrial production time-series in the European Union. Next, we study in detail the behaviour of each country time series with respect to the common factor(s).

It has to be noticed that we do not aim here at the identification of a causal relation. Indeed, also the opposite causal link might be at work and a common policy might induce higher business cycles synchronization (Artis and Zhang, 1997; Frankel and Rose, 1998).
Our analysis unveil important transformations that affected the EU business cycle dynamics in the past two decades. We observe that in the years that followed the introduction of the common currency and before the Great Recession, synchronization has slightly increased, with the eastern EU economies getting aligned with core European countries. In addition, business-cycle synchronization reached a peak in the Great Recession period, with the industrial production of all the EU economies experiencing a downward recessive phase. However, the strains of the Great Recession implied a change in the structure of business cycles co-movements within Europe and the emergence of two separated clusters of countries identifiable along the North-South axis. Expansions appeared in northern economies (Germany, Netherlands, Sweden, Poland, ...) while the recession has been prolonged in the southern ones (Italy, Spain, Greece, Portugal).

The above resurgence of asynchronization poses concerns for the well-functioning of the EU, characterized by a common monetary policy for most of its members and by conformation to common fiscal policy rules. Our results also call for further research aimed at investigating the main determinants of such a divergence in business cycles and also whether the current common policy framework has contributed to it or not.

The paper is organized as follows. Section 2 briefly reviews the relevant literature from the methodological and from the empirical perspectives. The Random Matrix Theory approach, which we extensively use in our empirical analysis is presented in section 3, also with explicit references to principal component analysis and dynamic factor models. Section 4 describes the econometric application, introduces the datasets used for the analysis as well as the results of our empirical exercise. Robustness checks are outlined in section 5. Section 6 concludes. Appendix A integrates the paper with additional material.

2 Literature Review

A vast majority of business cycle synchronization studies (see de Haan et al., 2008, for a detailed review) have measured synchronization intensity by means of indexes and averages built upon bivariate (Pearson) correlations of the cyclical components of the same variable for different economies (see Backus et al., 1993; Baxter, 1995) or for a set of countries against a reference country (see Artis and Zhang, 1999; Wynne and Koo, 2000). “Concordance indexes” have been created using a similar intuition. Such indexes allow one to measure synchronization as the percentage of periods in which two countries are in the same phase of the business cycle (see Artis and Zhang, 1999; Wynne and Koo, 2000). “Concordance indexes” have been created using a similar intuition. Such indexes allow one to measure synchronization as the percentage of periods in which two countries are in the same phase of the business cycle (see Harding and Pagan, 2002; Beine and Candelon, 2003). All these methods have the advantages of being simple, easily interpretable and ready-to-use also in more structured econometric regression exercises (see Imbs, 2004; Baxter and Kouparitsas, 2005; Di Giovanni and Levchenko, 2010). However, they all are based on the underlying assumptions that all the observed correlations are non-spurious. Here by spurious correlation we do not refer to the correlation of two \(I(1)\) variables, but to the idea that two stationary \(i.i.d\) variables with finite observations can display some correlation. The important question thus becomes how to distinguish whether

\[\text{In a similar vain, Artis et al. (2003) use diffusion indexes to count the percentage of EU countries that are in recession when a reference region (e.g. the whole Euro Area) is so.}\]
the distribution of the observed empirical correlations is different from the ones that one would have observed for finite \emph{i.i.d} variables.

A first option to solve the above issue is by assuming the existence of a set of latent stochastic processes governing the correlations among countries. This is the basic assumption underlying multivariate Markov-switching (MS) models, which have indeed been extensively used for the analysis of business cycle synchronization (Phillips, 1991). Typically these models evaluate the dependency relation among the latent variables governing the dynamics of the model. Two roads can be taken. The first imposes a specific \emph{a-priori} structural dependence amid latent variables as in the case of Smith and Summers (2005).\footnote{It is indeed sufficient to assume that all the model variables are a function of a single latent variable or that all the model variables are explained by the same number of independent latent variables with a specific correlation structure.} The second road instead, focuses on the assessments of the synchronization among different Markov-switching models \emph{a-posteriori}, providing estimates of average dependency relationships (see Guha and Banerji, 1998). Phillips (1991) shows that with a Markov-switching model, both the case of complete independence (e.g. two independent Markov processes in a bivariate specification) and the case of perfect synchronization (e.g. a unique Markov process for describing both variables) are naturally embedded. Following Phillips (1991) therefore, both Camacho and Perez-Quiros (2006) and Leiva-Leon (2014) focus on assessing whether the latent variables in multivariate models are either unsynchronized or perfectly synchronized. By modelling the data-generating process as a linear combination between the two cases, they evaluate the degree of synchronization. Despite the usefulness of this approach for an overall evaluation of synchronization, the underlying assumption of constant dependencies over time might be somehow heroic. With this assumption indeed, any assessments about endogenous changes in the structural relationship among the latent variables is impossible. The unique solution to this issue has recently been provided by Leiva-Leon (2017) that proposes an approach to endogenously infer structural changes in the relationship amid the latent variables governing multivariate MS models.

A second option comes instead from the collection of a large amount of time series whose correlations are used for reducing the dimensionality of the data and for studying only a small number of meaningful components. A paradigmatic example of this approach are dynamic factor (DF) models (Forni and Reichlin, 1998; Stock and Watson, 2011) which have extensively been employed for the study of business cycle synchronization. Dynamic factor models offer two advantages with respect to Markov-Switching models. First, they allow one to reduce the dimensionality of a large system of dynamic equations, by efficiently compressing information into a small number of factors which are able to explain a substantial portion of the variance of the whole system. Second, the factors can be used to decompose the total variance into different components, each representing a subset of the variables embedded in the whole system and, practically speaking, they can be used to perform shock-accounting exercises.\footnote{This allows one to explain the source of the co-movements among different economies as driven by exogenous shocks of different types – e.g. global shocks, country-specific shocks, industry-specific shocks, idiosyncratic shocks.} For these reasons, dynamic factor models have been used for the analysis of business cycle synchronization at the global level (Kose et al., 2003, 2012). However, in some cases they have also been employed for specific regional analyses, in particular for Europe (Eickmeier, 2009; Savva et al., 2012).
2010) or Asia (Moneta and Rüffer, 2009). One of the key debates on the dynamic factor model literature concerns the selection of the number of factors (see Barhoumi et al., 2013). Many alternatives are possible, and the approach we present in this paper, can also be seen as a new criterion for the selection of factors, in line with the approach proposed by Kapetanios (2010); Bai et al. (2015).

The variability in the approaches to study synchronization is also accompanied by some heterogeneity in empirical results. A long-standing controversy between Artis and Zhang (1997) and Inklaar and de Haan (2001) relates to the empirical evidence about synchronization in the European Union. The former authors argue that there has been an increase in integration and business cycle synchronization after the introduction of the European Exchange Rate Mechanism (ERM) in 1979. The evidence provided by the latter seems to suggest that the opposite holds true and that there has been a decrease in co-movements between 1979 and 1987. Furthermore, by employing a longer time-span, the works by Massmann and Mitchell (2005) and Giannone and Reichlin (2006) show that business cycle synchronization increased also in the EU during the 1970-2000 decades. However, after comparing the European Union with the United States, Clark and van Wincoop (2001) and Croux et al. (2001) find that in the US the business cycles of the federal states are much more synchronized than those of the European Union member countries over the same period. In line with Massmann and Mitchell (2005) and Giannone and Reichlin (2006) is the paper by Anagnostou et al. (2015), which supports the view that an increase in the levels of synchronization had occurred during the 1990s. Recent evidence based on wavelet analysis (Aguiar-Conraria and Soares, 2011) and on a DF model (Lehwald, 2013) however casts doubts on the degree of generalization of the previous results for the EU. These works indeed suggest that an increase in synchronization during the 1990’s has been evident only for the so called “core European countries” (i.e. the funding EU countries). Aguiar-Conraria and Soares (2011); Lehwald (2013) also report an increase in the degree of synchronization during the first seven years of the new century. Empirical evidence analysing the last decade, after the hit of the global financial crisis, is partially missing. The unique papers that, to our knowledge, analyse synchronization after the crisis are the one by Gächter et al. (2012) and Belke et al. (2017). Both works find a pronounced asynchronization of EU business cycles during the crisis period, in line with the convergence analysis by Borsi and Metiu (2015).

To sum up, we can conclude that the literature on synchronization has not yet reached a conclusive answer about synchronization patterns in the EU. An apparent common long-trend exists, suggesting an increase in synchronization among EU economies. Nevertheless, periods of asynchronization might occasionally appear. The variety of approaches used is certainly good news for policy makers who have at their disposal a portfolio of methods providing different views on the same phenomenon: each method indeed, observes at synchronization

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5For a recent selection procedure see Alessi et al. (2010).
6Also the average correlation among different EU countries cycles and the cycle of the aggregate EU increase in the same period according to Agresti and Mojon (2001), Altavilla (2004) and de Haan et al. (2008).
7Contrarily, looking at the synchronization in the nominal wages dynamics and using Markov switching models, Buscher and Gabrisch (2012) and William and C. (2018) find that asymmetries amid EU economies are still persistent even after the introduction of the Euro.
with different lenses and from a different perspective. Using all the results, would improve available information, and thus facilitate a global understanding of the phenomenon. At the same time, the lack of a clear answer to question of whether business cycles are getting more or less synchronized within the EU represent a source of uncertainty for an effective application of common macroeconomic policies within the union.

3 Methodology

We propose a general approach to quantify synchronization patterns among a set of $N$ variables, observed over a time-span $T$ and sampled at the same frequency $\Delta t$. Let us first denote with $\tilde{X}_{i,t}$ the stationary variable of interest. Before performing our analysis and without loss of generality, we also normalize our set of time series by applying the common transformation:

$$X_{i,t} = \frac{\tilde{X}_{i,t} - \mu(\tilde{X}_i)}{\sigma(\tilde{X}_i)}.$$ 

This allows one to rescale the $N$ series and to obtain a data matrix $X_{N \times T}$ with the two useful properties of null mean $\mu(X_i) = 0$ and unitary variance $\sigma^2(X_i) = 1$. The variance-covariance matrix is defined as

$$\Sigma_{N \times N} = \frac{1}{T}XX',$$

where $\sigma_{i,j} \in [-1, 1]$ and $\sigma_{i,i} = 1$ and where $X'$ denotes the transpose matrix. The $(i, j)$ element of the matrix $\Sigma$ represents the cross-correlation between the $i^{th}$ and $j^{th}$ variables over the whole time horizon $T$. To obtain finer description of the evolution of the correlations over different time periods, one can break the whole time series in $K$ rolling-windows of equal length $T_w$. This give rise to $K$ matrices $X(k)$ of size $N \times T_w$, where $k = 1, \ldots, K$ denotes each specific window. For each window-specific dataset $X(k)$, it is then possible to compute the corresponding correlation matrix $\Sigma(k)$. This simple data manipulation strategy allows one to analyse the evolution of the correlation matrix of manufacturing production indices over time.\(^8\) The division in shorter time-windows might however be useless. If the evolution of the correlation among the different variables is null indeed, the variance-covariance matrix of the whole time series will be equal to each sub-window variance-covariance matrix. To verify that this is not the case, and that studying time-evolution over sub-windows provides additional information vis-à-vis the complete sample, we employ a simple matrix statistics following Münnix et al. (2012). In particular, we define the similarity index $S(k, h)$ between a pair of variance-covariance matrices $\Sigma(k), \Sigma(h)$ as

$$S(k, h) = 1 - E(|\Sigma(k) - \Sigma(h)|).$$

Values of $S(k, h)$ close to 0 indicate a large difference between the two variance-covariance matrices observed in different time windows; this is a symptom for the occurrence of a major variation in the (linear) relations amid the $N$ variables over periods $k$ and $h$ and suggests

\(^8\)Notice that along the paper we will use both the terms correlation matrix and covariance matrix. They express the same concept here, since all the variables have been pre-whitened and renormalized.
that adopting a rolling-window approach provides relevant additional information about the evolution of the synchronization of the \(N\) variables. As detailed also in Section 2, the pairwise correlation is the basic metric employed to analyse the synchronization between cycles of different economies. Also in our approach, the covariance matrix is the starting point. Building on its properties, we define our approach and we quantify the degree of synchronization.

A variance-covariance matrix is by definition positive semi-definite, therefore its eigenvalues are all positive and distinct.\(^9\) Let us denote by \(\lambda_i\) with \(i = 1, \ldots, N\) the eigenvalues of \(\Sigma\) and by \(u_i\) the corresponding eigenvectors. The empirical density function of the eigenvalues is characterized by

\[
\rho_{\Sigma}(\lambda) = \frac{dn(\lambda)}{d\lambda},
\]

where \(n(\lambda)\) is the number of eigenvalues larger than \(\lambda\). It is possible to use the information stemming from the eigenvalues to study the process of synchronization among the \(N\) variables of interest. Indeed, one can compare the empirical density function of the eigenvalues with the theoretical density of the eigenvalues generated by a theoretical benchmark (i.e. a null model). Thus we can identify the number of empirical eigenvalues that significantly deviate from a theoretical benchmark that possesses some known and easily interpretable characteristics. The foregoing “Random Matrix Theory” (RMT) approach has successfully been employed in the empirical finance literature with the aim of understanding asset co-movements and asset portfolio overlaps (see Laloux et al., 1999, 2000; Plerou et al., 2002; Kim and Jeong, 2005; Bouchaud and Potters, 2009; Wang et al., 2011; Meng et al., 2014; Jiang et al., 2014, among others). This approach is also employed to identify the number of factors in dynamic factor models (see Bai et al., 2015).

The null model of reference is a relatively naive one. We indeed assume that under the null hypothesis, the theoretical dataset \(\hat{X}\) is composed by random observations \(\hat{x}_{i,t}\) which are independent and identically distributed. We also assume that the identical underlying distribution is a standard Normal.\(^{10}\) The following theorem defines \(\hat{\Sigma}\) the variance-covariance matrix of the data generated by the null model.

**Theorem - Marchenko-Pastur law.** For \(N, T \to \infty\) and \(Q = \frac{T}{N} \to a > 1\), the density function of the eigenvalues of \(\hat{\Sigma}\) is given by

\[
\rho_{\hat{\Sigma}}(\lambda) = \begin{cases} 
\frac{Q}{2\pi\sigma^2} \sqrt{\frac{\lambda_{\text{max}} - \lambda - \lambda_{\text{min}}}{\lambda}} & \text{for } \lambda \in (\lambda_{\text{min}}, \lambda_{\text{max}}) \\
0 & \text{else} 
\end{cases}
\]

(2)

where \(\lambda_{\text{max/min}}^{\text{RMT}} = \sigma^2 \left(1 \pm \sqrt{\frac{1}{Q}}\right)^2\) are the upper/lower bounds of the eigenvalues associated with a random matrix with the same variance \(\sigma^2\) and the same \(Q\) of the empirical observations.

The above theorem also states the lower and upper bounds for the eigenvalues generated by the null model, and thus corresponding to a variance-covariance matrix where correlations are purely random. The same theorem also states that the key condition for the Marchenko-

\(^9\)In order to keep the notation clean, we here drop the window-specific index \(k\).

\(^{10}\)This is not crucial for our analysis and we relax this assumption in section 5.1.
Pastur law to hold empirically is that the ratio between $T$ and $N$ converges toward a finite constant $a > 1$.\footnote{In a rolling windows context one shall simply substitute $T$ with $T_w$.}

However, in the presence of outliers observations in the empirical data, the empirical largest eigenvalue may significantly exceed the Marchenko-Pastur upper bound (Biroli et al., 2007; Bouchaud and Potters, 2009; Lux et al., 2019). In particular, if the largest absolute value of the empirical observations $X$ and defined by $\max(|x_{i,j}|) = S$ is larger than $(NT)^{1/4}$, the upper limit defined by the Marchenko-Pastur law shall be adjusted as follows:

$$
\lambda_{\text{adj}}^{\max} = \left(\frac{1}{Q} + \frac{S^2}{T}\right) \left(1 + \frac{T}{S^2}\right).
$$

Furthermore, if empirical values $|x_{i,j}|$ have a power-law distribution with exponent $\alpha > 0$ (i.e. $Pr(|x_{i,j}| > x) \sim x^{-\alpha}$) instead of the Normal distribution, the large values in the elements of $X$ might dominate the top largest empirical eigenvalues. In this case the upper bound of the null model shall become

$$
\lambda_{\text{pl}}^{\max} = N^{4/\alpha - 1} Q^{2/\alpha - 1}.
$$

In the cases where both $\lambda_{\text{adj}}^{\max}$ and $\lambda_{\text{pl}}^{\max}$ are greater than $\lambda_{\text{RMT}}^{\max}$, the effects of the extreme values can lead to anomalously large empirical eigenvalues with no genuine information. We test for these potential problems in a separate analysis in section 5.1 and show that they do not affect our empirical application.

The eigenvalues of a variance-covariance matrices can be interpreted as the portion of the variance that can be explained by a specific component/factor (see Stock and Watson, 2011; Wang et al., 2011). Hence, by comparing the empirical distribution of the eigenvalues and the theoretical counterpart generated by the null model specified above, one can study how many components matter for a broad explanation of the dataset variation. In particular, given the empirical variance-covariance matrix, two interesting possibilities might emerge:

1. $\lambda_{\text{RMT}}^{\max} > \lambda_1 > \cdots > \lambda_N$
2. $\lambda_1 > \cdots > \lambda_M > \lambda_{\text{RMT}}^{\max} > \cdots > \lambda_N$

In the first case, all the empirical eigenvalues are lower than the upper bound defined by the Marchenko-Pastur law. This means that the common components that track empirical data are not statistically different than the ones generated by a matrix with random correlations. Hence, one can conclude that there is no structural correlation and thus no synchronization among the $N$ variables of interest. The second case is instead the interesting one. It implies that the $M$ largest eigenvalues represent common drivers conveying information about the covariance structure of the empirical data better than what a null random matrix model could do. It is indeed easy to shown that each eigenvalue can be expressed as:

$$
\lambda_i = u'_i \Sigma u_i = u'_i \text{Cov}(X) u_i = \text{Var}(u'_i X_t)
$$

\footnote{In a rolling windows context one shall simply substitute $T$ with $T_w$.}
and the total variance as:

\[ \text{Var}(X_t) = \sum_{i=1}^{N} \text{Var}(X_i) = N = \sum_{i=1}^{N} \lambda_i = \sum_{i=1}^{N} \text{Var}(u'_i'X_t). \]  

(6)

This set of equations clearly shows that each eigenvalue \( \lambda_i \) describes a portion of the total variance of the data: in particular the portion explained by the \( i^{th} \) factor is \( f_i = u'_i'X_t \). The percentage of variance explained by the largest \( M \) eigenvalues is also called in the literature the absorption ratio and is formally defined as

\[ A_M = \frac{1}{N} \sum_{m=1}^{M} \lambda_m; \]

it captures the information embedded into the largest \( M \) components and, since the eigenvalues are sorted in a descending order, it will be true that \( A_1 < A_2 < \cdots < A_N = 1 \). (e.g. Pukthuanthong and Roll, 2009; Kritzman et al., 2011; Billio et al., 2012; Zheng et al., 2012; Meng et al., 2014).

Using the above-described strategy in a rolling-windows framework, it is therefore possible to investigate the temporal dynamics of the eigenvalues larger than \( \lambda_{\text{max}} \) with the aim of identifying periods with significant synchronization patterns among the \( N \) variables (i.e. not generated by random correlations). Furthermore, it is also possible to quantify the relation that each variable has with respect to the first \( M \) components using the information embedded into the corresponding eigenvectors. This is also called the loading matrix in the PCA framework and embeds the contribution that each of the \( N \) variables provides to the generation of the common factor.

Finally, one can also filter out the effects of the largest eigenvalue (i.e. the most important factor in terms of explained variance) on the variance-covariance matrix \( \Sigma \)

\[ \Sigma_1 = \lambda_1 u_1 u'_1 \]
\[ \Sigma_{\text{II}} = \Sigma - \Sigma_1. \]

This allows one to verify if significant information is left out after accounting for the contribution of the first component.

A final measure that can help one to better understand the evolution of synchronization among the \( N \) variables is the so-called inverse participation ratio (IPR). There are as many IPR as the number of eigenvalues and each is calculated building on the elements of the eigenvector associated to the \( i^{th} \) component. Formally, the \( IPR_i \) is defined as:

\[ IPR_i = \sum_{j=1}^{N} [u_i(j)]^4 \]  

(7)

where \( u_i(j) \) stands for the \( j^{th} \) element of the eigenvector associated with the \( i^{th} \) eigenvalue.\(^{12}\)

\(^{12}\)The fourth power is needed because for positive semi-definite symmetric matrices (like the covariance matrix) the sum of squares of the eigenvectors is always unitary; i.e. \( \sum_{j=1}^{N} u_i(j)^2 = 1 \).
Of course, the interest lays only on the IPR of the largest factors. The $IPR_i$ is interpreted as the number of significant variables contributing to the eigenvector corresponding to the $i^{th}$ factor and it is therefore defined in the interval $1 \leq IPR_i \leq N$. In particular we have that:

$$IPR_i = \begin{cases} 
1/N & \iff u_i(j)^2 = \frac{1}{N} \quad \forall j, \\
1 & \iff u_i(j)^2 = 0 \quad \forall j \text{ but one}
\end{cases}$$

In the case where the null model is rejected for the largest eigenvalue, the $IPR_1$ (i.e. inverse participation ratio derived from the eigenvector associated with this eigenvalue) can be interpreted as an indicator of synchronization among the $N$ variables: if the $IPR_1 = 1/N$, the $i^{th}$ factor is equally composed by the $N$ variables. This corresponds to “perfect synchronization” across countries. At the other extreme, when $IPR_1 = 1$, the $1^{st}$ factor explains only the variance of one single country’s time-series and all the other country-indexes are not correlated with this factor. The latter is the case “perfect asynchronization”.

## 4 Investigating Synchronization Patterns in the European Union

For our empirical exercise, we employ a monthly database from Eurostat describing the industrial production for the manufacturing industry in the EMU. The dataset covers the years from 2000 to 2017 at a monthly frequency, which allow us to capture also short-time deviations from long-run trends. The time-series we employ are plotted in Figure 1. Since the series present an index of manufacturing industrial production (with 2000-01 = 100 for all the economies), from Figure 1 we can only draw three types of information. First, industrial production volumes fell dramatically in almost all EU countries when the US financial crisis erupted in 2008. Second, since 2010, gaps in levels of industrial production among EU economies have widened. Indeed, some countries could not recover the pre-crisis volumes (e.g. Spain, Italy, Greece), while others (e.g. Germany) have rapidly recovered displaying stable growth paths. Finally, from Figure 1 it is clear that data are not stationary and shall therefore be pre-whitened. To do so, we use the Christiano and Fitzgerald (2003) version of the BandPass filter and following the Bry and Boschan (1971) routine, we identify the business cycles component of our series by excluding frequencies lower than 5 months and larger than 15 months.

### 4.1 Some descriptive statistics

The starting point of all the metrics described in section 3 is the Pearson correlation coefficient $\rho_{i,j}(t)$ calculated on the industrial production of all the pairs of countries $(i,j)$ over rolling windows of three years length. The distributions of the elements in the variance-covariance

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13 An alternative interpretation of the eigenvector components is the correlation that each of the $N$ variables has with the common factors.


15 We also performed augmented Dickey-Fuller tests and we could not reject the null hypothesis of at least one unit root in the data.
Figure 1
Manufacturing IP index for the EU-28 economies (UK included).
Source: Eurostat Data.

matrix $\Sigma$ are presented in Figure 2 for all the windows under scrutiny. The colours in the figure capture the values of the correlation coefficient, from perfect inverse correlation (purple colour, $\rho_{i,j}(t) = -1$) to perfect positive correlation (red colour, $\rho_{i,j}(t) = 1$). The figure reveals a steady increase in the degree of correlation among countries’ business cycles starting in 2003 and proceeding up to 2009. In particular, for the 2001-2003 window, the average correlation is 0.123, implying that at the beginning of the century, each EU country’s business cycle was scarcely correlated with the one of other EU economies. As the time went by, and the EU integration proceeded with the inclusion of other economies (e.g. Cyprus, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Malta, Poland, Slovakia have joined the EU in 2004), average correlation slightly increased to 0.128. However, it is the Great Recession period that is characterized by the highest correlation among EU business cycles (except Ireland, Lithuania and Poland). Figure 2 reveals how the mass of the coefficient distributions was clearly concentrated on high positive values in the years from 2008 to 2010. As we argue to a greater extent below, this high correlation was induced by the common negative shock that hit the EU in that period. All the countries have then experienced large drops in their manufacturing production volumes in the years after the Great Recession. Nevertheless, average correlation dropped back to 0.159 in the 2010-2012 window, indicating that different countries have been over different phase of the business cycle. Correlation among the EU economies therefore, seems to have regressed to the levels observable at the beginning of the century, and even below those levels in the final years of our sample. As figure 2 indicates, the left tail of the distribution of correlation has become fatter and the minimum correlation more negative than in the periods before the crisis.

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16 In appendix A.1, we also include a table with the first four moments of the correlation coefficients distributions over all the time windows.
Figure 2
Kernel densities of the correlation coefficients over the different rolling windows. Diagonal elements of the correlation matrices have been pruned for all the years to avoid a spike at 1 for every year.
Source: Eurostat Data.

From the correlation matrices we can also compute the similarity matrix as described in equation 1 and that we report in appendix A.2. The similarity matrix provides information about whether the correlation structure of the different EU economies has remained similar or it has experienced significant variations in different periods of time. We find that, on average, during the past two decades, the correlation structure among industrial productions of EU economies has smoothly evolved. The only exception to this pattern is represented by the period of the Great Recession, wherein the correlation structure experienced a sudden variation. This is indicated by the low values of similarity among the years in the period 2008-2011 and all the other years in our sample (see appendix A.2).17

4.2 Identifying common factors

After having observed and investigated aggregate correlation patterns by means of baseline metrics typically used in the synchronization literature, we turn in this section to develop the results from the application of the Random Matrix Theory approach presented in section 3. The main goals are: (i) identifying the number of significant and important factors; (ii) computing the fraction of total variance that each single factor is able to account for; (iii) describing the evolution of the degree of synchronization in EU over time.

The top panel of figure 3 shows all the $N = 28$ empirical eigenvalues calculated on the correlation matrices over the different windows and ordered from the largest to the smallest one. In the same panel, we also plot the RMT upper and lower bounds (i.e. $\lambda_{\text{min}}^{\text{RMT}}$ and $\lambda_{\text{max}}^{\text{RMT}}$), to

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17Similarity on the main diagonal is exactly equal to one by construction; very high levels close to the diagonal are also due to the rolling windows overlap.
compare the empirical values with the theoretical limits derived under the null model. The largest eigenvalue is always significantly larger than the theoretical upper bound, implying that there exist always sufficiently strong co-movements in observed industrial production to deviate from a purely stochastic Gaussian dynamic, represented by the null model. Also, we observe that at the beginning of the century and in the aftermath of the crisis, there are subsequent years in which also a second factor (and to a less extent a third one) which is relevant for the explanation of the correlation among industrial production of EU countries.

In the dynamic factor as well as in the RMT literatures, the first factor (the common EU factor henceforth) is commonly interpreted as a common market force (in this case a EU market force) that combines the effects brought about by trade openness (Frankel and Rose, 1998) and financial integration (Imbs, 2004, 2006) of the EU economies. However, it also incorporate the effects of common EU policies and common shocks affecting all EU countries. The second factor (the group factor henceforth) reflects instead other variables that are better able to characterize clus-

Figure 3
Top panel: eigenvalues evolution. Dashed lines represent the upper and lower bounds of the Marchenko-Pastur theoretical distribution. Bottom panel: absorption ratios of the two largest factors.
Source: Eurostat Data.
ters of economies. The bottom panel of figure 3 shows the fraction of the total variance that each significant factor is able to explain. The first factor alone (see figure 3) accounts for 50% of the total variance during the Great Recession period. The fraction of variance explained is instead much lower (between 20% to 25%) at the beginning and at the end of our sample, i.e. in the first years of the new century and in the aftermath of the Great Recession. The variance explained by the second factor is almost constant over time and around 15% of the total variance. We can therefore conclude that the systemic component of the EU business cycle co-movements has been high during the period 2007 to 2010 and that the additional components, explaining the co-movements of group of countries were relevant only at the beginning of the century and in the aftermath of the 2007-2009 recession. It follows that the emergence of clusters of EU economies having diverging business cycle patterns has mostly been important at the time of the introduction of the common currency and in the aftermath of the Great recession.\textsuperscript{18}

The above patterns are also confirmed by the analysis of the inverse participation ratio to the 1\textsuperscript{st} common factor, ($IPR_1$, see equation 7), which represents an alternative measure of the aggregate degree of synchronization. Figure 4 shows that the inverse participation ratio (solid line) increased in a first phase until 2007. It then started to decrease reaching the lowest level of around 0.045 in the 2009-2010 windows. After the great recession, the $IPR_1$ returned to the levels observable at the beginning of the century (around 0.07). It follows that synchronization was very high in the Great Recession period and weak in the periods before and after the crisis. Nevertheless, even at its peak (i.e. during the Great Recession) the observed degree of synchronization was far from the perfect synchronization, indicated by dashed line in figure 4 that represents the theoretical lower bound where perfect synchronization is reached ($1/N = 1/28$ for the Eurostat dataset).

\textbf{Figure 4}

\textit{Inverse Participate Ratio associated with the EU common factor.}

\textit{Source: Eurostat Data.}

\textsuperscript{18}We also include in the figure the contribution to the third factor, to show that is relatively minor and approximately invariant over time, meaning that after having filtered all the information brought about by the first two factors, there is no more statistical difference with respect to a null stochastic Gaussian model.
4.3 Synchronization clusters in the European Union

As a final step in our analysis we use the information contained into the eigenvector associated to the largest eigenvalue (i.e. associated to the common EU factor) to understand the strength of the correlation between each country and the common EU market mode. The information entailed in the eigenvector associated to \( \lambda_1 \) is the information of the correlation matrix after having filtered the original correlation structure with the information contained in the common EU factor and before removing the group factor, thus providing therefore an indication about how the different countries correlate with the first factor.

This also implicitly allows us to identify clusters of countries that strongly contribute to the factor or not. For this analysis, we show the eigenvectors related to the EU common component estimated using the trend-component (after we test for its stationarity).

Figure 5

*Eigenvector components corresponding to the contribution of each country to the EU common factor, estimated on the trend-component.*

*Source: Eurostat Data.*

Figure 5 provides a visual idea of the formation of synchronization clusters according to their degree of correlation with the largest factor. The colours in the map capture different degrees of correlation with the 1st common factor (and measured by the elements in the associated eigenvector), ranging for inverse correlation (negative values) to positive correlation with the factor. Clearly, countries that had almost zero or negative correlations were a-synchronized from the EU market mode in time-window, as had diverging patterns with respect to the 1st factor. The figure reveals interesting features that have characterized the business cycle dynamics in the European Union during the last two decades. First, it is possible to observe that at the beginning of the century, two clusters of countries emerged: the first cluster is characterized, among the others, by the presence of countries like Poland, Hungary, Slovakia, Bulgaria,
Estonia and Latvia; the second cluster instead is characterized by the presence of France, Italy, United Kingdom, Portugal, the Netherlands and, to a less extent, Germany. This suggests the presence of a clear division between the “Eastern Europe” block and the “Western Europe” block in terms of business cycle dynamics. Such a divide disappeared in the 2004-2006 rolling window (presented in the top-right panel), during which most of EU economies began to display high synchronization in their business cycle phase. All the EU economies (apart from Portugal and Greece) were similarly correlated with the EU common factor in this period, suggesting that the EU integration process started to display its effects as far as business cycle synchronization is concerned. Moreover, in the Great Recession period (bottom-left panel), the shock that originated in the US affected all the EU economies in a very similar fashion except for three economies (Poland, Hungary and Slovakia) which were less affected by the crisis. Notice that this is also the time-window where the share of variance explained by the 1st common factor was the highest (see the $IPR_1$ ratio in figure 4 and the previous section). Nevertheless, as we already stressed before, synchronization fell in the period after the Great Recession and clusters of countries emerged again. See the bottom-right panel in figure 5. In this last period of our analysis though the separation among countries in terms of business cycle phases can be identified along the North-South axis (rather than along the East-West axis). The map suggests that one group of countries is composed by Germany, France, Austria, United Kingdom, Poland, Sweden among the others. In the other group one can instead find Italy, Greece, Spain and Portugal. Interestingly enough, this second group also includes all the countries that were severely affected by the sovereign debt crisis during the 2010-2012 period.

5 Robustness checks

The results presented in the previous section have highlighted several interesting features about business cycles co-movements in the European Union. We highlighted how the overall dynamics of industrial production in the area is accounted for by at two significant common factors. We also highlighted that the degree of synchronization (as e.g. captured by the variance explained by the 1st common factor) has not been constant over the course of the last two decades. It was very high during the Great Recession, but low at the beginning of 21st century and in the aftermath of the crisis. We also studied the elements of the eigenvector associated with the 1st common factor in order to identify clusters of countries whose cyclical dynamics was diverging in the years where synchronization was low. We uncovered that also the composition of these clusters of countries was not the same across time and that in the more recent period the divide in business cycle dynamics can be localized along the North-South axis. Our synchronization analysis employs Random Matrix Theory and its results can be affected by a number of problems. In particular, one key hypothesis that we made is that the growth rates of industrial production in the null model are distributed as Normal $i.i.d.$ random variables. However, empirical evidence indicates that industrial production growth rates are far from being normal, and they are instead characterized by fat-tails (see e.g. Fagiolo et al., 2008). In addition, the presence of extreme values in the data can lead to an overestimation of the largest

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19 One explanation for this convergence might be due to the official inclusion in the EU in 2004 of some Eastern economies.
eigenvalue and of the statistical significance of synchronization. We address this important problem in section 5.1. In section 5.2 instead we replicate our analysis by using an alternative dataset for industrial production. As a matter of fact, it is important to use another index of industrial production, which comprises more sectors (but at the same level of aggregation) than the manufacturing one alone. The OECD industrial production dataset seems therefore appropriate as it refers to the volume of output generated by production units classified under all the industrial sectors (i.e. mining, manufacturing, electricity, gas and water) of the International Standard Industrial Classification of all economic activities (ISIC Rev. 4). This dataset is therefore more comprehensive than the Eurostat one. However, its correlation with GDP is lower, indicating that the Eurostat industrial production dataset represents a finer proxy of business cycle.

5.1 Controlling for the effect of extreme values

Results in section 4 have been obtained under the assumption that the growth rates of industrial production in the null model are distributed as Normal i.i.d. random variables. However in Section 3, we have observed that if the data display extreme values, then the largest eigenvalue might significantly exceed the Marchenko-Pastur upper bound and the upper bound shall be modified accordingly. In particular, Biroli et al. (2007) prove that if in the null model \( \hat{X}_{i,t} \) has a heavy tail distribution with same mean and variance and power law tail exponent \( \gamma \) and that the largest absolute deviation in the data – i.e. \( \max(|X_{i,t}|) \) – is lower than the threshold defined by \((NT)^{1/4}\), then the upper bound from RMT can be adjusted as:

\[
\lambda_{max} = \sigma^2(1 + \sqrt{1/Q})^2
\]  

We test the condition about the extreme values for all the rolling windows in figure 6, which is satisfied. Then, we show in figure 7 that our main interpretation of the results, in particular the selection of the number of factors and hence about the patterns of synchronization, does not relevantly change. In particular, the first factor is always significant while the second becomes relevant only in the first years of the century.
Evolution of the eigenvalues with standard RMT (blue dashed line) and with Biroli et al. (2007) corrected (red dashed line) upper bounds.

5.2 Results using an alternative datasets

Results in Section 4 were obtained using data collected from Eurostat. In this section we verify the robustness of the results with another dataset. In particular, we employ monthly OECD statistics on the total industrial production in 23 European economies.

Figure 8
Total IP index for EU-23 economies. Bulgaria, Croatia, Cyprus, Malta and Romania are missing.
Source: OECD Data.

In what follows, we simply present some of the key graphs that show the robustness with the analysis presented in the section 4 to a broader measure of industrial production that contains also industries which are possibly more volatile then manufacturing such as energy and gas. We present the $IPR_1$ in figure 9, which displays a downward jump and then a slowly decreasing patterns between 2005 and 2010 suggesting an increased synchronization with the early phase of the Euro; however from 2006 to 2015 synchronization displayed a reversed pattern. First a slow asynchronization, and then (in 2016) an abrupt and temporary increase in
the divergence. This partially overlaps with the implications drawn from our main results section. The IPR suggests a similar interpretation concerning the patterns of synchronization in the EU with the unique difference constituted by the very last rolling window. While the resynchronization of the last 3 years is only mild in the manufacturing sector, in the totality of industries, it seems stronger, possibly indicating some more positive signals for the future of the European Union industrial recovery.

Figure 9
Evolution of the IPR for the total industrial production.
Source: OECD Data.

Also looking at the pattern of the eigenvalues and at the absorption ratios in figure 10, we reinforce the idea that one factor alone (and only in some periods two factors) are sufficient to describe more than 75% of the EU variance in total industrial production. This is again coherent with the story of an increasing degree of synchronization in the early years of the century, reaching the peak during the crisis. Also, it is consistent with the fact that some clusters of countries emerge (i.e. in the period when two factors are significantly greater that the RMT theoretical upper bound). In particular, the clusters seemed to have a particular importance during the exit of the recession, confirming once again the evidence by Barigozzi et al. (2014) on the fact that different EU economies reacted differently to the common policies.

Finally, looking at which countries are positively/negatively correlated with the largest component (i.e. the common factor) we observe a difference with respect to the manufacturing sector. In particular the North-South clubs are less evident with Spain, Portugal and Greece correlating in the same direction of Germany. Italy, Sweden, Poland, Slovakia and Estonia correlate in the same direction of the French economy. We argue that this might be driven by the higher volatility present in some sectors (e.g. energy and gas, ...) that might confound the club convergence analysis. This is also the reason why we decided to employ IP in manufacturing only in the core of our analysis as presented in section 4.20

We believe that the main narrative of the paper remains therefore verified also with the OECD dataset. Indeed, on one side, there is a clear and robust pattern of synchronization detected by the $IPR_1$ temporal dynamics and confirmed in both the datasets; second, in the

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20Furthermore we have verified that the pairwise correlation between manufacturing production and GDP is larger than the correlation of total industrial production and GDP. This indicates that the manufacturing sector is a better proxy of the business cycle of a country than the total industrial production.
(a) Eigenvalues evolution. Dashed lines represent the upper and lower bounds of the Marchenko-Pastur distribution. Source: OECD Data.

(b) Absorption ratios of the three largest eigenvalues. Blue area represents the contribution to the variance of the 1st component; Green area the contribution of the 2nd component. Red area the contribution of the 3rd component. Source: OECD Data.

Figure 10
Evolution of all the eigenvalues (a) and of the three largest absorption ratios (b).

aftermath of the great recession a clear and rapid process of asynchronization took place, bringing the state of the union back to the one observed at the beginning of the century; this is indeed confirmed by the fact that the second eigenvector component becomes significantly different from the null model in both datasets and by a generalized decrease in the absorption ratio of the most important factor during the 2010-2014 period.

What shall instead be better investigated with future research is the identification of separate convergence clubs inside the EU. In particular, it seems that in the manufacturing sector, a clear North-South division is present; but for the total industrial production (i.e. considering also the energy, gas and mining industries), no clear geographical separation can be identified.
6 Conclusions

In this paper we have introduced a novel methodology with the aim of better measuring and understanding the process of synchronization among the business cycles of different economies and we have applied it to a sample of European economies. The method employs the Random Matrix Theory (RMT) approach, originally introduced by Wigner (1955) and which has strong links with Principal Component Analysis and Dynamic Factor Models. Using a Eurostat database that covers the 2000-2017 period and that describes the manufacturing industrial production in the EU we have uncovered the evolution of the patterns of synchronization in the EU and we have identified the emergence of synchronization clusters among different EU economies.

From our analysis two main results can be drawn: one about synchronization patterns and one about synchronization clusters. First, notwithstanding the literature has highlighted the presence of a positive long-run trend of synchronization, short-run deviations are the rule rather than the exception. We observe that the synchronization among the different EU countries have been increasing since the beginning of the century and up to the financial crisis, possibly because of a better integration between the eastern and the western economies; then the degree of synchronization reached its peak during the great recession, when all the EU countries have been affected by a common foreign shock; coordination among business cycle have however decreased in the aftermath of the global financial crisis, and the degree of synchronization (measured by the IPR) has regressed to the levels observable at the beginning of the century. Second, even if the level of synchronization observed in the early 2000s and in the aftermath of the crisis are similar, a structural transformation has affected the European Union: the groups of countries with high synchronization profiles have changed. From a polarization between east-west blocks (at the beginning of the century) to a polarization between
north-south blocks (in the aftermath of the crisis).

From a policy perspective, our results suggest that notwithstanding the increase in the integration have allowed the eastern block to better synchronize with some core EU economies such as Germany, France and the Netherlands, some additional policy efforts are needed in order to better coordinate the business cycles of the member countries. The danger from asynchronization periods stands indeed in the inability of a common policies to be equally effective in all the member states. Moving from here, future research shall be aimed at better understanding and testing different hypotheses about the origins of the observed synchronization patterns. This is indeed the key for providing better policy suggestions and for helping policy makers in their decision processes.

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References


A Appendix

A.1 Additional tables

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Table 1
Correlation matrix descriptive statistics.

A.2 Additional figures

In this figure we present the similarity matrix, computed as suggested in equation 1. The values of \( S(k, h) \) (theoretically bounded between 0 and 1) are always larger than 0.5 in our sample. This indicates that the difference between two covariance matrices observed at different time windows are not extremely different in general. However, there is a clear pattern of similarity around the diagonal (elements on the diagonal are exactly equal to one by construction) due to the overlapping of two adjacent rolling windows. Furthermore, we notice a strong similarity block at the centre, covering the years between 2006 and 2011. This indicates that the crisis period has been a particular period. Also the low values for the similarities (the two white areas with values around 0.5) between the crisis period and both the pre-crisis and the post-crisis phases, corroborate this hypothesis.

![Similarity matrix](image)

Figure 12
Similarity matrix.
Source: Eurostat Data.