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The Evolution of the Business Cycles and Growth Rates Distributions

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The Evolution of the Business Cycles and Growth Rates Distributions *

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Abstract

This paper performs an empirical analysis of the international cross sectional distribution of gross domestic product (GDP) growth rates and business cycles. We consider a balanced panel of 91 countries in the period 1960-2010 and two different measures of GDP fluctuations: the logarithmic growth rates and the Hodrick-Prescott cycles. Both measures are characterized by fat-tailed distributions and strong heteroscedasticity. The latter is the result of a scale relation between the variance of the fluctuations and the size of the country. The analysis of the time evolution of these properties shows that distribution tails become asymmetrically fatter during the period of study, suggesting an increased probability of finding high amplitude fluctuations in more recent years. Moreover, we observe significant changes in the scale parameter characterizing the relation between volatility and country size. These findings enrich the discussion about robust properties of business cycles and reveal more evidence about scaling-law relations in economic systems.

Keywords: Growth Rates Distribution; International Business Cycles; Scaling-laws in Economics.

JEL codes: E10, C31, R11.

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1 Introduction

The growth of countries along history has followed irregular patterns in the short and in the long run. The identification of the causes and effects of the persistent nature of these irregularities, or economic fluctuations, has challenged and still challenges the macroeconomic theory. For instance, both the dynamical stochastic general equilibrium models of recent introduction (Clarida et al., 1999; Woodford, 2010) and more traditional convergence theories (Barro, 1998; Islam, 2003) usually employ the GDP per capita growth rates as a proxy of economic fluctuations. However, they fail at explaining changes in frequency and amplitude, even for a single country.

One of the most important issues, often neglected, in cross section analysis is the relevance of world countries' heterogeneity. In the case of growth volatility, several important conclusions have emerged from the study of the distribution of growth rates. For instance, it has been argued that political instability reduces growth, such giving importance to the institutional and political structures (Alesina et al., 1992; Ramey and Ramey, 1995); that part of the volatility faced by poor countries is related to their diversification and specialization profiles (Koren and Tenreyro, 2007; Easterly et al., 2000); that international trade and financial institutions are the origin of the interdependence in countries aggregate growth (Easterly et al., 2000). In a nutshell, it is widely accepted that short term volatility has important implications for long-term performance.

Interestingly, and to some extent far from the traditional economic understanding, the empirical evidence in the cross section indicates that the probability density function of GDP growth rates exhibits fat-tails, excess kurtosis and heteroscedasticity (see: Canning et al., 1998; Lee et al., 1998; Castaldi and Dosi, 2009). This statistical finding is consistent with the possibility of having sudden slumps and booms. Moreover, it has been shown that fat-tails are present in the residuals of the GDP time series of a single country also after detrending with most of the traditional techniques, implying that extreme events are observed at different frequency levels (see: Fagiolo et al., 2008, 2009).

The heteroscedastic nature of GDP fluctuations is related to the existence of a scale relation between their variance and the size of the country. In general richest countries face lower volatility than poorer ones (see, for example, Ramey and Ramey, 1995; Fiaschi and Lavezzi, 2003). This relationship and the fat-tailed nature of the shocks also characterize economic growth at a "micro" scale, when single business firms are considered (Bottazzi and Secchi, 2006a). These similarities are intriguing. They suggest the existence of a common mechanism governing the growth process at different scale which could explain why these features survive to aggregation (see, for example, Brock, 1999; Castaldi and Dosi, 2009).

This paper aims at characterizing the dynamic behavior of the cross sectional probability distribution of GDP fluctuations. We are interested in the evolution of the parameters that characterize its shape and in the scale relation that determines the volatility profile faced by countries. For this purpose, we propose an econometric method that, at the same time, takes into account the heteroscedasticity, the autocorrelation and rescales the fluctuations to obtain a homoscedastic distribution. The estimation procedure is based on a non linear Least Absolute Deviation (LAD) estimator, which is generally understood to provide more robust estimates and is in line with the empirical finding of persistent fat-tailed fluctuations.

Our results confirm that growth rates are heteroscedastic and fat-tailed, with a distribution remarkably similar to the Laplace. Besides, the analysis of the temporal evolution of the parameters of the distribution allows us to conclude that left and right tails evolved differently with a remarkable tendency to get fatter in the long term, suggesting a greater probability of finding high amplitude fluctuations in the recent years. We detect periods of asymmetry in the distribution, that is periods in which the nature of fluctuations above and below the modal value had different distributional properties. We also show that the scaling of variance is quite nonreactive over time, apart from a slow reduction (in absolute terms) since the 1990s, indicating a reduction in the comparative volatility faced by high and low income economies. The properties we are able to identify and their time evolution are basically the same when using log growth rates or detrended cycles to define fluctuations. The insensitivity of the results to the filtering technique suggests that heterogeneity not only survives aggregation but it is also present at different perturbation frequencies of the time series.

The remaining of this paper is organized as follows. Section 2 presents the data and the definitions of the variables. In Section 3, we study the cross sectional properties of the distribution of growth rates, pooling all the years of the sample together. We start with the traditional analysis and then move to the non-linear LAD regression. The same methodology is applied in Section 4 to study the time evolution of the distribution of GDP shocks. As a robustness check, we replicate the same analysis in Section 5 but using HP-cycles instead of growth rates. Finally, Section 6 concludes summarizing our major findings.

2 Data and definitions

The following analysis is based on the gross domestic product (GDP) at constant prices (2005 reference year) of a balanced panel of 91 countries between 1960 and 2010 as reported in the Penn World Table version 7.1, (see Appendix A for a list of countries). Let $y_{i,t}$ be the natural logarithm of GDP of country i in year t. In general, one assumes that the time series $y_{i,t}$ can be additively decomposed in a trend $\theta_{i,t}$ and in a volatile, or cyclical, component $c_{i,t}$

$$y_{i,t} = \theta_{i,t} + c_{i,t} \ . \tag{1}$$

The latter component is commonly associated to a residual defined through a detrending technique. In what follows we are interested in the properties of the probability density function (PDF) of the cyclical part $\{c_{i,t}\}$ and its evolution. Obviously, any assumption made on the trend ends up by affecting the properties of the cycles. In this paper we are not specifically interested in discussing what is the most appropriate way to remove the trend from the series. Our strategy is thus to present the results obtained by using two definitions frequently employed in the literature: the growth rates (or first order differences) and the cycles obtained through the application of the Hodrick-Prescott (HP) filter to the original time series.[1]

The main argument behind the use of first order differencing is that the GDP grows

[1] We also performed the analysis on the residuals of a bandpass filter as defined in Baxter and King (1995) and Christiano and Fitzgerald (1999). Results are analogous and available upon requests.

following a unit root process. Hence, each GDP time series is a different realization of a random walk and in equation (1) it can be simply assumed $\theta_{i,t} = y_{i,t-1}$, so that fluctuations becomes equal to the standard logarithmic growth rate

$$r_{i,t} = y_{i,t} - y_{i,t-1} \ . \tag{2}$$

Conversely, the HP-filter captures a non-linear trend by filtering low frequency fluctuations. The trend component is obtained by minimizing the expression

$$\min_{[\theta_t]_{t=1}^{T}} \left\{ \sum_{t=1}^{T} c_{i,t}^2 + \lambda \sum_{t=2}^{T} ((\theta_{i,t+1} - \theta_{i,t}) - (\theta_{i,t} - \theta_{i,t-1}))^2 \right\} \ , \tag{3}$$

where T is the sample size, λ is a strictly positive parameter that penalizes the trend variability and $c_{i.t}$ are defined in (1). The trend identified through (3) is stochastic, smooth, and non correlated with the cycles (Kydland and Prescott, 1990). The properties, advantages and shortcomings of the HP-filter are widely discussed in the literature. In particular, it is well known that the value of λ affects the trend smoothness and determines, at least in part, the statistical properties of the cycles (e.g. their variance). It is still an open question whether the parameter λ has to be chosen exogenously, as suggested by Kydland and Prescott (1990), or determined endogenously.[2] For the time being, we estimate cycles setting $\lambda = 6.25$, a widely accepted choice for annual data (Uhlig and Ravn, 1997).

Notice that according to the definition above, GDP fluctuations are derived from each country time series separately.[3] Both growth rates and HP-cycles usually display short term autocorrelation. By definition, growth rates are very sensitive to high frequency movements and are characterized by a higher variability. Conversely, the HP filter removes lower frequencies and leads to stationary cycles. The amplitude of HP-cycles depends in general on the parameter λ, but it is bounded from above by the amplitude of growth rates.[4] What is

[2] In fact it has been shown by Nelson and Plosser (1982) that part of the variability attributed to the cyclical component belongs to the trend.

[3] This should be confronted with Castaldi and Dosi (2009) where a cross sectional detrending is applied instead.

[4] See Stock and Watson (1999) for a more complete and formal characterization and comparison of these and other filters.

important for our purposes is that these two definitions provide somehow “extremal” reference systems in which GDP fluctuations can be analyzed. As we will see in Section 5, however, the distributional properties of the cycles are largely independent from the definition adopted.

Figures 1(a)-(b) show the evolution of the first four cumulants of the cross sectional distribution of growth rates, computed for the different countries. As expected, they do not evolve smoothly. Nonetheless, a few features appear which can be easily singled out. Regarding the average, the higher performance is observed in the 1970s while three dramatic slumps, of increasing strength, are observed in 1975 (2%), 1983 (1%), and 2009 (0%). The variance has an apparent slow tendency to decrease in the long term. Skewness fluctuates around zero. In periods of around ten years, there are seemingly 4 to 6 changes in its sign. Negative peaks tend to be higher in magnitude than positive ones, revealing a non permanent symmetry. The kurtosis, which is probably the most distinctive of all statistics, has many peaks above the level of 3, which is the kurtosis of the normal distribution. In fact, since 1980s, dramatic changes are frequent and each observed peak is markedly above the reference.

The evolution of the first four moments of the distribution of HP-cycles reported in Figures 1(c)-(d) has similar characteristics. The average oscillates around zero, as it might be expected from the stationary nature of the cycles. Notice that for this variable the kurtosis reaches larger values as well, in particular in mid 1970s, mid 1990s and early 2000s.

3 Pooled growth rates

This section identifies several cross-sectional properties of the distribution of GDP growth rates obtained pooling the observations across the whole period under analysis. We start by applying a procedure which has been adopted in several studies (see: Canning et al., 1998; Lee et al., 1998; Castaldi and Dosi, 2009) and we are able to replicate, by a large extent, their findings. We will see, however, that this procedure contains several limitations. Hence, we propose a new procedure which is able to overcome these limitations and we compare the new results with the old ones.

Consider the empirical distributions obtained by pooling the growth rates across all years but splitting the countries in three groups according to the income level (GDP per capita), following the classification of the World Bank.[5] The first group consists of high-income countries (including OECD and non-OECD countries). The second group is composed of middle-income countries, those classified by the World Bank as upper-middle income. Finally, the third group is composed of low-income countries and contains those countries which according to the World Bank are poor or of low-middle income. The empirical probability density functions (PDF) of the growth rates for the three groups of countries is reported in Figure 2(a). The plot is in a semi-log scale in order to expose the behavior of the tails. The observed tent shape suggests that growth rates follow a Laplace (double exponential) density. The support of the density, however, is different for the three groups and is greater the lower the income of the economy, suggesting that growth rates are characterized by a significant heteroscedasticity. The origin of this phenomenon can be better understood by looking at the relationship between the size of the economy and the volatility of its growth rates. To see it, in each year we split the countries in different bins according to their relative size $s_{i,t} = y_{i,t} - \overline{y_t}$ where $\overline{y_t}$ is the yearly cross-sectional average country size.[6] In the subplot of Figure 2(a), the variance of the growth rates for the countries in each bin is computed and is plotted against the average bin size on a semi-log scale. The plot suggests a negative exponential relation among the variance of growth rates and the average size (in log)

$$\ln(\sigma) \sim \beta s \; . \tag{4}$$

Performing a simple OLS estimation on the binned statistics one gets $\beta = -0.076(0.011)$. In line with several previous studies (see, for example, Canning et al., 1998; Castaldi and Dosi, 2009), we can conclude that larger economies face a lower volatility than smaller ones.[7] Concerning the heteroscedasticity of the cross-sectional distribution, the most common strategy

[5] Details about the classifications are available at `databank.worldbank.org`. For a description of the methodology used see `data.worldbank.org/about/country-classifications/world-bank-atlas-method`

[6] In this way, the variable $s_{i,t}$ has zero average. Countries with low GPD have negative relative sizes while larger than average economies have positive sizes. The merit of this measure is that it eliminates a possible common trend for all countries allowing inter temporal comparisons.

[7] A similar behavior can be observed also for single firms inside a given economy, see Bottazzi and Secchi (2006a) and reference therein.

is to standardize the growth rates by removing the central tendency and rescaling them by a factor proportional to $\exp(\beta s)$ (Lee et al., 1998). In this way, a homoskedastic distribution of residuals is obtained which can be studied by pooling together the whole sample of countries.

The procedure above, however, might be quite inefficient. Firstly, the value of β depends on the definition (and number) of bins used in the estimation of (4). Secondly, the procedure ignore the fact that growth rates are not in general independent across time and are characterized by a significant autoregressive structure. To overcome these difficulties, we propose an econometric method which removes the heteroscedasticity and directly estimates the rescaled homoscedastic residuals while taking into account the autoregressive structure of the growth rates. Consider an autoregressive process for r described as

$$r_{i,t} = \alpha + B(L)\, r_{i,t} + u_{i,t}, \tag{5}$$

where α is a constant term and $B(L) = \sum_{j=1}^{N} \phi_j L_j$ is a polynomial of degree N in the lag operator $L_j r_{i,t} = r_{i,t-j}$. The heteroscedasticity of growth rates implies that the shocks u are not independent from the past realizations of r. We assume for the heteroscedastic structure the functional form in (4), so that the error term can be written as $u_{i,t} = e^{\beta s_{i,t}} \varepsilon_{i,t}$, where the ε's are identically and independently distributed according to a common distribution with zero median.[8] Finally, Equation 5 is estimated via Least Absolute Deviation (LAD) so that the problem reduces to

$$\{\beta, \phi, \alpha\} = \underset{\beta,\phi,\alpha}{\arg\min} \sum_i \sum_{t \in \tau} \left| \frac{r_{i,t} - B(L) r_{i,t} - \alpha}{e^{\beta s_{i,t-1}}} \right| , \tag{6}$$

where the parameter ϕ is the vector of coefficients of the polynomial B. The LAD estimator is, in general, characterized by good asymptotic properties and is preferred to OLS when outliers are present or when the distribution of residuals is non normal and posses a high kurtosis. In addition, LAD is the maximum likelihood estimator when the residuals ε are distributed according to a Laplace law, which based on Figure 2(a) seems to be a good quali-

[8]We also considered a version with a quadratic term proportional to $s_{i,t}^2$, but the coefficient resulted non significantly different from zero.

tative description of growth rates themselves. Figure 2(b) shows the empirical density of the residuals obtained through (6) with $N = 3$, split according to the income class of the associated countries.[9] As can be seen, the LAD estimation effectively removes the size-dependent heteroscedasticity: the density functions of different groups of countries have similar patterns and the distance between left and right tails are roughly the same. Moreover, the residual clearly display an approximated double exponential shape, confirming the adoption of the LAD regression. To obtain a quantitative assessment of this claim, we consider the general asymmetric exponential power (EP) density introduced in Bottazzi and Secchi (2011)

$$f(x, a, b, m) = \begin{cases} \frac{1}{A} e^{-\frac{1}{b_l}\left|\frac{x-m}{a_l}\right|^{b_l}} & x < m \\ \frac{1}{A} e^{-\frac{1}{b_r}\left|\frac{x-m}{a_r}\right|^{b_r}} & x > m \end{cases} \tag{7}$$

where

$$A = a_l b_l^{1/b_l} \Gamma(1 + 1/b_l) + a_r b_r^{1/b_r} \Gamma(1 + 1/b_r).$$

The scale parameters $a_{\{l,r\}}$ characterize the width of the left and the right tails, while the shape parameters $b_{\{l,r\}}$ describe their asymptotic behavior. The parameter m is the position of the mode, which in this case is set to zero. The symmetric version of this density is recovered when the left and right parameters are equal. For instance, the Normal and Laplace distributions are obtained with $a_l = a_r$ and $b_l = b_r = 2$ and $b_l = b_r = 1$, respectively. We fit both (7) and its symmetric version on the pooled residuals via maximum likelihood estimation (the details of the maximization procedure are described in Bottazzi (2004)). The resulting asymmetric density is drawn in Figure 2(b) (continuous line).

Table 1 reports the shape coefficients of the symmetric and asymmetric EP estimated on the pooled growth rates (first column), the growth rates rescaled via (4) (second column) and the residual of (6) (third column), together with the estimates of (4) and (5). As can be seen, the LAD estimate of β differs from the value obtained through the binned regression in a significant way. Moreover, the lag autoregressive parameters are significant and large.

[9] The inclusion of higher autoregressive terms produce non significant estimates for the ϕ_j with $j > 3$ and does not change significantly the values of $\{\phi_j\}_{j \leq 3}$.

The joint effect of the difference in β and the removal of the autoregressive component is that the Laplace law ($b = 1$) is a much better approximation for the LAD residuals than for the rescaled growth rates. The asymmetric parametrization, however, reveals a significant difference in the behavior of the two tails. While the left tail is heavier and follows a Laplace law, the right tail shows a faster decrease. The estimated values for the scale parameters are $a_l = 0.033(0.001)$ and $a_r = 0.031(0.001)$, reinforcing the idea that the residuals in the left half of the density are spread over a larger support.

In conclusion, we show that also when the autoregressive and heteroscedastic nature of the growth rates are appropriately taken into account, the probability of observing fluctuations far away from the average growth rate is much higher than what implied by a normal law. Moreover, since the left part of the distribution is heavier, the probability of observing very large negative shocks is higher than the probability of observing positive shocks of the same magnitude.

4 Dynamic of the growth rates distribution

The use of a binned regression for the identification of the heteroscedastic structure of growth rates makes the procedure extremely demanding in terms of the size of the sample. Conversely, the new procedure introduced above based on non-linear LAD regression has the major advantage of requiring a relatively small number of observations. In this section, we exploit this property to analyze the time evolution of the cross-sectional distribution of growth rates. Basically, we will repeat the analysis of the previous section, but considering only observations in a limited temporal window instead of the whole sample. In fact, a serious problem when pooling many years is that one might mix different macro phenomena from different periods, as for instance technology shocks, spread of crisis, changes in policies, etc. Shorter periods of analysis guarantee an increased homogeneity of the phenomena under study. In order to obtain historical pictures as consistently as possible, without sacrificing statistical significance, we use moving windows of eight years. Hence, we have 91×8 observations available for each estimation. For each time period, we estimate a set of model parameter $\beta_\tau, \phi_\tau, \alpha_\tau$ by (6) and, using the residuals, a set of distributional parameters of the symmetric and asymmetric EP distributions. In what follows, the time label τ refers to the time interval $(\tau-7,\tau)$. For example ϕ_{1988} is the autoregressive coefficient computed using data from year 1981 to year 1988.

Figures 3(a)-(b) report the time evolution of β and ϕ_1.[10] Figures 3(c)-(d) summarize the symmetric and asymmetric EP estimates. In order to expose the degree of symmetry, we plot the symmetric estimates b and a and the differences between the asymmetric estimates (b_r-b_l) and (a_r-a_l). The average of the latter is roughly equivalent to the former.

First of all, notice that the most characteristic qualitative traits of the full sample are also present in each time window: the inverse exponential relation between volatility and country size, the approximate Laplace law for the distribution of residuals and the significant first order correlation are robust features persistent over the whole period of analysis. Nonetheless, the quantification of these effects vary according to the period considered even if some parameters show comparatively greater changes than others. To discuss these changes it is helpful to split the sample in the three separate periods: from the 1970s to mid 1980s; from mid 1980s to mid 1990s; and lastly, from mid 1990s to 2010s.

During the first period, the parameter β does not undergo any change while the autocorrelation parameter ϕ_1 and the shape of the PDF exhibit significant changes. The ϕ_1 parameter doubles its magnitude, reaching the value 0.35 in mid 1980s. The shape of the PDF is statistically symmetric, however the point estimates highlights periods where the left tail is fatter. The highest asymmetry is observed in 1982, $b_r=1.5$ and $b_l=1.1$. Later a symmetric scenario is recovered with two fat-tails having $b_l=b_r\sim 1.1$. Thus, in this period, the growth process becomes more persistent, so that good past performers are more likely to be good performers also in the future, while the probability of obtaining a relatively large shock becomes independent from the sign of the shock. The moderate increase of the parameter a signals a slightly larger overall volatility toward the end of the period but since β remains constant, the cross country difference is untouched and we can conclude that the economic slow down

[10] Higher autoregressive coefficients are significant in all time windows, and exhibit lower changes that mainly co-move with ϕ_1. Without loss of generality and to facilitate the presentations of the results we omit them. Results are available upon request.

was somewhat ‘fair’, in the sense that no country class faced a comparatively higher or lower volatility.

The mid 1980s to mid 1990s period begins with a bad performing World economy, with an average growth rate of around 0.03. In this period the standard deviation of the distribution of growth rates decreases while its kurtosis increases. Both β and ϕ_1 are stationary while the distribution of residuals shows a tendency of having fatter tails (decreasing b) but a smaller support (decreasing a). These changes are in accordance with the dynamics observed for the cumulants, in particular with the decreasing volatility, and suggest a relative abundance of smaller shocks accompanied, though, by few extreme events. This is the beginning of the so-called “great moderation” (Clarida et al., 2000). Many explanations have been put forward to interpret the following period of calm. For instance, it was suggested that structural change has endowed economies with the ability of absorbing shocks (Davis and Kahn, 2008) or that better financial systems and policy makers were playing an important stabilizing role (see: Bernanke, 2004). These theories has been so blatantly disproved by recent events that any further discussion seems superfluous.

The last period, after mid 1990s, is possibly the most dynamic, as the estimated values of all parameters change considerably. The PDF is symmetric, its support shrinks, and it gets fatter than the Laplace law ($b < 1$); this effect is connected with the already mentioned jumps in the level of kurtosis. Even if GDP is getting less volatile, as more shocks fall near the center of the ‘tent-shape’, the probability of having high amplitude fluctuations increases. But the truly interesting changes are the decrease in the parameter β and the rapid movements of ϕ_1. In this scenario, all countries, rich and poor, would face similar low levels of volatility; at the same time, the growth dynamics becomes less persistent so that we observe a sort of convergence in the dynamics of shocks between countries with different income levels. The convergence of the growth rates of course does not imply a similar dynamics in terms of GDP level as differences among rich and poor remain evident (see, Quah, 1996).

The reduction of β (in absolute terms) might be a sign of higher interdependence of the different economies. If the performance of small countries is more strictly related to the performance of their larger commercial partners, then fluctuations are more likely to be of the same scale. Bottazzi and Secchi (2006b) explain the different volatility of large and small economic entities with a difference in their diversification patterns. In this sense, it could be that the higher globalization has made the specialization structure of large and small countries more similar. However, the data we have does not allow us to pursue this argument further.

5 The analysis of HP-cycles

As a robustness check we replicate the previous analysis, both at an aggregate level and inside a moving window, using the residuals of the HP-filter instead of the first log differences as a measure of growth rates. The PDF of the HP-cycles for the three groups of countries defined at the beginning of Section 3 is shown in Figure 4(a). The support is different but the shape is virtually identical to the one observed for growth rates. Using bin statistics, it is possible to confirm that the same inverse exponential relation among the standard deviation and the relative country size is still there (see subplot inside figure). The results of the non-linear LAD regression (6) are reported in Table 2 (third column), together with the direct estimation of the EP densities on the HP-cycles (first column) and the cycles rescaled by the exponential law identified through the binned regression (second column). Comparing the new estimates with the old ones in Table 1 it is immediate to see that they are practically identical, apart of course from the value of α which in the case of HP-cycles is essentially set to zero by the filter itself.[11] And this is more so for the LAD regression. The adoption of an appropriate procedure makes the identification of the cross sectional and autoregressive properties of the shocks insensitive to the actual definition adopted.

As discussed in Section 2, the computation of the HP-cycles depends on the specification of the parameter λ. The role of this parameter is to penalize the high frequency variations, so higher values for λ implies smoother trends and, consequently, more fluctuating cycles. Figure 5 reports the estimates of the asymmetric EP on the residual of the LAD regression

[11] The estimate is not precisely zero because the constant of a LAD regression refers to the median, not the mean, of the dependent variable. However given the rather strong symmetry, the result is the same.

as a function of λ. As can be seen, the behavior is smooth. Small variations around the reference level $\lambda = 6.25$ do not produce significant differences. As expected, when larger values are considered, the increased amplitude of the cycles translates in higher values for both the scale and shape parameters. Conversely, the estimates of the b's and of β are not affected by the value of the smoothing parameter used in the HP-filter (see subplot).

Finally, the estimates of the parameters inside the moving windows are shown in Figure 4. Here we keep our original reference level $\lambda = 6.25$. All dynamic patterns are rather similar to those observed with the growth rates, with a minor differences: the β parameter starts moving towards zero in mid 1980s.

6 Conclusions

This paper studies the cross sectional properties of the distribution of GDP shocks. We use two different definitions for the shocks: the log growth rates and the residuals of the HP-filter. We show that the adoption of a non-linear LAD regression as a mechanism to identify simultaneously the heteroscedastic and autoregressive nature of the shocks makes the results insensitive to the actual definition adopted. This means that heterogeneity not only survives aggregation but it is also present at different frequencies of the time series. Indeed, firstly an inverse exponential relation between volatility and country size was confirmed for both growth rates and HP-cycles. Secondly, we find that the growth shocks distribution approximates well a double exponential shape. This implies that the probability of observing fluctuations far away from the average is much higher than what implied by a normal law. Thirdly, the analysis of the evolution of the distribution reveals significant patterns. In recent years, more than in the past, rich and poor countries are characterized by similar levels of volatility. In addition, at the beginning of the new millennium we observe a significant reduction in the persistence of the growth process. This persistence seems however to increase during the last three/four years, reaching the top level of the mid 1990s. Finally, the tails of the PDF become increasingly fatter over time. This result not only contrasts with the general idea of the existence of a "great moderation", but also suggests that convergence theories should be more aware of the distributional properties of business cycles.

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A List of countries

High-income countries: Switzerland, Luxembourg,United States, New Zealand, Sweden, Netherlands, Australia, Canada, United Kingdom, Norway, Denmark, Austria, Iceland, Belgium, France, Finland, Italy, Israel, Ireland, Spain, Greece, Japan, Portugal, Korea, Republic of, Barbados, Trinidad & Tobago, Puerto Rico, Singapore, Hong Kong, Cyprus, Taiwan.

Middle-income countries: Venezuela, Argentina, Jamaica, Costa Rica, Uruguay, Mexico, South Africa, Chile, Peru, Turkey, Namibia, Brazil, Ecuador, Jordan, Colombia, Dominican Republic, Mauritius, Panama, Romania, Malaysia, Thailand, China.

Low-income countries: El Salvador, Guatemala, Bolivia, Honduras, Fiji, Paraguay, Nigeria, Senegal, Philippines, Cameroon, Cape Verde, Egypt, Cote d'Ivoire, Congo, Republic of, Sri Lanka, Morocco, Pakistan, India, Indonesia, Lesotho, Haiti, Central African Republic, Kenya, Guinea, Madagascar, Bangladesh, Benin, Togo, Comoros, Niger, Uganda, Nepal, Burkina Faso, Mali, Tanzania, Ethiopia, Mozambique, Burundi.

Sources: The World Bank and Penn World Table Version 7.1.

growth rates, pooled sample

Parameters	Non-scaled	Bin-scaled	LAD-scaled
β	-	-0.076(0.011)	-0.100(0.006)
α	-	-	0.020(0.001)
ϕ_1	-	-	0.284(0.011)
ϕ_2	-	-	0.117(0.011)
ϕ_3	-	-	0.108(0.011)
	Subbotin Estimation		
b	1.107(0.031)	1.151(0.033)	1.038(0.030)
b_l	0.980(0.039)	1.015(0.042)	0.978(0.039)
b_r	1.279(0.056)	1.342(0.061)	1.127(0.049)

Standard errors are reported in parenthesis

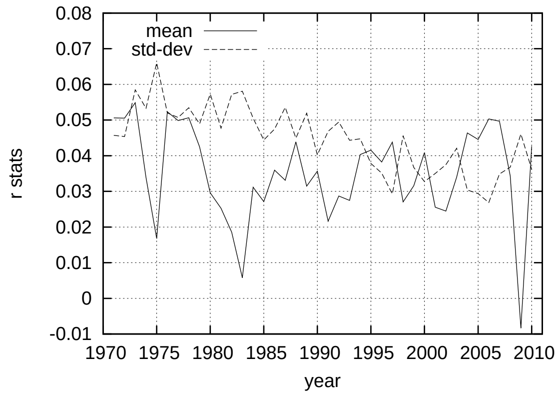
Table 1: Estimated parameters of the stochastic process and Subbotin paramters of rescaled distribution, for the period $(1960, 2010)$, using OLS on the binned statistics and the LAD regression

HP cycles, pooled sample

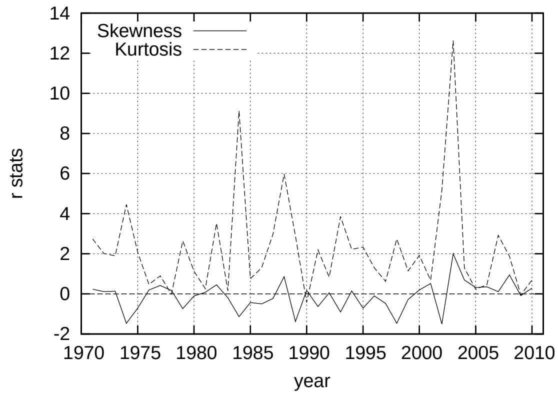
Parameters	Non-scaled	Bin-scaled	LAD-scaled
β	-	-0.096(0.013)	-0.109(0.006)
α	-	-	0.000(0.000)
ϕ_1	-	-	0.331(0.011)
ϕ_2	-	-	-0.169(0.011)
ϕ_3	-	-	-0.213(0.011)
	Subbotin Estimation		
b	1.028(0.028)	1.087(0.031)	1.089(0.032)
b_l	0.874(0.033)	0.930(0.037)	0.974(0.040)
b_r	1.189(0.047)	1.242(0.051)	1.232(0.054)

Standard errors are reported in parenthesis

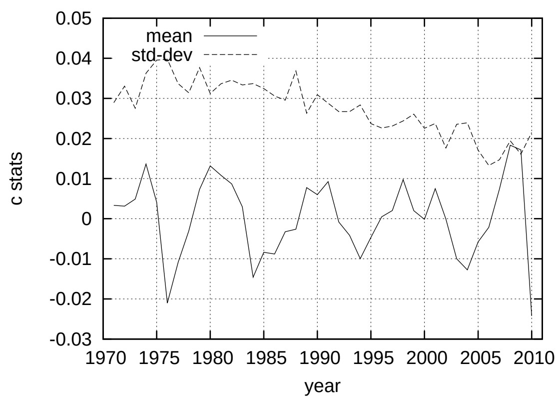
Table 2: Estimated parameters of the stochastic process and Subbotin paramters of rescaled distribution, for the period $(1960, 2010)$, using OLS on the binned statistics and the LAD regression



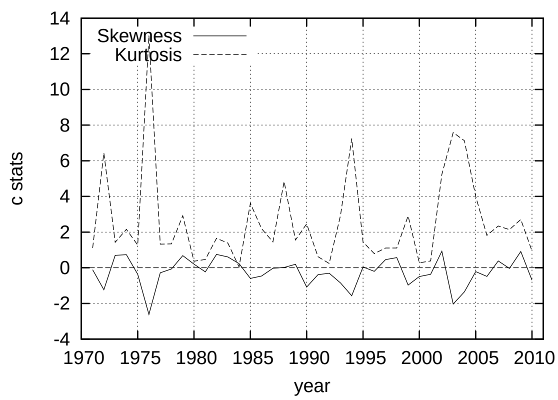
(a)



(b)

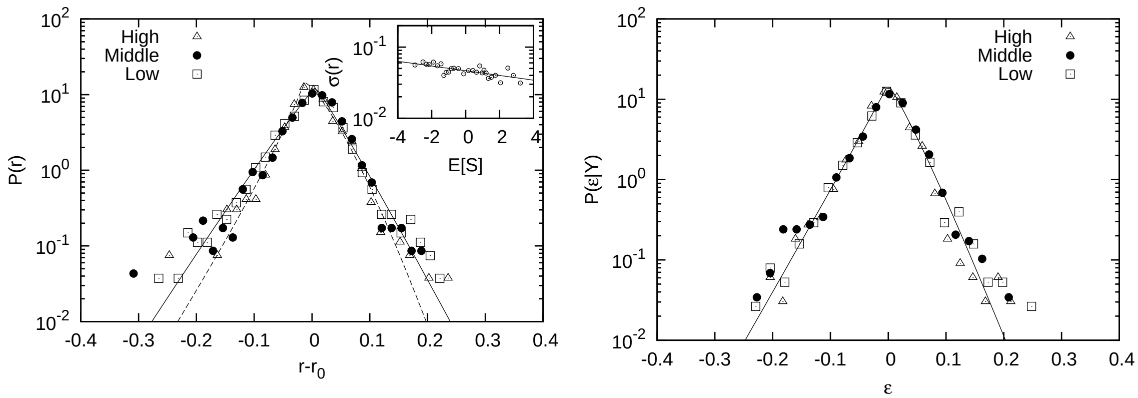


(c)



(d)

Figure 1: Evolution of statistical moments of growth rates (a) and (b) and for HP-cycles (c) and (d).



(a) (b)

Figure 2: (a) Empirical PDF of growth-rates for groups of countries with different income levels. The PDF is estimated with binned frequencies. In the subplot volatility vs. the average of country sizes using bin statistics. (b) Empirical PDF of rescaled growth rates for different country classes. The continuous and dashed lines in plots represents the Subbotin estimation of the corresponding PDF.

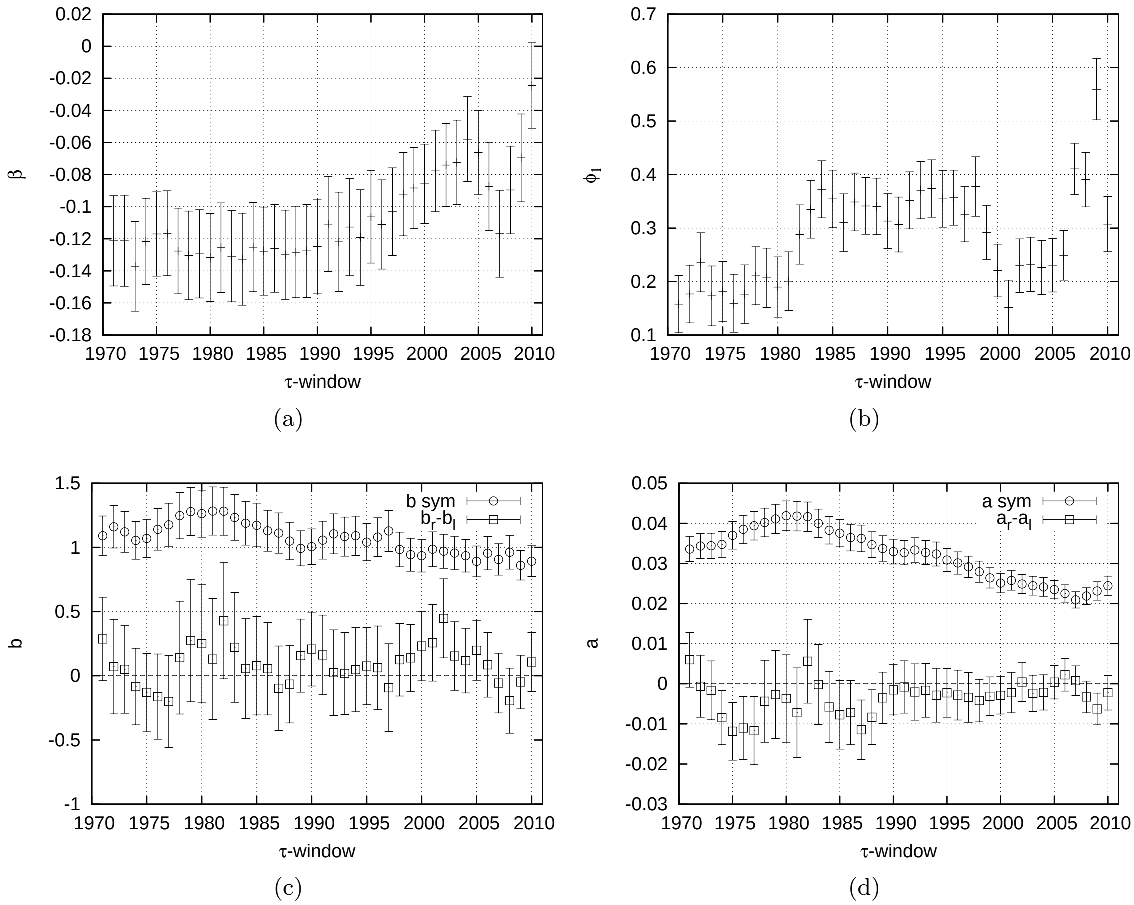
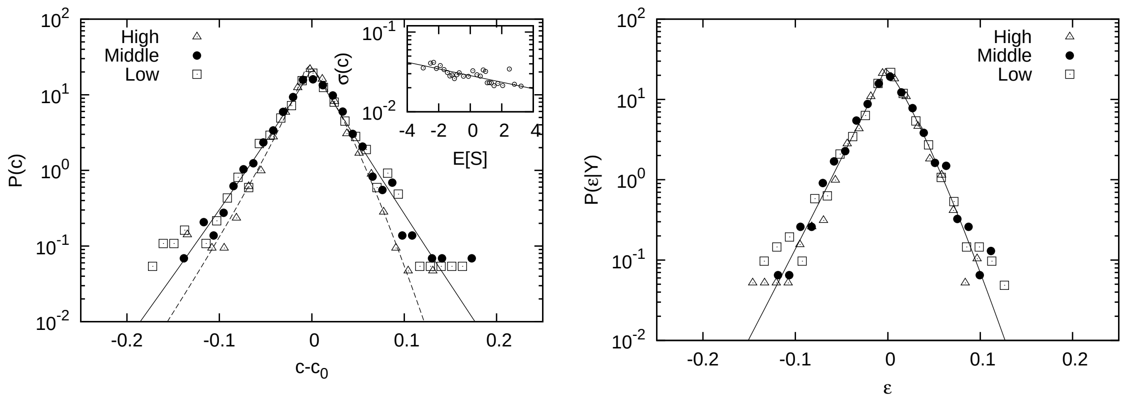
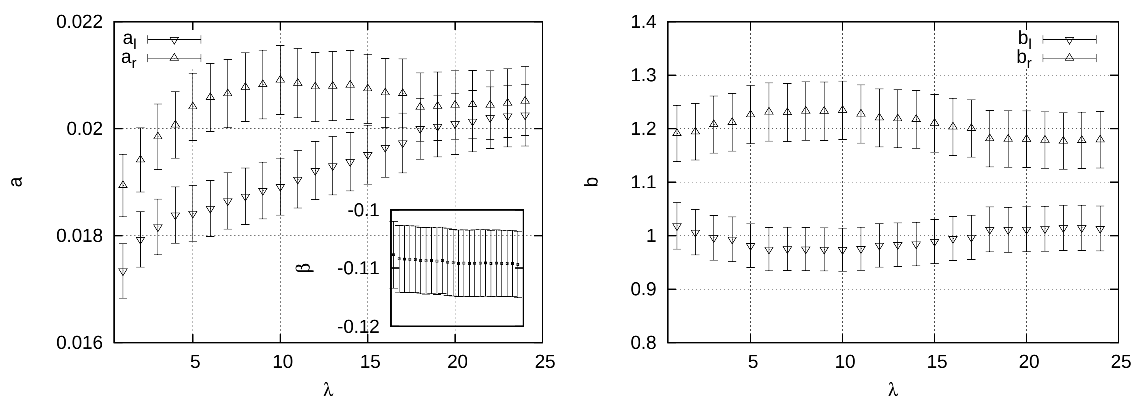


Figure 3: Dynamics of the estimated parameters for the rescaled growth rates. (a) Scaling β parameter. (b) Autoregressive ϕ_1 parameter. (c) Estimated Subbotin shape parameters, symmetric b and asymmetric comparison $b_r - b_l$. (d) Estimated Subbotin variance parameters, symmetric a and asymmetric comparison $a_r - a_l$. Error bars correspond to +/− two standard errors.



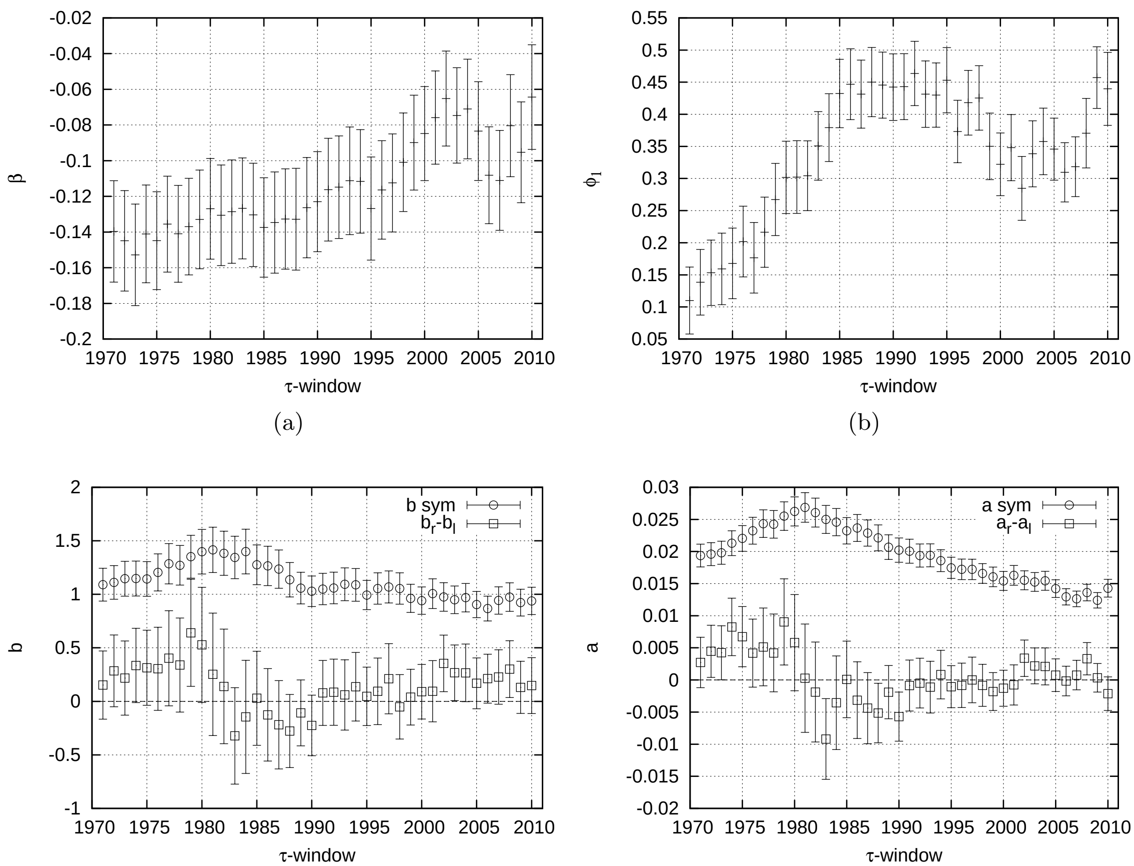
(a) (b)

Figure 4: (a) Empirical PDF of HP-cycles for different country classes; the subplot inside plots volatility vs. average of country sizes using bin statistics. (b) Empirical PDF of HP-cycles for different country classes. The continuous and dashed lines in plots represents the Subbotin estimation of the corresponding PDF.



(a) (b)

Figure 5: (a) Estimated EP parameters a_r and a_l vs. HP-Filter parameter λ; in the subplot the scaling parameter β vs. λ. (b) Estimated EP parameters b_r and b_l vs. λ. Error bars correspond to $+/-$ two standard errors.



(a) (b)

(c) (d)

Figure 6: Dynamics of the estimated parameters for HP-cycles. (a) Scaling β parameter. (b) Autoregressive ϕ_1 parameter. (c) Estimated Subbotin shape parameters, symmetric b and asymmetric comparison $b_r - b_l$. (d) Estimated Subbotin variance parameters, symmetric a and asymmetric comparison $a_r - a_l$. Error bars correspond to $+/-$ two standard errors.