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Working Paper Series

Nonfundamental Representations of the Relation between Technology Shocks and Hours Worked

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2008/09 April 2008

ISSN (online) 2284-0400

Nonfundamental Representations of the Relation between Technology Shocks and Hours Worked

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April 2008

Abstract

Estimating the response of hours worked to technology shocks is often considered as a crucial step for evaluating the applicability of macroeconomic models to reality. In particular, Galí [1999] has considered the conditional correlation between employment and productivity as a key tool for building an empirical evaluation of Real Business Cycle theories and New-Keynesian models. Impulse-response functions are often identified by means of Structural Vector AutoRegressive models. However, a structural Moving Average model of the economy cannot be estimated by VAR techniques whenever the agents' information space is larger than the econometrician's one, that is when we face a problem of nonfundamentalness. We consider how factor models can be seen as an alternative to VAR for assessing the validity of an economic model without having to deal with the problem of nonfundamentalness. We apply this method to the well known business cycle model by Galí [1999], which originally was estimated using a VAR, and retrieve alternative nonfundamental representations of the relation between technology shocks and hours worked. Such representations always yield a positive correlation between productivity and hours worked when conditioning on a technology shock. This result is more robust than the results by Christiano et al. [2004], because it is independent of the transformation used for hours worked and moreover is perfectly consistent with the unconditional correlation observed between the common components of the variables considered.

Keywords: technology, hours worked, factor models.
JEL-classification: C52, E24, E60 .

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We thank Marco Lippi for helpful comments and suggestions. The usual disclaimer applies.

1 Introduction

We find a positive conditional correlation between hours worked and labour productivity when estimating the impulse responses to a technology shock using a dynamic factor model. A positive correlation is found also when conditioning on a nontechnology shock. This result is independent of the transformation used for hours worked and is perfectly consistent with the positive unconditional correlation observed between the common components of the variables considered, as obtained by factor decomposition. If we consider the idiosyncratic (i.e. residual) component as a simple measurement error, then the relation between common components is the one we should be interested in for policy making. Our results are in contrast with those obtained by Galí [1999] and are more robust with respect to those obtained by Christiano et al. [2004] using VAR. The difference is imputable to the presence of nonfundamental representations of the model considered.

We typically face the problem of nonfundamentalness when dealing with structural models of the economy. Such a problem arises when the agents' information space is larger than the econometrician's one. Therefore, it is impossible for the latter to use standard econometric techniques, as Vector Autoregression (VAR), to estimate economic models. In this paper, we restate the conditions under which it is possible to invert a Moving Average (MA) representation in order to get a VAR. We then consider how factor models can be seen as an alternative to VAR for assessing the validity of an economic model without having to deal with the problem of nonfundamentalness. We apply this new method to the well known business cycle model by Galí [1999], which originally was estimated by means of a VAR model.

Galí proposes a model of the economy with sticky prices that is the benchmark of all the successive macroeconomic models aimed at explaining business cycle fluctuations. The main motivation of Galí is to provide a theory able to replicate the almost null observed unconditional correlation between labour productivity and labour input growth rates. Real-business-cycle (RBC) theorists assume a positive correlation when conditioning on a technology shock. In order to replicate these empirical findings, RBC models necessitate another shock (e.g. on government purchases or on preferences). The results on US data and the model by Galí instead point towards a negative correlation when conditioning on a technology shock and a positive one when conditioning on a nontechnology (monetary) shock. These results seem to be robust for different proxies of labour input. Namely, Galí considers hours per worker differentiated or not and total labour force always differentiated. However, more recently Christiano et al. [2004] show contradictory results for the impulse response of hours per worker to a technology shock. Differences appear when estimating the original VAR using hours per worker differentiated or in levels. The impulse response has a negative impact coefficient if we differentiate, while the impact coefficient is positive if we take levels.

In the next section we state the definition of fundamentalness in square systems. In section 3 we consider factor models as the tools for identifying economic shocks in structural models and we show how with this approach the problem of nonfundamentalness becomes nongeneric. In section 4 we present a method for estimating impulse response functions when using the factor models. In section 5 we review the results by Galí [1999] and Christiano et al. [2004], when considering the response of labour input to a technology shock. These results are compared with those obtained when using factor models. In section 6 we give some ideas for possible future developments of this work.

2 Nonfundamentalness and VAR models

When estimating structural models, we must always take into account the possibility of having a problem of nonfundamentalness, that is of representations that cannot be identified by the econometrician by means of VAR techniques. Consider an n-dimensional covariance stationary zero-mean vector stochastic process $\mathbf{x}_t$ of observable variables, driven by a q-dimensional unobservable vector process $\mathbf{u}_t$ of structural (i.e. with economic meaning) shocks. We can always write

$$\mathbf{x}_t = \mathbf{B}(L)\mathbf{u}_t \ , \tag{1}$$

where $\mathbf{B}(L) = \sum_{k=0}^{\infty} \mathbf{B}_k L^k$ is a one-sided polynomial in the lag operator L, in principle of infinite order. The shocks are orthogonal white noises: $\mathbf{u}_t \sim \text{w.n.}(0, \mathbf{\Gamma}_0^u)$, with $\mathbf{\Gamma}_0^u$ diagonal. In all what follows we assume that $\mathbf{x}_t$ has rational spectral density and therefore the entries of $\mathbf{B}(L)$ are rational functions of L. We define the k-th lag impulse response of the variable x_{it} to the shock u_{jt} as the (i, j)-th element of the matrix $\mathbf{B}_k$. Whenever $\mathbf{u}_t \in \overline{\text{span}} \{\mathbf{x}_{t-k}, k \geq 0\}$, we say that $\mathbf{u}_t$ is fundamental with respect to $\mathbf{x}_t$. If $n < q$ then it is almost impossible to obtain $\mathbf{u}_t$ from the present and past values of observed data, since we observe fewer series than the shocks we want to recover. Thus a necessary condition for fundamentalness is $n \geq q$.

We start by considering the square systems (in which the number of shocks is equal to the number of observed variables) and we provide the sufficient condition for fundamentalness.

Definition 1 (Fundamentalness in square systems) *Given a covariance stationary vector process $\mathbf{x}_t$, the representation $\mathbf{x}_t = \mathbf{B}(L)\mathbf{u}_t$ is fundamental if*

1. $\mathbf{u}_t$ *is a white noise vector;*
2. $\mathbf{B}(L)$ *has no poles of modulus less or equal than unity, i.e. it has no poles inside the unit disc;*
3. $\det \mathbf{B}(z)$ *has no roots of modulus less than unity, i.e. all its roots are outside the unit disc*

$$\det \mathbf{B}(z) \neq 0 \quad \forall z \in \mathbb{C} \quad \text{s.t.} \ \ |z| < 1 \ .$$

If the roots of $\det \mathbf{B}(z)$ are outside the unit disc, we have invertibility in the past (i.e. the inverse representation of (1) depends only on nonnegative powers of L) and we have fundamentalness. We can thus estimate a VAR for $\mathbf{x}_t$ and the residuals, once identified, are the real economic shocks we are looking for. However, if at least one of the roots of $\det \mathbf{B}(z)$ is inside the unit disc, we still have invertibility, and we also have nonfundamentalness. Since in this case the inverse representation of (1) depends also on negative powers of L, we can speak of invertibility in the future, and we thus cannot use standard techniques as VAR to identify the model. Finally, if there is one root on the unit circle, the representation is still fundamental but it is not invertible.

Nonfundamentalness appears in the literature in two ways: endogenously or exogenously. In the first case the model is by definition nonfundamental, this is the case of permanent income models (see Blanchard and Quah [1993] and Fernandez-Villaverde et al. [2007]) and rational expectations (see Hansen and Sargent [1980]). While in the exogenous case it is the way in which the dynamics of exogenous variables is specified which makes the model fundamental or not. An example of this latter case is in the debate between Blanchard and Quah [1989] and

Lippi and Reichlin [1993]. For a survey of the literature on nonfundamentalness, see Alessi et al. [2007].

3 Nonfundamentalness and factor models

Although the literature often considers nonfundamentalness as a minor problem at least in all practical cases, ruling out nonfundamental representations might hide the econometrician a large number of alternative possible meaningful representations of a given model. Typical models used by central banks, such as Dynamic Stochastic General Equilibrium (DSGE) models, involve expectations and are therefore potentially nonfundamental. When validating these models by means of VAR, there is a serious problem of identification of the structural shocks. We would like to find econometric models that do not have to bother with the problem of nonfundamentalness, but still are able to achieve identification of structural shocks. Dynamic factor models are a good tool for this latter purpose. In this section we outline how these models are built and how they deal with nonfundamentalness.

From the linearization of structural models, as for example DSGE models, we usually come up with a state space form for the observable variables $\mathbf{x}_t$ and the unobservable state-variables $\mathbf{F}_t$:

$$\begin{aligned} \mathbf{x}_t &= \mathbf{\Lambda}\mathbf{F}_t \, , \\ \mathbf{A}(L)\mathbf{F}_t &= \mathbf{H}\mathbf{u}_t \, . \end{aligned} \tag{2}$$

The dimension of $\mathbf{F}_t$ is $r \times 1$. The static rank of the system (i.e. the rank of the covariance of $\mathbf{x}_t$) is at most r and it depends on the size of the vector of exogenous shocks $\mathbf{u}_t$, which is q, and on the number of lags for each shock included in the model, which is s, the order of $\mathbf{A}(L)$. Therefore, the static rank depends on the structure of the economy. Empirically, we find reduced static rank, i.e. $r < n$, in the form of common cycles and, for this reason, it is assumed in most DSGE models. We can invert the second of (2) to obtain its MA representation

$$\mathbf{x}_t = \mathbf{\Lambda}(\mathbf{I} - \mathbf{A}(L))^{-1}\mathbf{H}\mathbf{u}_t = \mathbf{B}(L)\mathbf{u}_t \, . \tag{3}$$

From this equation we see that the dynamic rank of $\mathbf{x}_t$ (i.e. the rank of its spectral density matrix $\mathbf{\Sigma}^x(\theta)$) is q, it therefore depends on the number of exogenous forces. In general, for macroeconomic datasets $q < n$, which means that there is collinearity among the n variables. About the ranks notice that

$$\mathbf{\Sigma}^x(\theta) = \mathbf{B}(e^{-i\theta})\mathbf{\Gamma}_0^u\mathbf{B}(e^{i\theta})' \, ,$$

and since $\operatorname{rank}\mathbf{B}(L) = q$ the dynamic rank is q, while

$$\mathbf{\Gamma}_0^x = \mathbf{\Lambda}\mathbf{F}_t\mathbf{F}_t'\mathbf{\Lambda}' \, .$$

Therefore the maximum static rank is r. The reduced static and dynamic ranks are restrictions that come from the theory and that could be tested.

In principle we could now estimate the VAR $\mathbf{D}(L)\mathbf{x}_t = \boldsymbol{\varepsilon}_t$ where $\boldsymbol{\varepsilon}_t = \mathbf{H}\mathbf{u}_t$. However, to estimate this VAR we need $r = n$ in order to invert $\mathbf{\Gamma}_0^x$, which is almost never the case. Thus VAR estimation is not possible due to the reduced static rank of macroeconomic datasets.

There are two alternatives: either estimating a VAR only on blocks of r variables, or adding measurement errors. In the latter case we eliminate the collinearity among variables and we can estimate the full system, thus either we estimate a VARMA on the whole system, or we estimate a dynamic factor model. The last case is the one that we are interested in (see Giannone et al. [2006] for details on this and the other cases).

When adding orthogonal measurement errors $\boldsymbol{\xi}_t$, we lose the collinearity of the variables and we can write (3) as a dynamic factor model

$$\mathbf{x}_t = \mathbf{B}(L)\mathbf{u}_t + \boldsymbol{\xi}_t = \boldsymbol{\chi}_t + \boldsymbol{\xi}_t \ , \tag{4}$$

where $\mathbf{u}_t$ is the q-dimensional vector of common shocks. The usual assumptions hold, therefore $\mathbf{u}_t \sim \text{w.n.}(0, \mathbf{I}_q)$, and $\boldsymbol{\xi}_t$ is an idiosyncratic n-dimensional process of measurement errors s.t. ξ_{it-k} is orthogonal to u_{jt} for any i, j, and k. Moreover, the q largest eigenvalues of the spectral density matrix of $\mathbf{x}_t$ diverge as n goes to infinity, while the $(q+1)$-th is bounded almost everywhere for $\theta \in [-\pi, \pi]$. These assumptions are reasonable since measurement errors are supposed to vanish when considering linear combinations of many collinear variables. As a consequence, the common component $\boldsymbol{\chi}_t$ has reduced dynamic rank $q < n$, while $\boldsymbol{\xi}_t$ has full dynamic rank: this is how we break collinearity. Notice that the need of large cross sections to apply the factor model is perfectly consistent with the standard practice of central banks, which use all the available information when making decisions.
We can also add measurement errors to the state space form (2)

$$\begin{aligned} \mathbf{x}_t &= \boldsymbol{\Lambda}\mathbf{F}_t + \boldsymbol{\xi}_t \ , \\ \mathbf{A}(L)\mathbf{F}_t &= \mathbf{H}\mathbf{u}_t \ . \end{aligned} \tag{5}$$

Once again, given the previous assumptions, we have a common part with reduced static rank and an idiosyncratic part with asymptotically vanishing covariance that has full static rank. Therefore, when dealing with large cross sections we still have asymptotically reduced dynamic and static rank of the whole dataset $\mathbf{x}_t$. Hereafter we call $\mathbf{F}_t$ the static factors while $\mathbf{u}_t$ are the dynamic factors that correspond to the structural shocks of the economy. We want to identify $\mathbf{u}_t$ and the impulse responses that they generate.

What about nonfundamentalness? We can show that actually this is no more a generic problem in factor models and, under reasonable assumptions, we can always guarantee that the dynamic factors $\mathbf{u}_t$ are fundamental for $\mathbf{x}_t$ (see Forni et al. [2007]). In factor models we always have $n > q$, therefore we first need a definition of nonfundamentalness that generalizes definition 1 to the case of singular systems. It is indeed the singularity of dynamic factor models that makes the property of nonfundamentalness non generic.

Definition 2 (Fundamentalness in singular systems) *Given a covariance stationary vector process $\mathbf{x}_t$, the representation $\mathbf{x}_t = \mathbf{B}(L)\mathbf{u}_t$ is fundamental if:*

1. $\mathbf{u}_t$ *is a white noise vector;*

2. $\mathbf{B}(L)$ *has no poles of modulus less or equal than unity, i.e. it has no poles inside the unit disc;*

3. $\mathbf{B}(L)$ *has full rank inside the unit disc*

$$\text{rank } \mathbf{B}(z) = q \quad \forall\, z \in \mathbb{C} \quad \text{s.t. } |z| < 1 \,.$$

Alternatively, we can restate this last condition in terms of the roots of $\det \mathbf{B}(z)$*. We ask that the determinants of all the* $q \times q$ *submatrices of* $\mathbf{B}(z)$ *have no common roots inside the unit disc. More precisely, if we call* $\mathbf{B}_j(L)$ *the submatrices contained in* $\mathbf{B}(L)$ *and we define the set of indexes* $\mathbb{I} = \left\{ j \in \mathbb{N} \text{ s.t. } j = 1, \ldots, \binom{n}{q} \right\}$*, the definition of nonfundamentalness requires that*

$$\nexists\, z \in \mathbb{C} \;\text{ s.t.} \begin{cases} |z| < 1 \\ \\ \det \mathbf{B}_j(z) = 0 \quad \forall\, j \in \mathbb{I} \,. \end{cases}$$

As an example, consider the case $q = 1$. If $n = 1$ we are back to definition 1 and for fundamentalness we require that no root of $\mathbf{B}(z)$ is smaller than one in modulus. If instead $n > 1$ we have n polynomials $\mathbf{B}_j(z)$ and from definition 2 the representation is nonfundamental if they have a common root inside the unit disc. Thus, if $n = q$, nonfundamentalness is generic since if it holds in a point then, for continuity of the roots of $\mathbf{B}(z)$, it holds also in its neighborhood. While if $n > q$ nonfundamentalness is nongeneric because to have a common root we must satisfy $\binom{n}{q} - 1$ equality constraints. In singular models we usually have highly heterogeneous impulse responses of the variables to the few structural shocks, therefore it is highly improbable to have a common root for all of them, although it is not unlikely to have common roots for some submatrices $\mathbf{B}_j(L)$. Roughly speaking, although in principle the econometrician has a smaller information set than the agents' one (i.e. there is nonfundamentalness), he can supply the lack of information by observing additional series, and, if dynamic heterogeneity is guaranteed, then these series contain useful information. In macroeconomic datasets this is very likely to happen, thus fundamentalness in factor models is a reasonable property.

Summing up, the advantages of factor models, when we want to identify structural models of the economy, are mainly two.

1. Factor models are a natural representation of structural models as DSGE. $\mathbf{x}_t$ contains the observed variables of the model and some proxies of the state variables which are often unobserved and can be estimated as the latent static factors $\mathbf{F}_t$. Indeed, the typical macroeconomic variables included in the panel are indicators of economic activity built by aggregation, which can be seen as linear combinations of unobserved state variables (and their lags) plus some measurement errors. It is possible to impose structural relations between the observed $\mathbf{x}_t$ and the unobserved $\mathbf{F}_t$, i.e. to impose restrictions on $\mathbf{\Lambda}$. This is the approach used for example by Boivin and Giannoni [2006].

2. Structural models like DSGE make often use of expectations therefore their linearized state space form may suffer of the problem of nonfundamentalness. Factor models supply the econometrician with the information needed to achieve identification even of nonfundamental representations. This is possible thanks to the large cross section and heterogeneity in the responses of the variables to the structural shocks. Nonfundamentalness becomes then a nongeneric problem so that we do not need to be concerned about it anymore.

4 Estimation of impulse responses

Consider a large dataset $\mathbf{x}_t$ of dimension $n \times T$, containing the variables of interest for some economic model and other variables correlated with them. We can assume a factor structure written in the usual state-space form

$$\begin{aligned} \mathbf{x}_t &= \mathbf{\Lambda F}_t + \boldsymbol{\xi}_t \\ \mathbf{F}_t &= \mathbf{AF}_{t-1} + \mathbf{Hu}_t \,. \end{aligned} \tag{6}$$

The q-dimensional vector $\mathbf{u}_t$ is the vector of structural shocks we want to identify. We know from Stock and Watson [2006] that a consistent estimate $\hat{\mathbf{F}}_t$ of the r static factors is given by the static principal components of the observed variables, i.e. they are the projection of $\mathbf{x}_t$ on the eigenvectors $\mathbf{S}$ of the estimated covariance matrix. Following Giannone et al. [2004] and Forni et al. [2007], we obtain the estimates for the parameters of the model, the dynamic factors and their loadings as in the following:

$$\begin{aligned} \hat{\mathbf{\Lambda}} &= \mathbf{S}' \,, \\ \hat{\mathbf{A}} &= \mathbf{S}'\hat{\mathbf{\Gamma}}_1^x\mathbf{S}(\mathbf{S}'\hat{\mathbf{\Gamma}}_0^x\mathbf{S})^{-1} \,, \\ \hat{\mathbf{\Gamma}}_0^{Hu} &= \mathbf{S}'\hat{\mathbf{\Gamma}}_0^x\mathbf{S} - \hat{\mathbf{A}}\mathbf{S}'\hat{\mathbf{\Gamma}}_0^x\mathbf{S}\hat{\mathbf{A}}' = \mathbf{HH}' = \mathbf{M\Phi M}' \,, \\ \hat{\mathbf{H}} &= \mathbf{M\Phi}^{1/2} \,, \\ \hat{\mathbf{u}}_t &= \mathbf{\Phi}^{-1/2}\mathbf{M}'(\mathbf{I}_r - \hat{\mathbf{A}}L)\hat{\mathbf{F}}_t \,, \\ \hat{\mathbf{B}}(L) &= \hat{\mathbf{\Lambda}}(\mathbf{I}_r - \hat{\mathbf{A}}L)^{-1}\hat{\mathbf{H}} \,, \end{aligned}$$

where for a generic variable $\mathbf{y}$ the matrices $\hat{\mathbf{\Gamma}}_0^y$ and $\hat{\mathbf{\Gamma}}_1^y$ represent respectively the contemporaneous and lagged estimated covariance matrices.

In practice we truncate the MA(∞) representation of the last equation and we obtain a dynamic model for the common component $\boldsymbol{\chi}_t = \hat{\mathbf{B}}(L)\hat{\mathbf{u}}_t$. None of the terms on the right hand side is identified unless we impose $q(q-1)/2$ restrictions. Imposing restrictions is equivalent to fix a rotation matrix $\mathbf{\Omega}$. A typical restriction is long-run neutrality of one of the shocks.

As an example consider the case with $q = 2$, we thus need only one restriction to achieve identification. We impose neutrality of the second shock on the first variable. The required restriction can be restated as an equation for the rotation angle ϑ of the matrix

$$\mathbf{\Omega} = \begin{bmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{bmatrix} \,.$$

If we call $\hat{\mathbf{C}}(L)$ the structural impulse responses then $\hat{\mathbf{C}}(L) = \hat{\mathbf{B}}(L)\mathbf{\Omega\Omega}'\hat{\mathbf{u}}_t$, and the rotation angle satisfies

$$\operatorname{tg}\theta = -\left[\frac{\hat{\mathbf{B}}_{12}(1)}{\hat{\mathbf{B}}_{11}(1)}\right] \,.$$

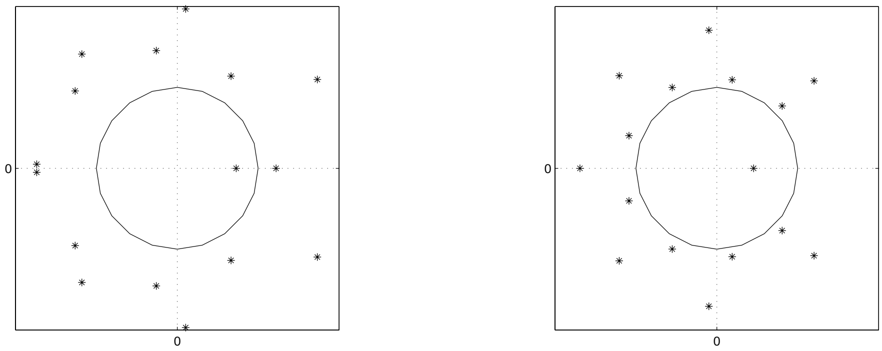
Once the impulse responses are identified, we can compare the result with a simple VAR by taking only the rows of $\hat{\mathbf{B}}(L)\mathbf{\Omega}$ corresponding to the variables of interest. In this way we restrict to a squared system analogous to the one obtained with a VAR. However, the way in which these functions are estimated implies that possible nonfundamental representations are taken into consideration.

With the method illustrated in this section we do not have to invert a VAR of observable variables in order to get impulse responses. In fact, we invert a VAR also in this case, but for static factors (i.e. the second equation of (6)). However, we know from Forni et al. [2007] that fundamentalness of dynamic factors with respect to static ones is a justifiable assumption provided that we have enough heterogeneity in the impulse responses. Indeed this fact translates into the assumption that in factor models the fundamentalness of shocks for the whole panel $\mathbf{x}_t$ is innocuous, as shown in the previous section, while nonfundamentalness for subpanels is still possible. Thus it may be the case that the squared subsystem that we interested in has a determinant with roots inside the unit disc.

5 The response of labour to a technology shock

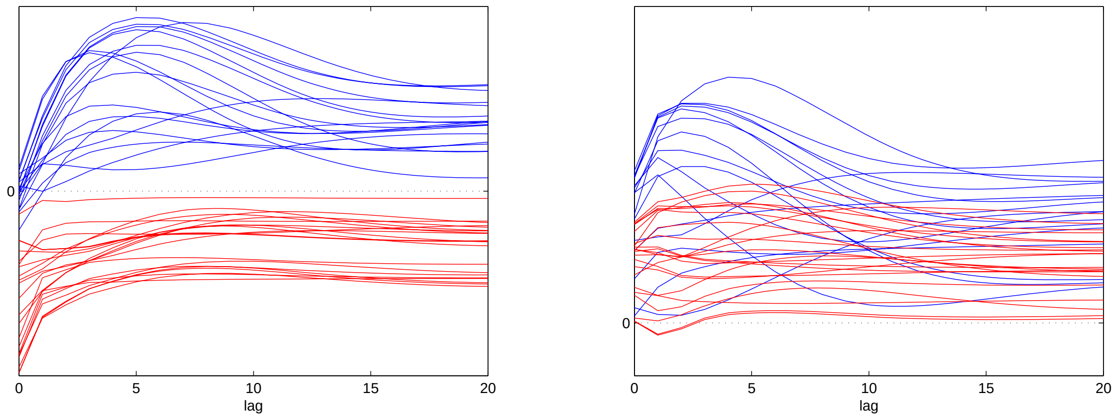
We look at the impulse response of labour input to a technology shock when using total employment (always differentiated), hours per worker differentiated and in levels. All the three series of labour input are downloaded from the US Bureau of Labor and Statistics, namely as total employment we take Total Civil Labour Force over 16 years old, while as hours worked we take the Index of Weekly Hours, and hours per worker are computed as hours worked divided by total employment.[1] We add to the original two variables a US macroeconomic dataset containing 135 monthly series (simlar to the ones used in factor model papers, e.g. Giannone et al. [2004]) and the series of quarterly GDP. We transform all monthly data in quarterly data, and the sample period chosen is from 1964:Q1 to 2006:Q12. The logarithm of the series of labour input and the logarithm of GDP are indicated respectively as l_t and y_t. The corresponding logarithm of labour productivity is computed as $\pi_t = y_t - l_t$. All variables are transformed to achieve stationarity, except for l_t when measured as hours per worker, which is taken either in levels or in first differences. In all three cases the criterion for the number of dynamic factors by Hallin and Liška [2007] suggests the presence of two common shocks, in agreement with the hypothesis made by Galí. The number of static factors r is, as usual, a more controversial issue, however results are robust for $r = 6, \dots, 18$. We report here results only for the case $r = 10$, which corresponds to the hypothesis that each dynamic factor is loaded by the observable series with 4 lags (i.e. one year). The variance explained by the common component is 79% and 68% for the series of hours worked in levels and differentiated respectively, while is just 31% for total employment. We thus concentrate mainly on the results obtained when using hours worked as labour input. We then apply to the whole dataset the estimation technique explained in previous section with the identification restriction of a vanishing long-run impulse response of productivity to the nontechnological shock.

[1] These series are not exactly the same used by Galí, due to unavailability of the original data. However, they are very similar. Moreover, the sample chosen is more recent, but, for a theory to be general enough, its predictions should hold for different sample periods.



(a) Hours in difference.   (b) Hours in levels.

Figure 1: Evidence of nonfundamental representations.

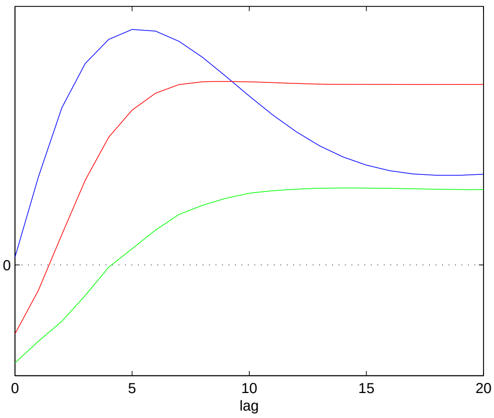


(a) Technology shock.   (b) Nontechnology shock.

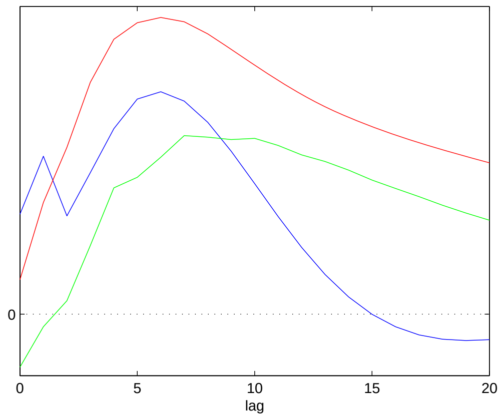
Figure 2: Evidence of dynamic heterogeneity in the impulse responses. Industrial production series: blue. Price series: red.

In figure 1 we plot the roots of det $\hat{\mathbf{C}}(z)$ once that identification is achieved.[2] In both cases at least one root always lies inside the unit disc. Different results from the ones in Galí [1999] and Christiano et al. [2004] can only be due to the presence of nonfundamental representations that can be identified thanks to the additional information coming from the other series in the panel. To show the dynamic heterogeneity in the responses of the observed variables to the structural shocks, we plot in figure 2 the impulse responses for two groups of variables: industrial production and price indexes. Within a group a variables the responses are similar, while between groups they differ. When we concentrate on the impulse responses of labour input to a technology shock we find interesting differences between our estimation method and the results obtained with a standard VAR. Indeed, when using factor models, the impact response of labour to a technology shock is always positive, while with VAR we obtain a negative response both when using employment and when using hours in first difference, and a positive response

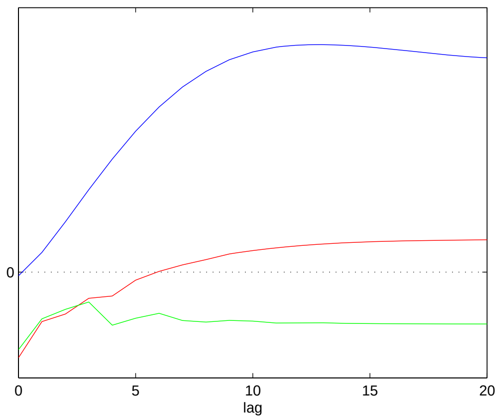
[2] Notice that identification is not a necessary step to determine if the roots are inside the unit disc. Indeed if a representation is nonfundamental before applying the orthogonal transformation $\boldsymbol{\Omega}$ it is still nonfundamental after. A fundamental representation can be obtained from a nonfundamental one (and viceversa) only by applying more complex transformations called Blaschke matrices, as for example in Lippi and Reichlin [1993].



(a) Hours in difference.



(b) Hours in levels.



(c) Employment.

Figure 3: Impulse responses to a technology shock. Factor model: blue. VAR: red. VAR on idiosyncratic component: green.

when using hours in levels. This result is well known in VAR literature (see Christiano et al. [2004]). Figure 3 shows the impulse responses, where we plot also those obtained by a VAR on the idiosyncratic components. Galí always finds a negative correlation in growth rates between labour productivity and labour input when conditioning on the technology shock. He explains the observed very small correlation between growth rates by introducing a monetary shock such that, when conditioning on it, we have positive correlation between the variables. Unconditional correlations are computed always on growth rates: $\text{corr}(\Delta y_t, \Delta l_t)$. Correlations conditional on the i-th shock are computed as in Galí [1999]. If the estimated impulse response matrix is $\hat{\mathbf{C}}(L) = \sum_{j=0}^{K} \hat{\mathbf{C}}_j L^j$, for a given maximum truncation lag K, then the conditional correlations are

$$\text{corr}(\Delta y_t, \Delta l_t | i) = \frac{\sum_{j=0}^{K} \hat{\mathbf{C}}_{j,1i} \hat{\mathbf{C}}_{j,2i}}{\sqrt{\sum_{j=0}^{K} (\hat{\mathbf{C}}_{j,1i})^2 \sum_{j=0}^{K} (\hat{\mathbf{C}}_{j,2i})^2}} .$$

We can also infer a theoretical correlation between the two variables defined as [3]

$$\text{corr}(\Delta y_t, \Delta l_t) = \frac{\sum_{j=0}^{K} \hat{\mathbf{C}}_{j,11}\hat{\mathbf{C}}_{j,21} + \sum_{j=0}^{K} \hat{\mathbf{C}}_{j,12}\hat{\mathbf{C}}_{j,22}}{\sqrt{\sum_{j=0}^{K}((\hat{\mathbf{C}}_{j,11})^2 + (\hat{\mathbf{C}}_{j,12})^2)\sum_{j=0}^{K}((\hat{\mathbf{C}}_{j,21})^2 + (\hat{\mathbf{C}}_{j,22})^2)}}.$$

When using factor models, we always find positive correlation when conditioning on the technology shock, which cannot be explained by the model of Galí. It seems that the results obtained with VAR estimation are mainly due to idiosyncratic components. Notice that when using hours in levels for the VAR we do not find the original result by Galí but the more recent result by Christiano et al. [2004]. This is a controversial issue raised in recent literature where the differentiation or not of hours leads to opposite conclusions when looking at the impact of technology on labour. If hours are taken in differences the negative conditional correlations point towards New Keynesian models with sticky prices, while if hours are taken in levels real-business-cycle seems to be the right paradigm for the positive conditional correlation.
We confirm the result of previous literature concerning the response to a monetary shock, but we find a different result for the response to a technology shock. Our results seem to be at odds with the observed unconditional correlation between growth rates of the whole series. However, if we consider the unconditional correlation between the estimated common components, we find a positive correlation. This happens when using as labour input hours per worker both in level or differentiated, while our model fails to explain the negative correlation between common components when using employment. The reason lies in a good factor decomposition when using hours per worker, and a bad decomposition when using total employment (only 30% of explained variance). Thus the issue is: are we interested in the impulse responses for the whole series or just for the common components? The answer depends on the meaning we attach to the idiosyncratic parts. When estimating a VAR on these latter, the conditional correlations are those observed in the literature. Table 1 summarizes all the results on correlations.

Our main result is: independently of the transformation used, a technology shock creates a positive correlation between hours per worker and productivity growth rates. Moreover, with factor models we replicate the unconditional correlation between common components, while with a VAR on idiosyncratic components we replicate the unconditional correlation between the whole series, as it happens with the traditional VAR by Galí. If we believe in the factor decomposition, the real indicator of the variable is given by its common part, while the idiosyncratic part is just noise or measurement error. We think that economic models should be aimed at studying the dynamics of the common component separated from the idiosyncratic one.

6 Further research

The results provided are only preliminary. We need now to check the significance of the correlation coefficients and to compute confidence intervals for the impulse response functions. Very often, models that apparently lead to completely different responses turn out to be statistically equivalent due to very large confidence bands. This fact again suggests the necessity of discussing the utility of VAR and impulse responses in general for discriminating between

[3] When using hours in levels the estimated impulse response is not relative to the labour input growth rate, thus first we transform the impulse response and then we compute the conditional correlation.

Labour input	**Unconditional empirical**		**Conditional on technology**		**Unconditional theoretical**		
	$\mathbf{x}_t$	χ_t	**VAR**	**Factor**	**VAR**	**Factor**	**idio**
Hours Difference	-0.02	0.41	-0.45	0.71	-0.02	0.72	-0.35
Hours Levels	-0.02	0.33	0.27	0.33	-0.15	0.14	-0.29
Employment	-0.29	-0.13	-0.77	0.20	-0.16	0.96	-0.36

Table 1: Unconditional and conditional correlations.

different models. We believe in factor decomposition given that it seems to be the most natural theory-free representation of macroeconomic data, usually it does not to bother with the problem of nonfundamentalness, and allows to use all the available information. In the empirical exercise presented above, the presence of nonfundamental representations seems to be the reason for finding new impulse responses which cannot be retrieved when using VAR. However, the direct relation between nonfundamental representations and different shapes of impulse responses is still in progress and is the main subject of our future research in this field.

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