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Towards an Evolutionary Interpretation of Aggregate Labor Markets Regularities

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Towards an Evolutionary Interpretation of Aggregate Labor Market Regularities*

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Abstract

Three well-known aggregate regularities (i.e. Beveridge, Wage, and Okun curves) seem to provide a quite complete picture of the interplay between labor market macro-dynamics and the business cycle. Nevertheless, existing theoretical literature still lacks micro-founded models which are able to account jointly for these three crucial stylized facts. In this paper, we present an agent-based, evolutionary, model trying to formalize from the bottom up individual behaviors and interactions in both product and labor markets. We describe as endogenous processes both vacancy and wage setting, as well as matching and bargaining, demand and price formation. Firms enjoy labor productivity improvements (technological progress) and are selected on the basis of their revealed competitiveness (which is also affected by their hiring- and wage-setting behaviors). Simulations show that the model is able robustly to reproduce Beveridge, Wage and Okun curves under quite broad behavioral and institutional settings. Moreover, the system generates endogenously an Okun coefficient greater than one even if individual firms employ production functions exhibiting constant returns to labor. Monte Carlo simulations also indicate that statistically detectable shifts in Okun and Beveridge curves emerge as the result of changes in institutional, behavioral, and technological parameters. Finally, the model generates quite sharp predictions about how system parameters affect aggregate performance (i.e. average GDP growth) and its volatility.

Keywords: Labor Markets, Dynamics, Aggregate Regularities, Beveridge Curve, Okun Curve, Wage Curve, Matching Models.

JEL Classification: J63, J64, O12, J41.

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1 Introduction

Over the last couple of decades, a quite large literature has been trying to investigate the process through which firms and workers meet in the labor market, how this matching process affects wage setting and (un)employment dynamics, and the extent to which unemployment and output interact over the business cycle¹.

Three well-known empirical aggregate regularities seem to provide a quite complete picture of the interplay between the forces at work. First, the Beveridge curve predicts a negative relationship between rates of vacancies and rates of unemployment. Second, the Phillips curve suggests that changes in wage rates are negatively related to unemployment rates. Alternatively, the Wage curve predicts a negative correlation between levels of real wages and unemployment. Third, the Okun curve posits a more than proportional increase in real GDP for every one percentage point reduction in the unemployment rate.

Most of the literature has tried so far to explain these phenomena on the grounds of a standard “toolbox” based on micro-foundations which postulate hyper-rational firms and workers. The “representative individual hypothesis” is often employed to overcome difficulties entailed by aggregation of heterogeneous agents. Moreover, static equilibrium conditions are largely used to interpret macroeconomic dynamics.

Despite their formal sophistication, the degrees of success of this class of models is, at best, mixed. In particular, existing literature seems to lack a joint explanation of the foregoing three aggregate regularities.

In this paper, we propose a radically different interpretative strategy. The model that we present in the following might be taken as an exploratory attempt to provide a micro-foundation of the interactions between labor-market and output dynamics from an evolutionary perspective².

The underlying philosophy builds on the acknowledgement that both firms and workers live in complex systems which evolve through time and might be characterized by endogenous, persistent, novelty. Agents are heterogeneous in their endowments, wealth, and, possibly, in their behavioral rules and rationality skills. Given the complexity of the environment they have to cope with - which changes endogenously as the outcome of individual behaviors - agents can only be boundedly-rational and hold an imperfect understanding of the system (Dosi, Marengo, and Fagiolo, 2004).

Expectations employed to revise control variables (e.g. demanded and offered wages,

¹For a quite exhaustive overview of the state-of-the-art of both theoretical and empirical labor market literature, cf. Ashenfelter and Layard (1986), Ashenfelter and Card (1999) and Petrongolo and Pissarides (2001).

²More on the general *Weltanschauung* of the evolutionary approach is in Dosi and Nelson (1994) and Dosi and Winter (2002). The model we present has large overlappings with the “Agent-Based Computational Economics” (ACE) approach (Tesfatsion, 1997; Epstein and Axtell, 1996; Aoki, 2003), as well as with self-organization models of labor markets pioneered by Lesourne (1992).

output produced, etc.) are typically assumed to be adaptive. Workers and firms interact directly and their choices are affected by those undertaken in the past by other agents. Interaction networks (e.g. matching rules in labor market) are themselves endogenous and may change across time. Firms interact both in the labor market and in the product market, wherein their revealed “competitiveness” is affected also by their hiring and wage-setting behaviors.

Macroeconomic dynamics is generated in the model via aggregation of individual behaviors. Typically, non-linearities induced by heterogeneity and far-from-equilibrium interactions induce a co-evolution between aggregate variables (employment, output, etc.). Statistical properties exhibited by aggregate variables might then be interpreted as emergent properties grounded on persistent micro disequilibria.

Consequently, even when some equilibrium relationship exists between aggregate variables (e.g. inflows and outflows from unemployment), the economy might persistently depart from it and follow some disequilibrium path. The observed stable relations amongst those same aggregate variables might emerge out of turbulent, disequilibrium, microeconomic interactions.

Here, making use of a model built on such premises, we shall address two types of questions. First, we shall ask whether the model is able to reproduce robustly over a large set of behavioral and institutional settings the main aggregate regularities that we observe in real-world labor-market data. For instance: Does our model generate jointly Beveridge, Wage, and Okun curves for a sufficiently large region of system parameters? Notice that this would lend support to a disequilibrium foundation of aggregate regularities: despite the fact that the economy always departs from equilibrium (if any), aggregate regularities emerge as the outcome of decentralized interactions, adaptive behavioral adjustments, and imperfect coordination.

Second, we shall try to map different behavioral and institutional settings into statistically distinct patterns of labor market dynamics. For example: Are there institutional and technological settings wherein the economy is *unable* to display robustly a downward-sloping relation between vacancy and unemployment rate? Under which conditions can one observe shifts of the Beveridge curve? And, similarly: Under which technological regimes the Okun curve displays a greater than one absolute elasticity?

The paper is organized as follows. In Section 2 we start by briefly surveying the main empirical findings about the foregoing three aggregate regularities. Next, we discuss how mainstream economic theory has been trying to provide explanations of such stylized facts. In Section 3, we present the model and we discuss the extent to which it departs from existing theoretical frameworks. Section 4 presents the results of simulation exercises. Finally, Section 5 draws some concluding remarks.

2 Individual Behaviors, Interactions, and Aggregate Regularities in Labor Market Dynamics: An Assessment of the State of the Art

2.1 A Brief Overview of Empirical Regularities

When dealing with labor market dynamics, a familiar angle of inquiry regards the extent to which “rigidities” and “frictions” are able to account for the observed unemployment levels (Phelps, 1972; Blanchard and Wolfers, 2000). In this respect, the Beveridge curve (BC) is a good starting point. The BC postulates a negative relationship (over time) between the rate of unemployment u and the rate of vacancies v , where rates are defined in terms of total employment³.

The intuition is simple: if an economy exhibits higher level of vacancies - in turn plausibly corresponding to a higher level of aggregate demand - it is easier for workers to find a job. Thus, one should also observe a lower level of unemployment. Movements along the curve should be typically induced by the business cycle. For instance, contractions should imply - *ceteris paribus* - a reduction in aggregate demand. This should in turn induce a decrease of vacancies and an increase in unemployment.

Moreover, the position of the BC in the (u, v) space is typically related to the degree of “frictions” existing in the labor market and, more generally, to its institutional setting: the closer the curve to the axes, the lower - *ceteris paribus* - market “frictions”. Shifts of the curve are attributed to all factors influencing: (a) *directly* the process of matching between vacancies and unemployed workers (e.g. unemployment benefit system, employment protection laws, active labor market policies, etc.); and (b) *indirectly* the (u, v) relationship via the impact they have on the wage rate (union power, union coverage, degree of coordination of wage bargaining, etc.)⁴.

Empirical findings seem to be quite controversial (Blanchard and Diamond, 1989; Nickell, Nunziata, Ochell, and Quintini, 2001; Belot and Ours, 2000; Fitoussi, Jestaz, Phelps, and Zoega, 2000). In fact, casual inspection of scatter plots of rough (u, v) data across countries does not show any clear-cut negative relationships. Even if some weakly downward-sloping curves do emerge across sub-samples, it seems that shifts and twists prevail. Heterogeneous cross-country patterns also emerge⁵. However, once controls for institutional

³Observation of reliable proxies for actual vacancies entails many empirical problems, especially in Europe, see Solow (1998). For instance, one is typically bounded to observe only *ex-ante* vacancies (i.e. job openings). *Ex-post* vacancies (i.e. unfilled job openings) are much more affected by frictions than *ex-ante* ones and thus should be in principle preferred as objects of analysis.

⁴On these points, cf. Nickell, Nunziata, Ochell, and Quintini (2001).

⁵All this would demand a careful discussion on what does we mean by “aggregate regularities” (i.e. “Is it any BC only in the eye of the observer?”) and their relationship with theory. This is, however, beyond the scope of this paper.

factors, time, and country dummies are introduced in panel-data regressions, then quite robust, statistically significant, negative elasticities between u and v typically emerge. That is, BCs emerge within data cells containing homogeneous groups of observations. Together, one observes that the impact of variables that *indirectly* affect unemployment through the wage rate is not significant. As far as shifts are concerned, it seems that all OECD countries present a shift to the right of the BC over time (implying higher “rigidities”). Nevertheless, after the 80’s, some countries, including Germany, Sweden, and Japan, seem to exhibit reverse patterns.

Econometric analyses have helped in highlighting the role of institutional variables in shaping the dynamics of jobs and vacancies. However, current analyses still display some major drawbacks. First, on the methodological side, econometric testing is typically not parsimonious. This could lead to the emergence of negative relationships only in over-homogeneous cells, thus weakening the “robustness” of the regularity. Second, unemployment and vacancy rates are computed as ratios between non-stationary variables, possibly entailing too much variability over time⁶. Finally, the role of technical progress is typically not investigated in econometric analyses and almost always treated as a business cycle effect.

A complementary empirical regularity is the famous Phillips curve, or the alternative Wage curve (Blanchflower and Oswald, 1994). As is known, they both posit the existence of co-movements between unemployment and wages. Two almost alternative worlds can be envisaged. If an economy experiments a negative relationship between *changes* of the wage rate and the unemployment rate, one is in a Phillips curve (PC) regime. Conversely, a Wage curve (WC) world is characterized by a negative relationship between *levels* of the wage rate and unemployment rates (Blanchard and Katz, 1997; Card and Hyslop, 1996)⁷.

Some remarks are in order. While the WC is typically taken as a proposition about homogeneous areas (e.g. regions or location-specific labor markets), the PC is assumed to bear a more general validity. Hence, the two may not be mutually exclusive: it is possible to think of homogeneous areas characterized by contemporaneous co-movements of both wage growth and levels in response of unemployment shifts. However, empirical studies (Blanchflower and Oswald, 1994; Card, 1995) show that, in homogeneous areas, WC is in general valid, while PC is not. This seems to be a quite robust finding, holding true across regions, countries, etc.. At the same time, the elasticity of wage levels to unemployment rates varies - although not dramatically - across regions and countries. Notice that since

⁶The denominator of both vacancy and unemployment rates is total employment (instead of labor force or population), which does not appear to be $I(0)$; cf. however Layard, Jackman, and Nickell (1991) for an alternative point of view. The choice of total employment is required if one wants to keep a tight relation with the BC theoretical counterpart modeled through a homogenous of degree one matching function (see below). In the model which follows, we define all rates in terms of total population (or labor force).

⁷For additional evidence on the wage vs. Phillips curves debate - especially concerning “wage-price spirals” - see Flaschel, Kauermann, and Semmler (2003) and references therein.

different wage-unemployment elasticities imply different degrees of responsiveness of wages to labor market conditions (as reflected by unemployment rate), workers can earn different wages - holding other conditions fixed - when they choose to work in regions with high or low unemployment rates.

As the WC pertains to homogeneous data cells, one cannot “see it” in rough data. Panel data estimation must be performed in order to control for variables such as personal characteristics of workers, labor market institutions, “fixed” effects allowing discrimination among sectors or regions, etc. . A strong results here is that a statistically significant, negative, relationship between the wage and unemployment rate still holds across different institutional setups (Borsch-Supan, 1991; Bleakley and Fuhrer, 1997).

The interpretation of a WC is quite controversial. In fact, Card (1995) prefers to argue about what a WC *is not*. In particular, a WC *is not* a Phillips curve, because it does not emerge as a misspecification of a PC regression. Moreover, a WC *is not* a supply function, as it cannot be obtained as a short-run inverted labor supply function (i.e. a relationship linking wage and unemployment through a given supply of labor in the short-run).

Nevertheless, once one has acknowledged the fact that the WC robustly emerges as an aggregate empirical regularity, some important implications follow. On the one hand, the market-clearing (equilibrium) interpretation underlying a PC cannot be invoked anymore. On the other hand, the competitive equilibrium framework does not easily account for WC emergence. In fact, a competitive labor market with all its canonical features would lead to a *positive* correlation between unemployment and the wage rate. Climbing up a downward demand for labor schedule - i.e. raising wage - would indeed induce higher levels of unemployment, as the unmet supply of labor would grow.

The third aggregate regularity we address here - i.e. the Okun curve (OC) - characterizes the interplay between labor markets and economic activity (Okun, 1962, 1970). In fact, one typically observes a negative, linear, relationship between changes in unemployment rate and GDP growth rates, with an absolute value of the slope larger than one. The standard interpretation⁸ runs as follows. Suppose that in the economy there is a under-utilization of labor resources with respect to the full employment level (i.e. unemployment rates are higher than the “natural” level). Then, the effect on economic activity of the cost associated to such under-utilization is more than proportional.

Therefore, whatever the causes, empirical evidence suggests an amplifying feedback between unemployment dynamics and output dynamics. A decrease of one percentage point in the unemployment rate - *ceteris paribus* - is associated with a growth rate of GDP of about two to three percentage points (according to original Okun estimations). Notice that a coefficient greater (less) than one entails some form of increasing (decreasing)

⁸Notice that an alternative interpretation can be given in terms of labor-productivity / unemployment changes (i.e. as a cyclical “Verdoorn-Kaldor” type of law), displaying rising productivity as unemployment falls.

returns. The “*ceteris paribus*” assumption is, however, far from innocent: it means that, over different periods of expansions and recessions, all other variables affecting GDP growth should remain nearly stable.

The debate about the existence (and the slope) of the Okun curve is not yet settled. First, the empirical value of the Okun coefficient (i.e. absolute value of the slope of the regression between changes in unemployment rate and GDP growth) is still a subject of controversy. So, for example, Prachowny (1993) challenges the “*ceteris paribus*” assumption and shows that taking into account all variables that increase GDP (e.g. changes in weekly hours, movements in capacity utilization, labor productivity) leads us to a decreasing returns regime: in Prachowny’s exercises, a 1-point decrease in unemployment rate only induces an increase in GDP of 0.66%. Conversely, Attfield and Silverstone (1997), by taking into account cointegration relationships between I(1) variables, recover an Okun coefficient in line with an increasing returns economy. Moreover, they show that additional control variables introduced by Prachowny are no longer significant when ECM (Error Correction Models) are employed and estimates are computed using dynamic OLS.

A second issue concerns whether the Okun coefficient is stable over time and across countries (Moosa, 1997; Sögner and Stiassny, 2000). Evidence shows that Okun coefficients are weakly stable over time but quite heterogeneous across countries. Moreover, the Okun relationship seems to be stronger in North-America than in Europe.

From a methodological point of view, the interpretation of the Okun curve must be carefully spelled out. The traditional interpretation is a static one. The joint bivariate process simply implies an invariant relationship with an implicit causality arrow going from economic activity to unemployment. Blanchard and Quah (1989) and Evans (1989) have instead challenged this interpretation and introduced some dynamics in a stationary bivariate VAR framework. Their aim was to consider the reversed effects from unemployment back to economic activity. Despite the fact that their estimations seem to support Okun’s conclusions, the bivariate system does not exhibit a clear-cut structural value for the elasticity between economic activity and unemployment. An implication is that the OC does not seem to be very robust in a dynamic perspective.

Finally, as it happens to both BC and WC, one typically faces a few important, data-related, problems. For example, while many econometric studies employ as measures of unemployment and GDP changes their deviations from some equilibrium values (i.e. “natural” unemployment rate and potential GDP, respectively), Okun’s original analysis was in terms of growth rates (Okun, 1962). In turn, the contemporary re-formulation might entail many estimation biases, e.g. those related to the estimation of “natural” levels. Furthermore, one has to assume that the unemployment rate and GDP are stationary around a deterministic trend (which instead might be stochastic). For all those reasons, in the following we shall use one-period growth rates instead of deviations.

2.2 Theoretical Explanations of Aggregate Labor-Market Regularities

Mainstream economic theory has been trying to explain the foregoing aggregate regularities in the familiar *equilibrium-cum-rationality* framework, building the explanation on the shoulders of hyper-rational, maximizing, representative worker and firm. Hence, any aggregate regularity is interpreted as the equilibrium outcome of some maximizing exercises carried out by such agents. Thus, even when the sign in the equilibrium correlation between any two aggregate variables (e.g. vacancy and unemployment rates) is derived from an intertemporal optimization problem, the hyper-rationality assumption allows one to compress the entire (infinite) stream of choices in a unique, simultaneous, decision implying non reversible, consistent, choices.

A paradigmatic example of such modeling strategy can be found within the theoretical literature aimed at micro-founding and explaining the BC. Suppose we start from a standard “matching model” (Pissarides, 2000; Blanchard and Diamond, 1989). Then, the total number of hires from unemployment (i.e. the number of matches) M in the economy can be given by $\varepsilon \cdot m(cU, V)$, where U is unemployment, V is the number of vacancies, c is search effectiveness of unemployed workers and ε is matching efficiency.

All search and matching, which in reality is an inherently dynamic process, is thus described in a static setting by means of a deterministic matching function m , which is assumed to be well-behaved, homogeneous of degree one, and increasing in both arguments. In equilibrium, given employment level N and the exogenous inflow rate into unemployment s , it is assumed that $sN = M = \varepsilon \cdot m(cU, V)$. Exploiting constant returns to scale, one thus gets a BC:

$$s = \varepsilon \cdot m(c \cdot u, v), \tag{1}$$

where $u = U/N$ and $v = V/N$ are unemployment and vacancy rates.

It is worthwhile noticing that the BC relationship is directly implied by the functional form and the parametric assumptions of the matching function m . In particular, the BC is treated here as a static (long-run) equilibrium locus in the $u - v$ space, requiring that all flows in and out of unemployment must always compensate⁹. Needless to say, this is at odds with any empirical observation.

Moreover, in order to get the desired results, many over-simplifying assumptions are required. *First*, the environment must be strictly stationary, ruling out any form of technological and organizational change, as well as any type of endogenous selection amongst firms and workers. *Second*, the presence of a hyper-rational, representative individual rules out the possibility of accounting for any form of heterogeneity across firms and workers.

⁹On the contrary, the model we present below allows the economy to evolve on a permanent disequilibrium path.

More than that: it excludes the very possibility of analyzing any *interaction* process among agents¹⁰. *Third*, as a consequence, one is prevented from studying the dynamic outcomes of multiple (reversible) decisions of hiring, firing, quitting, and searching which unfold over time.

Similar critiques also apply to the purported micro-foundations of Wage and Okun curves¹¹. Consider the Wage curve first. Since a competitive equilibrium market framework cannot account for a downward sloping equilibrium relationship between wage and unemployment rates (Blanchflower and Oswald, 1994; Card, 1995), other frameworks departing from perfect competition have to be devised in order to provide a rationale for this robust piece of aggregate evidence. Models generating a WC belong to two strands. First, *bargaining models* build on the idea that higher levels of joblessness produce lower bargaining power for workers and thus a reduced ability to elicit some kind of surplus. This effect can be amplified by the existence of a union in the labor market. This interpretation employs *implicit contract theory* and assumes that a contract does not only consist of a wage level, but also of some implicit temporary insurance against unemployment. Second, *efficiency wage models* (Shapiro and Stiglitz, 1984) assume that unemployment functions as a “discipline device” for workers. Other things being equal, higher unemployment levels induce a higher probability of job loss. Therefore, rational employees should exert a higher effort in the high-unemployment equilibrium, even if they receive a lower wage.

Note that, in these alternative WC models, what varies are the assumptions on what causes the departures from the perfect competition set-up, but they all continue to share a *rationality-cum-equilibrium*, static framework. Similar considerations apply to the *state-of-the-art* of contemporary interpretations of the Okun curve. Also in this case, the evidence is hard to reconcile with the “pure” neoclassical view in which one assumes that markets always clear: in such a setting, there is no easy way to generate downward-sloping relationships between unemployment changes and economic activity.

Since only structural and frictional unemployment is allowed to exist, a negative relation between unemployment and GDP growth is hard to sustain, insofar as it is difficult to assume that structural or frictional unemployment declines in upswings and increases in downswings. In general, theoretical explanations must rely on a careful and often *ad hoc* modeling of expectation formation. For instance, one could assume that in an upswing people searching for a new job still hold low wage aspirations and are therefore more willing to take a particular job. This should result in shorter search times in upswings and lower unemployment¹².

¹⁰In this respect, the far-reaching observations by Kirman (1992) on the pitfalls of any “representative agent” reduction of market interactions fully apply also to most contemporary models of the labor market.

¹¹Cf. Hahn and Solow (1997) for a thorough discussion on this and related points.

¹²See also Aghion and Howitt (1994) and Schaik and Groot (1998) for attempts to explain the OC within the framework of endogenous growth models.

Conversely, both a old-fashioned and a new Keynesian perspective allow us to explain Okun law in more straightforward ways. A possibility is to assume fixed prices and wages. Then, changes in aggregate demand induce firms to alter their output plans; labor demand changes and hence the unemployment rate is affected. Another possibility is to consider models of monopolistic competition (Blanchard and Kiyotaki, 1987) with menu costs (nominal rigidity) on the market for goods and real rigidities on labor market (e.g. efficiency wages): there, changes in aggregate demand can be easily shown to affect output and therefore unemployment¹³.

Notwithstanding the existence of some competing, although not entirely persuasive, interpretations of each of the three aggregate regularities *taken in isolation*, the economic literature witnesses a dramatic lack of theories attempting *jointly to explain* Beveridge, Okun and Wage curves. The over-simplifying assumptions needed in order to derive analytically-solvable models (to repeat: hyper-rational, optimizing representative agents, static frameworks, commitment to equilibrium, etc.) strongly constrain the possibility of providing a unified theory of the interplay between the microeconomics of labor market dynamics and the macroeconomics of unemployment and economic activity.

In the following, we begin indeed to explore a radically different path and study the properties of a model in which the most stringent assumptions of standard formalizations are abandoned, and we explicitly account for the processes of out-of-equilibrium interaction among heterogeneous agents.

3 An Evolutionary Approach to Labor Market Dynamics

3.1 The Model

Consider an economy composed of F firms and N workers¹⁴. Time is discrete: $t = 0, 1, 2, \dots$ and there is a homogeneous, perishable good g whose price is $p_t > 0$. In each period, a firm $i \in \{1, \dots, F\}$ produces q_{it} units of good g using labor as the sole input under a constant returns to scale (CRTS) regime:

$$q_{it} = \alpha_{it} n_{it}, \tag{2}$$

where α_{it} is the current labor productivity of firm i and n_{it} is the number of workers hired at t by firm i . Workers are homogeneous as far as their skills are concerned. If the firm

¹³An interesting by-product of this type of models is that productivity shocks can lead to OC as well. Indeed, GDP and employment move in the same direction as long as the effects of productivity shocks on efficiency-wages are not too strong.

¹⁴The ratio between the number of workers and the number of firms (N/F) can be interpreted as a measure of the concentration of economic activity.

offers a contractual wage w_{it} to each worker, current profits are computed as:

$$\pi_{it} = p_t q_{it} - w_{it} n_{it} = (p_t \alpha_{it} - w_{it}) n_{it}. \quad (3)$$

Contractual wages offered by firms to workers are the result of both a matching and a bargaining process. We assume that any firm i has at time t a “satisficing” wage w_{it}^s it wants to offer to any worker. Similarly, any worker $j \in \{1, \dots, N\}$ has at time t a “satisficing” wage w_{jt}^s which he wants to get from firms. Moreover, any worker j can only accept contractual wages if they are greater or equal to his *reservation wage* w_j^R , which we assume to be constant over time for simplicity.

We start by studying an economy where jobs last only one period. Hence, workers must search for a new job in any period. Job openings are equal to labor demand and, at the same time, to “ex-ante” vacancies. However, workers can be unemployed and firms might not satisfy their labor demand.

Let us turn now to a brief description of the flow of events in a generic time-period. We then move to a detailed account of each event separately.

Dynamics

Given the state of the system at the end of any time period $t - 1$, the timing of events occurring in any time period t runs as follows.

1. Firms decide how many jobs they want to open in period t .
2. Workers search for a firm posting at least one job opening and queue up.
3. Job matching and bargaining occur: firms look in their queues and start bargaining with workers who have queued up (if any) to decide whether to hire them or not.
4. After hiring, production takes place according to eq. (2). Aggregate supply and demand are then formed simply by aggregating individual supplies and demands. Subsequently, a “pseudo-Walrasian” price setting occurs. We assume that the price of good g at t is given by:

$$p_t Q_t = W_t, \quad (4)$$

where $Q_t = \sum_{i=1}^F q_{it}$ is aggregate (real) output and $W_t = \sum_{j=1}^N w_{jt}$ is total wage. Thus, total wage equals aggregate demand, as we assume that workers spend all their income to eat good g in any time period. Then, firms make profits:

$$\pi_{it} = (p_t \alpha_{it-1} - w_{it}) n_{it}.$$

5. Given profits, firms undergo a selection process: those making negative profits ($\pi_{it} < 0$) exit and are replaced by entrants, which, as a first approximation, are simply “average” firms (see below).
6. Firms and workers update their satisficing wages (w_{it-1}^s and w_{jt-1}^s).
7. Finally, technological progress (if any) takes place. We assume that in each period labor productivity may increase at rates which are exogenous but firm-specific (see below).

Job Openings

At the beginning of period t , each firm creates a queue of job openings. Since in reality only *ex-ante* vacancies (i.e. new job positions) can be empirically observed, we will employ throughout the term job openings as a synonym of (ex-ante) vacancies. “Ex-post” vacancies will be computed as the number of unfilled job-openings.

Let us then call v_{it} the number of new positions opened by firm i at time t . As far as the firm’s decision about how many vacancies to open is concerned, we experiment with two alternative “behavioral” scenarios.

In the first one, a firm simply observes current (i.e. time $t-1$) price, quantity produced and contractual wage offered, and sets vacancies v_{it} as:

$$v_{it} = \bar{v}_{it-1} = \left\lceil \frac{p_{t-1}q_{it-1}}{w_{it-1}} \right\rceil, \quad (5)$$

that is, it creates a queue with a number of open slots equal to the “ceiling” of (i.e. the smallest integer larger than) the ratio between revenues and the contractual wage offered in the last period. We call this job opening scenario the “**Wild Market Archetype**”, in that no history-inherited institution or behavioral feature is built into the model.

In the second “behavioral” scenario (which we shall call the “**Weak Path-Dependence**” scenario), we introduce some rather mild path-dependence into the vacancy setting. We suppose that: (a) jobs opened by any firm at time t are a non-decreasing function of last-experienced profits growth rate; and (b) cannot exceed \bar{v}_{it-1} . More formally:

$$v_{it} = \min\{\bar{v}_{it-1}, v_{it}^*\}, \quad (6)$$

and:

$$v_{it}^* = \begin{cases} \lceil v_{it-1}(1 + |X|) \rceil, & \text{if } \frac{\Delta\pi_{it-1}}{\pi_{it-1}} \geq 0 \\ \lceil v_{it-1}(1 - |X|) \rceil, & \text{if } \frac{\Delta\pi_{it-1}}{\pi_{it-1}} < 0 \end{cases}, \quad (7)$$

where X is an i.i.d. random variable, normally distributed with mean zero and variance $\sigma_v^2 > 0$, and $\lceil x \rceil$ denotes the ceiling of x . Notice that the higher σ_v , the more firms react

to any given profits growth rate by enlarging or shrinking their current queue size. Hence a *higher* σ_v implies *higher* sensitivity to market signals. Notice that, in both scenarios, firms always open at least one vacancy in each period.

Job Search

In our model, workers can visit in any time period only one firm. Similarly to job opening, we consider two “behavioral” scenarios for the job search procedure employed by workers to find a firm that has just opened new job positions. In the first one, called “**No Search Inertia**”, each worker j simply visits any firm i in the market with a probability proportional to the last contractual wage w_{it-1} offered. If the selected firm has places still available in the queue, the worker gets in and demands a wage equal to the “satisficing” one, i.e. w_{jt-1}^s .

In the second scenario, which we label “**Search Inertia**”, we introduce some stickiness (loyalty) in firm visiting. If worker j was employed by firm i in period $t - 1$, he visits first firm i . If i still has places available in the queue, the worker gets in and demands w_{jt-1}^s . Otherwise, the worker employs the random rule above (“No Search Inertia”) to select among the remaining $F - 1$ firms.

In both scenarios, a worker becomes unemployed if he chooses a firm that has already filled all available slots in its queue.

Job Matching and Bargaining

After workers have queued up, firms start exploring workers wage demands to match them with their *desiderata*. Suppose that, at time t , firm i observes $0 < m_{it} \leq N$ workers in its queue. Then, it will compute the average wage demanded by those workers:

$$\bar{w}_{it} = \frac{1}{m_{it}} \sum_{h=1}^{m_{it}} w_{j_h t-1}^s, \quad (8)$$

where j_h are the labels of workers in i 's queue. Next, it sets the contractual wage for period t as a linear combination of \bar{w}_{it} and the satisficing wage w_{it-1}^s . Thus:

$$w_{it} = \beta w_{it-1}^s + (1 - \beta) \bar{w}_{it}, \quad (9)$$

where $\beta \in [0, 1]$ is an institutional parameter governing firms' strength in wage bargaining. A higher β implies a higher strength on the side of the firm in wage setting. If $\beta = 0$, firms just set contractual wage as the average of wages demanded by workers in the queue. If $\beta = 1$, firms do not take into account at all workers' *desiderata*.

Once the firm has set the contractual wage at which it is willing to hire workers in the queue, any worker j in the queue will accept the job only if w_{it} exceeds the reservation

wage w_j^R .

As soon as a worker j accepts the job, he temporarily changes his satisficing wage to keep up with the new (actual) wage earned, i.e. $w_{jt-1}^s = w_{it}$. Similarly, a firm who has filled at least a job opening will replace w_{it-1}^s with w_{it} ¹⁵.

Given the number of workers n_{it} hired by each firm, production, as well as price setting and profits determination occur as explained above. *Ex-post* firm i 's vacancies are defined as $\tilde{v}_{it} = m_{it} - n_{it}$.

Selection, Exit, and Entry

Suppose that - given the new contractual wage, price p_t , and current productivity α_{it-1} - firm j faces negative profits, i.e. $p_t \alpha_{it-1} < w_{it}$. Then selection pressure makes firm j exit the market.

Each exiting firm is replaced by a new firm which starts out with the average “characteristics” of those firms still in the market at t (i.e. those making non-negative profits)¹⁶. Notice that this entry-exit process allows to keep an invariant number of F firms in the economy at each t .

Satisficing Wages Updating

Surviving firms, as well as the N workers, will then have the opportunity to revise their satisficing wage according to their perceptions about the outcome of market dynamics.

- **Firms:** We assume that each firm has an invariant desired ratio of filled to opened jobs $\rho_i \in (0, 1]$ which it compares to the current ratio:

$$r_{it} = \frac{n_{it}}{v_{it}}.$$

If firm i hired too few workers (as compared to the number of job positions it has decided to open), then it might want to increase the wage it is willing to offer to workers. Otherwise, it might want to decrease it. We capture this simple rule by positing that:

$$w_{it}^s = \begin{cases} w_{it-1}^s(1 + |Y|) & \text{if } r_{it} < \rho_i \\ w_{it-1}^s(1 - |Y|) & \text{if } r_{it} \geq \rho_i \end{cases}, \quad (10)$$

where Y is an i.i.d. random variable distributed as a standard normal. Notice that w_{it-1}^s is equal to w_{it} (i.e. contractual wage just offered) if the firm has hired at least one worker.

¹⁵These new values of satisfying wages will then be employed in the updating process. Since satisfying wage can be interpreted as (myopic) expectations, satisfying wage updating plays in the model the role of expectation formation process.

¹⁶All results we present in the next Section are robust to alternative assumptions concerning entry and exit.

- **Workers:** If worker j remains unemployed after matching and bargaining, he might want to reduce his satisficing wage (without violating the reservation wage threshold). Otherwise, he might want to demand a higher wage during the next bargaining session. We then assume that:

$$w_{jt}^s = \begin{cases} \max\{w_j^R, w_{jt-1}^s(1 - |Y|)\} & \text{if } j \text{ unemployed} \\ w_{jt-1}^s(1 + |Y|) & \text{if } j \text{ employed} \end{cases}, \quad (11)$$

where Y is an i.i.d. random variable distributed as a standard normal. Again, $w_{jt-1}^s = w_{jt}$ if j has been just hired.

Technological Progress

The last major ingredient of the model regards labor productivity dynamics. Here, we experiment with two “technological scenarios”. In the first one (“**No Technological Progress**”), we study a system where labor productivity does not change through time (i.e. $\alpha_{it} = \alpha_i, \forall i$)¹⁷. In the second scenario (“**Technological Progress**”), we allow for an exogenous, albeit firm-specific, dynamics of labor productivities. We start with initially homogeneous labor coefficients ($\alpha_{i0} = \alpha$) and we let them grow stochastically over time according to the following multiplicative process:

$$\alpha_{it} = \alpha_{it-1}(1 + Z), \quad (12)$$

where Z , conditionally on $Z > 0$, is an i.i.d. normally distributed random variable with mean 0 and variance $\sigma_Z^2 \geq 0$ ¹⁸. The latter governs the opportunity setting in the economy. The larger σ_Z , the more likely firms draw large productivity improvements. Notice that if we let $\sigma_Z = 0$ we recover the “**No Technological Progress**” scenario.

3.2 Initial Conditions, Micro- and Macro-Dynamics

The foregoing model, as mentioned, genuinely belongs to an evolutionary/ACE approach. Given its behavioral, bottom-up, perspective, one must resort to computer simulations to explore the behavior of the system¹⁹. One of the main goals is to look for meta-stable properties (and rarely to equilibria in the traditional sense) which emerge as the result of the co-evolution among individual behaviors over time and persist for sufficiently long time spans.

¹⁷Labor productivity may in turn be either homogeneous across firms ($\alpha_i = \alpha$) or not.

¹⁸Hence, there is a probability 0.5 to draw a neutral labor productivity shock ($Z = 0$), while positive shocks are distributed as the positive half of a $N(0, 1)$.

¹⁹Simulation code is written in C++ and is available from the Authors upon request.

In our model, the dynamics of the system depends on four sets of factors. *First*, we distinguish behavioral (e.g. concerning job opening and job search) and technological scenarios. We call such discrete institutional and technological regimes “system setups”. *Second*, a choice of system parameters (F/N , σ_v , β , σ_Z) is required (see Table 1).

Parameter	Range	Meaning
N/F	R_{++}	Concentration of economic activity (Number of Workers / Number of Firms)
σ_v	R_{++}	Sensitivity to market signals in vacancy settings (only in a Weak Path-Dependence Scenario)
β	$[0, 1]$	Labor-market institutional parameter governing the strength of firms in wage-setting
σ_Z	R_+	Technological parameter tuning the availability of opportunities in the system (= 0 means no technological progress)

Table 1: System Parameters

Third, one should explore the would-be importance of different initial conditions²⁰. Since simulations show that the latter do not dramatically affect the long-run properties of aggregate variables, we typically define a “canonical” set of initial conditions. All results presented below refer to this benchmark choice. Finally, individual updating by firms and workers induces a stochastic dynamics on micro-variables (e.g. contractual wages, desired production, desired employment, etc.). By aggregating these individual variables over firms and workers, one can study the properties of macro-dynamics for the variables of interest. We will focus on unemployment:

$$U_t = N - \sum_{i=1}^F n_{it}, \quad (13)$$

vacancies:

$$V_t = \sum_{i=1}^F v_{it}, \quad (14)$$

output price p_t , total wages:

$$W_t = \sum_{j=1}^N w_{jt}, \quad (15)$$

²⁰In the model this implies defining initial values $(n_{i0}, \alpha_{i0}, w_{i0}^s, w_{i0})_{i=1}^F$ for firms and $(w_{j0}^s)_{j=1}^N$ for workers. Moreover, an initial price p_0 , and some distributions for desired ratios $(\rho_i)_{i=1}^F$ and reservation wages $(w_j^R)_{j=1}^N$ have to be chosen.

and (real) GDP:

$$Q_t = \sum_{i=1}^F q_{it}, \quad (16)$$

as well as its growth rate:

$$h_t = \Delta \log(Q_t). \quad (17)$$

Related Literature on Matching and Labor-Market Dynamics: A Necessary Digression

One of the key features of the foregoing model is an explicit microfoundation - within an evolutionary framework - of labor market dynamics regarding the processes governing e.g. job opening, job search, matching, bargaining, and wage setting.

Standard theoretical literature on matching in labor markets, as mentioned above, has typically abstracted from any explicit account of decentralized interaction patterns. For example, matching models based upon a “search equilibrium” framework²¹, while stressing the existence of frictions and imperfect information in labor markets, have implicitly assumed a sort of centralized, equilibrium, device matching the “representative firm” and the “representative worker” (eq. 1 stands precisely for that). Wage setting is then often assumed to be a Nash bargaining process. Given these strong assumptions, as well as the restrictions on the shape of the matching function itself, it is not surprising that the model delivers, e.g., Beveridge curves.

The bottom line of the exercises belonging to the “pure equilibrium” *genre* is that they turn out to be unable, almost by construction, to account for involuntary unemployment or even endogenous changes in the “equilibrium” rates of unemployment. Important advances, incrementally departing from the standard model, have tried to incorporate agents’ informational limitations, in order to account for phenomena such as endogenous fluctuations in aggregate activity and persistent involuntary unemployment (see e.g. the seminal work by Phelps and Winter (1970) and Phelps (1994)).

More recently, some efforts have been made to depart from exogenous and deterministic matching devices and assume some “endogenous matching” mechanism to describe the (Walrasian) decentralized process governing the meetings between firms and workers in the labor market²². For instance, Lagos (2000) studies an ex-ante frictionless and random decentralized matching process, while Peters (1991) describes wage offers as a sequential game with incomplete information where firm strategies can influence the search behavior of the workers. The main goal of these contributions is to study under what conditions a centralized, well-behaved, matching function can be ex-post generated, in equilibrium, by

²¹See *inter alia* Pissarides (2000), Petrongolo and Pissarides (2001), Mortensen (1986) and Mortensen and Pissarides (1994).

²²See Lagos (2000), Peters (1991), Cao and Shi (2000), Burdett, Shi, and Wright (2001), Smith and Zenou (2003) and Julien, Kennes, and King (2000).

some decentralized, endogenous matching function. An important conclusion is that, if such centralized matching device exists, then its properties heavily depend on the fine details of market organization and institutional setups (and thus also on policy interventions).

This is certainly a point our model takes on board in its full importance, and it does so through an explicit account of the (disequilibrium) unfolding of the interaction process. In this respect, our model has three important antecedents in labor market literature. *First*, the out-of-equilibrium, interaction-based perspective that we pursue is a distinctive feature of “self-organization” labor market models²³. They assume heterogeneous, boundedly rational workers and firms meeting at random over time in institutionally-shaped labor markets. For given institutional arrangements, the system self-organizes in long-run configurations where different unemployment and wage levels emerge as the result of individual choices and interactions. *Second*, the ACE model in Tesfatsion (2001) also assumes many heterogeneous, interacting agents, characterized by “internal states” and behavioral rules, who exchange information in the market. Matching occurs in a decentralized way through a one-sided offer auction and individual work-site payoffs are modeled as in a Prisoner-Dilemma game. *Third*, Aoki (2003) extends the ACE model of fluctuations and growth proposed in Aoki and Yoshikawa (2003) to allow for unemployment dynamics. Similarly to our model, co-evolution between product and labor market dynamics is explicitly taken into account and simulations allow to reproduce (albeit in some benchmark parameterizations) Okun curves. However, matching and wage bargaining are not incorporated in the model as endogenous processes. Therefore, no implications about wage and Phillips curves can be derived from simulation exercises.

Notwithstanding many overlappings with “self-organization” and ACE formalizations, our model proposes advances, *vis-à-vis* the state of the art in this area, on at least four levels. *First*, it accounts for the co-evolutionary dynamics between the labor market and the product market. More specifically, we try to nest labor market interactions in what one could call a “general disequilibrium” framework with endogenous aggregate demand. This feature allows us to study market properties associated with an endogenous business cycle. *Second*, we explicitly model (as endogenous processes) job opening, matching, wage bargaining, and wage setting. *Third*, we allow for technical progress and the ensuing macroeconomic growth. *Fourth*, in the analysis of the results, we go beyond an “exercise in plausibility” and we explicitly compare the statistical properties of the simulated environments with empirically observed ones, specifically with respect to the emergence of Beveridge, Wage, and Okun curves.

²³Cf. Lesourne (1992) and Laffond and Lesourne (2000). Self-organizing processes are discussed in Witt (1985).

4 Simulation Results

The general strategy of our simulation experiments runs as follows. First, we attempt to identify some general conditions (i.e. setups and parameters choices) under which the model is able *jointly* to replicate the three aggregate regularities characterizing labor markets dynamics and economic activity discussed in Section 2.

Second, in order to wash out stochastic effects in micro- and macro-dynamics specific to single sample paths, we perform Monte Carlo exercises so as to understand how the statistical properties of labor-market dynamics and economic activity change across different parameterizations and setups.

4.1 Simulation Setups

All simulation exercises we present in the paper refer to (and compare) the following behavioral and institutional scenarios, and combinations thereof:

1. **Walrasian Archetype (WA)**: This economy is characterized by the “**Wild Market Archetype**” scenario as far as job opening is concerned and the “**No Search Inertia**” scenario for workers’ job search. In this world, there is no path-dependence in job openings, nor in job search. Workers visit firms at random, while the latter open a number of new positions in each period without being influenced by past experienced profits.
2. **Institutionally-Shaped Environment (ISE)**: In this economy, workers and firms face some path-dependence in job opening and job searching. We assume that firms open new job positions within a “**Weak Path-Dependence**” scenario (i.e. they adjust job openings according to last profits growth), while workers search for a firm under the “**Search Inertia**” scenario (i.e. they try to stick to the last firm in which they were employed).

Each of the two foregoing behavioral choices can be associated with a different technological scenario (with or without technological change), in order to define a “system setup”. Table 2 summarizes the four “worlds” which we extensively explore in our simulation exercises²⁴.

²⁴In all exercises that follow, we set the econometric sample size $T = 1000$. This time span is sufficient to allow for convergence of the recursive moments for all variables under study.

Setups	Label	Job Opening	Job Search	Tech. Progress
1	Walrasian Archetype w/o Tech. Progress	Wild Market Archetype	No Search Inertia	NO
2	Walrasian Archetype w/ Tech. Progress	Wild Market Archetype	No Search Inertia	YES
3	Institutionally- Shaped Environment w/o Tech. Progress	Weak Path- Dependence	Search Inertia	NO
4	Institutionally- Shaped Environment w/ Tech. Progress	Weak Path- Dependence	Search Inertia	YES

Table 2: System Setups

4.2 Some Qualitative Evidence

We start by investigating from a qualitative perspective the emergence of Beveridge, Wage, and Okun curves in an economy characterized by the “Walrasian Archetype”.

In this world where agents decide myopically and do not carry over past information, the system does not allow the recovery of any aggregate, statistically significant, negative relationship between vacancy and unemployment rates. Simulations show that, irrespective of the technological scenario, the Beveridge curve does not emerge (cf. Figs. 1 and 2) in a large region of the system parameters $(F/N, \beta, \sigma_Z)$ space.

Notwithstanding the fact that matching and search do not seem to affect the (u, v) relation, the unemployment rate turns out to be negatively related to wage levels. Moreover, higher unemployment growth entails smaller GDP growth. Therefore, both Wage and Okun curves robustly emerge no matter whether technological progress is shut down or not. Notice that if $\sigma_Z = 0$, the economy works as a dynamic allocation device trying to match in a decentralized and imperfect way individual labor demand and supply *for given resources*. It is then easy to see that both Okun and Wage relationships are a consequence (and not an emergent property) of the joint assumptions of quasi-Walrasian price-setting and constant returns to scale. Indeed, from (2) and (4), one gets: $W_t = -p_t U_t + p_t(N - N_t + \sum_i \alpha_i n_{it})$ and $Q_t = -U_t + (N - N_t + \sum_i \alpha_i n_{it})$. Thus, if $\alpha_i \neq \alpha$, both curves are implied by the assumptions. In particular, one should observe a unit coefficient for the wage curve. If labor productivities are heterogeneous, one should instead observe for both WC and OC some noise around negatively sloped lines.

If, on the contrary, technological progress occurs in a WA scenario, there is no apparent reason to expect both OC and WC to emerge robustly. Yet, as simulations show, they both characterize system dynamics for a large region of the parameter space, even if no path-dependent behavior drives the economy (cf. Figs. 3 and 4).

Consider now an economy in which firms are influenced by past profits when they

adjust vacancies and workers try to stick to previous employers (i.e. what we call an “Institutionally-Shaped Environment”). Then, irrespective of the technological regime, the model is able robustly to generate Beveridge curves with statistically significant (negative) slopes: see Figs. 5 and 6. Furthermore, when technological progress is present, both Wage and Okun curves still characterize macro-dynamics as robust, emergent, properties of the system, cf. Figs. 7 and 8.

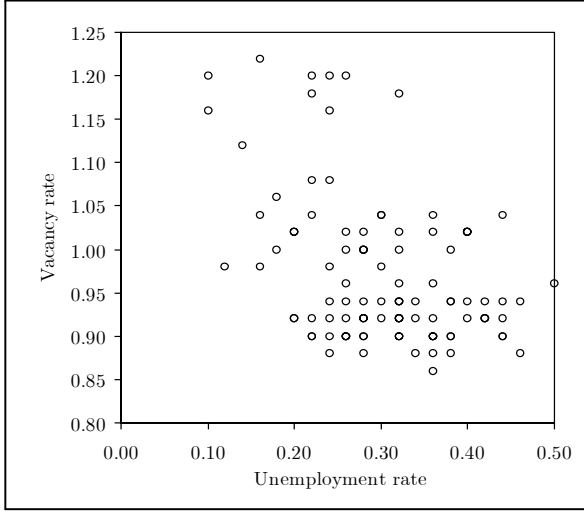


Figure 1: Vacancy vs. Unemployment Rate in a “Walrasian Archetype” Economy **without** Technological Progress. Parameters: $N/F = 5$, $\beta = 0.5$.

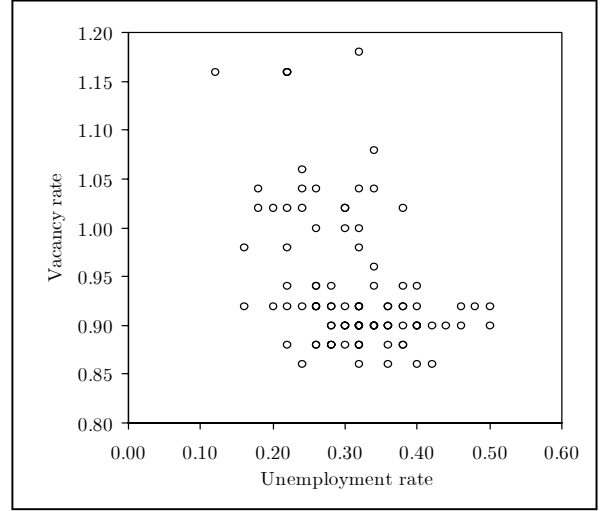


Figure 2: Vacancy vs. Unemployment Rate in a “Walrasian Archetype” Economy **with** Technological Progress. Parameters: $N/F = 5$, $\beta = 0.5$, $\sigma_Z = 0.1$.

Table 3 summarizes our main qualitative results about the emergence of aggregate regularities. Notice that some path-dependence seems to be a necessary condition for a Beveridge relationship. Moreover, a standard Okun curve seems to be in place even when technological progress persistently boosts available production capacity.

Setup	Tech. Change	Aggregate Regularity			
		Beveridge	Wage	Phillips	Okun
WA	No	No	Yes*	No	Yes*
WA	Yes	No	Yes	No	Yes
ISE	No	Yes	Yes*	No	Yes*
ISE	Yes	Yes	Yes	No	Yes

Table 3: Emergence of Aggregate Regularities: Qualitative Results. (*) The associated aggregate regularity can be (partly) explained by the assumptions made in the model about micro-behaviors.

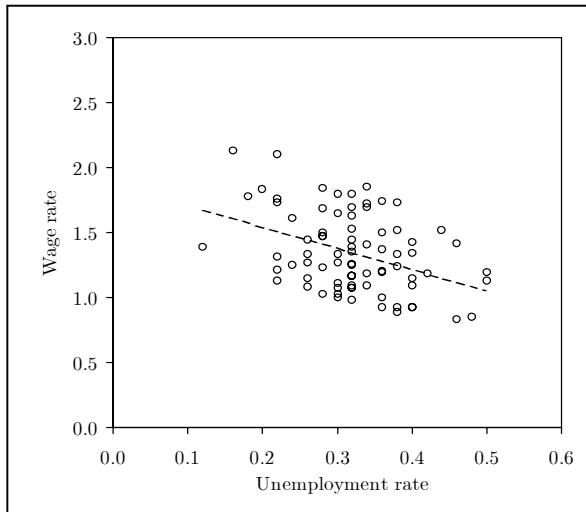


Figure 3: Emergence of Wage curve in a “Walrasian Archetype” Economy **with** Technological Progress. Parameters: $N/F = 5$, $\beta = 0.5$, $\sigma_Z = 0.1$.

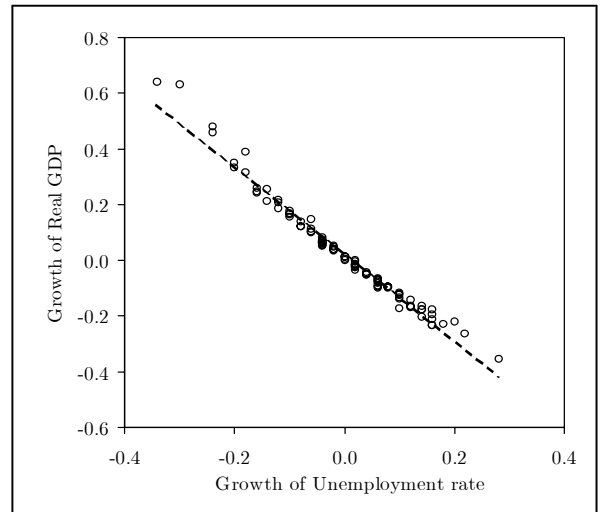


Figure 4: Emergence of Okun curve in a “Walrasian Archetype” Economy **with** Technological Progress. Parameters: $N/F = 5$, $\beta = 0.5$, $\sigma_Z = 0.1$.

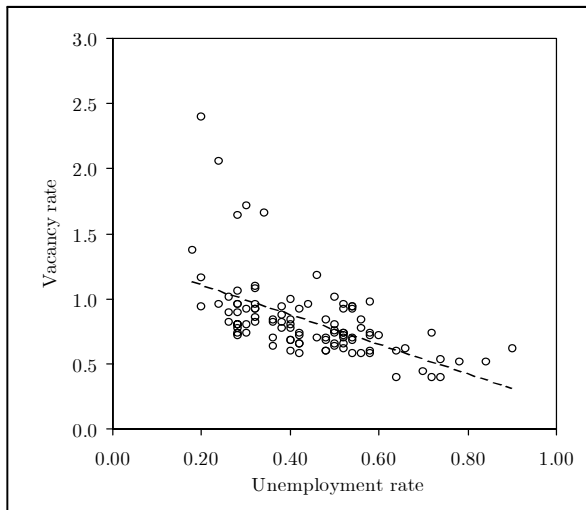


Figure 5: Emergence of Beveridge curve in a “Institutionally-Shaped” Environment **without** Technological Progress. Parameters: $N/F = 5$, $\beta = 0.5$.

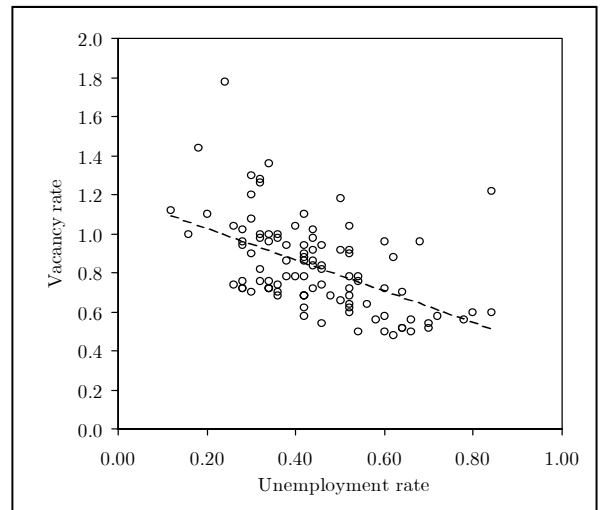


Figure 6: Emergence of Beveridge curve in a “Institutionally-Shaped” Environment **with** Technological Progress. Parameters: $N/F = 5$, $\beta = 0.5$, $\sigma_Z = 0.1$.

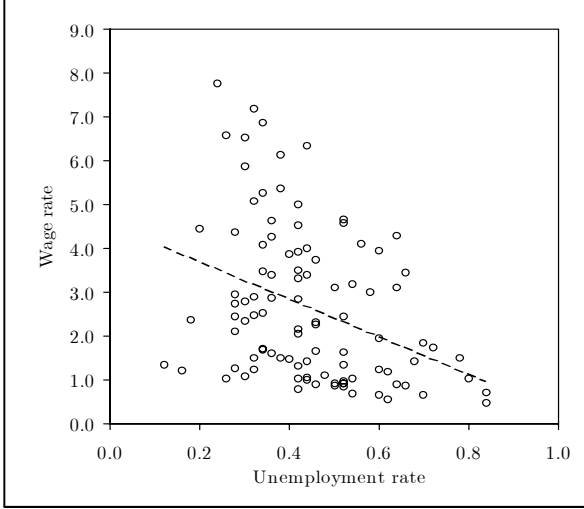


Figure 7: Emergence of Wage curve in a “Institutionally-Shaped” Environment **with** Technological Progress. Parameters: $N/F = 5$, $\beta = 0.5$, $\sigma_Z = 0.1$.

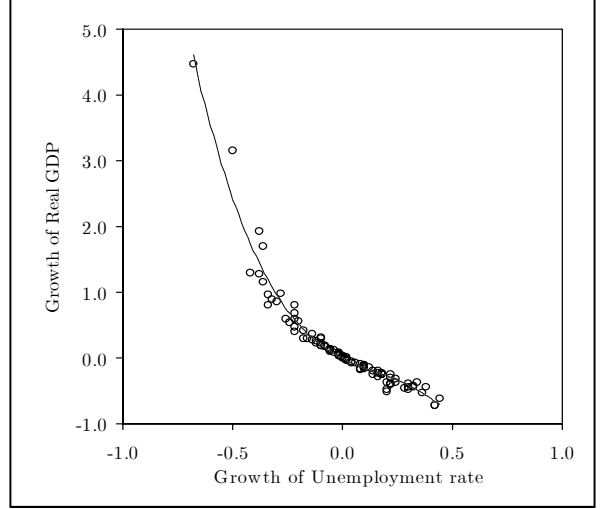


Figure 8: Emergence of Okun curve in a “Institutionally-Shaped” Environment **with** Technological Progress. Parameters: $N/F = 5$, $\beta = 0.5$, $\sigma_Z = 0.1$.

Finally, despite persistent heterogeneity arising endogenously from labor productivity dynamics, Phillips-curve type of regularities are typically rejected by the simulated data in favor of a Wage curve relationship.

4.3 Monte Carlo Experiments

In the last section, we singled out some broad behavioral and technological conditions under which aggregate regularities of interest emerge for a sufficiently large sub-region of the parameter space. We now turn to a more detailed and quantitative study addressing the robustness of emergence results. We present here two sets of exercises.

First, we study whether the implications summarized in Table 3 are robust, for any given parametrization, across independent realizations (i.e. time-series). To this end, in each of the four main “setups” under study, we identify a “benchmark” setting for system parameters, and we generate M independent (Monte Carlo) simulations. We then study the moments of the distributions of the statistics of interest. We focus in particular on test statistics for the significance of coefficients in Beveridge and Okun regressions, the magnitude of the Okun coefficient, as well as test statistics discriminating between Wage and Phillips curves.

Second, we will perform some simple “comparative dynamics” exercises to investigate what happens to emergent regularities when one tunes system parameters within each “setup”. We are in particular interested in detecting shifts (if any) in the Beveridge curve and changes in the Okun coefficients. Once again, we will discuss the outcome of Monte Carlo statistics coming from independent time-series simulation runs for any

given parametrization²⁵.

Emergence of Aggregate Regularities: Robustness Tests

To begin with, consider the emergence of Beveridge curves. Suppose that, for any setup under analysis, a benchmark parametrization under which the results in Table 3 hold is given. Following existing empirical literature, we computed, for each of M independent simulated time-series, estimates (and R^2) for the simple time-series regression:

$$u_t = b_0 + b_1 v_t + \epsilon_t, \quad (18)$$

where ϵ_t is white-noise, u_t is the unemployment rate and v_t is the vacancy rate (both defined as activity rates). We also performed two-tailed test statistics for the null hypothesis $b_1 = 0$ and computed the percentage of rejections (i.e. frequency of emergence of Beveridge curve, in case of a negative estimate). We then computed the Monte Carlo average and standard deviation of estimates \widehat{b}_1 , of their standard errors $\sigma(\widehat{b}_1)$ and goodness-of-fit R^2 , together with the maximum value of the distribution of tail-probabilities for the test $b_1 = 0$.

As Table 4 shows, estimates for the Beveridge coefficient are, on average, negative. In more detail, the “institutionally-shaped environment” entails a 100% percentage of rejections for the test (i.e. a statistically significant Beveridge curve always emerges). However, when a WA is assumed, the frequency of rejections dramatically decreases. In this case, the distribution of tail probabilities is considerably shifted to the right as compared to a WA economy. This means that the emergence of a Beveridge curve in a WA economy may be considered as a quite rare event. This result is also confirmed by looking at goodness-of-fit: average R^2 are much lower in the WA case than in the ISE. Furthermore, the dispersion of the Monte Carlo distribution of estimates increases when one moves towards an “institutionally-shaped” system. Interestingly enough, the presence of technological progress seems to allow for an even more robust emergence of a BC: when $\sigma_Z > 0$, R^2 are higher and the average magnitude of the coefficient increases.

While the Beveridge curve tends to emerge robustly only in an “institutionally-shaped” economy, simulations show that a Wage curve always characterizes our system in all four setups. In particular, statistical tests aimed at discriminating between a Phillips and a Wage world, show that the latter is almost always preferred. Following Card (1995), we perform the lagged regression:

$$\Delta \log \widetilde{W}_t = g_t + a_1 \log u_t + a_2 \log u_{t-1} + \Delta e_t, \quad (19)$$

²⁵All Monte Carlo experiments are undertaken using a Monte Carlo sample size $M = 100$. Initial conditions are always kept fixed (see above).

	Setups			
	WA		ISE	
	$\sigma_Z = 0$	$\sigma_Z > 0$	$\sigma_Z = 0$	$\sigma_Z > 0$
MC Average of \widehat{b}_1	-0.176 (0.095)	-0.263 (0.178)	-0.422 (0.072)	-0.524 (0.078)
MC Average of $\sigma(\widehat{b}_1)$	0.118 (0.015)	0.158 (0.013)	0.055 (0.007)	0.053 (0.007)
R^2	0.024 (0.011)	0.039 (0.016)	0.375 (0.066)	0.431 (0.082)
1st Quartile of Tail Prob. Distr. for $H_0:b_1=0$	0.081	0.050	*	*
3rd Quartile of Tail Prob. Distr. for $H_0:b_1=0$	*	*	0.000	0.000
Percentage of rejections ($H_0:b_1=0$) at 5%	10%	25%	100%	100%

Table 4: Emergence of the Beveridge curve in alternative setups. WA = “Walrasian Archetype”. ISE= “Institutionally- Shaped Environment”. Estimation of $u_t = b_0 + b_1 v_t + \epsilon_t$. Monte Carlo Standard Errors in parentheses. Monte Carlo sample size $M = 100$. Benchmark parametrization: $N/F = 5$, $\beta = 0.5$, $\sigma_Z = 0.1$ (when >0), $\sigma_v = 0.1$ (under ISE).

where \widetilde{W}_t is the wage rate, u_t is the unemployment rate, g_t is a time trend, and first-differences are taken to avoid serial correlation in e_t . As Card (1995) shows, the Wage curve hypothesis implies $a_1 = -a_2$ (together with $a_1 < 0$), while the Phillips curve hypothesis requires $a_2 = 0$. Table 5 reports Monte Carlo testing exercises in our four setups. Notice that the percentage of rejections of a Phillips world is quite high, while we tend not to reject the hypothesis that wage levels are negatively correlated with unemployment rates in almost all simulations.

	Setups			
	WA		ISE	
	$\sigma_Z = 0$	$\sigma_Z > 0$	$\sigma_Z = 0$	$\sigma_Z > 0$
MC Average of \widehat{a}_1	-0.814 (0.025)	-1.643 (0.093)	-1.019 (0.072)	-2.329 (0.225)
MC Average of \widehat{a}_2	0.781 (0.019)	1.520 (0.083)	0.977 (0.020)	2.134 (0.169)
R^2	0.985 (0.003)	0.906 (0.023)	0.978 (0.017)	0.914 (0.026)
% of rejections ($H_0 : a_2=0$) at 5%	100%	99%	99%	100%
% of rejections ($H_0 : a_1=-a_2$) at 5%	10%	5%	5%	1%

Table 5: Emergence of the Wage curve in alternative setups. WA = “Walrasian Archetype”. ISE= “Institutionally- Shaped Environment”. Functional form tested: $\Delta \log \widetilde{W}_t = g_t + a_1 \log u_t + a_2 \log u_{t-1} + \Delta e_t$. Rejecting Phillips curve hypothesis means rejecting $H'_o : a_2 = 0$. Rejecting Wage curve hypothesis means rejecting $H'_o : a_1 = -a_2$. Monte Carlo Standard Errors in parentheses. Monte Carlo sample size $M = 100$. Benchmark parametrization: $N/F = 5$, $\beta = 0.5$, $\sigma_Z = 0.1$ (when >0), $\sigma_v = 0.1$ (under ISE).

The R^2 is very high in all setups. This might be an expected result when $\sigma_Z = 0$, because, without technological progress, a Wage curve follows from price-setting and con-

stant returns. However, when $\sigma_Z > 0$, the goodness-of-fit remains high (and standard errors very low). Our model seems to allow for well-behaved Wage curves also when technological progress induces persistent heterogeneity in labor productivity dynamics. Furthermore, a quite general and robust result (see also below) concerns the effect of technological progress upon the slope of the curve. As discussed above, the latter is expected to be around -1.0 when $\sigma_Z = 0$, but nothing can in principle be said about the expected slope when $\sigma_Z > 0$. Our results suggest that, even when technological progress is present, the Wage curve robustly emerges. Indeed, wage rates become even more responsive to unemployment than in the $\sigma_Z = 0$ case.

As with the Wage curve, the Okun curve, too, turns out to be a robust outcome of our labor market dynamics. Evidence of this effect simply appears by linearly regressing GDP growth rates against changes in the rates of unemployment:

$$\Delta \log(Q_t) = c_0 + c_1 \Delta \log(u_t) + \epsilon_t. \quad (20)$$

We computed Monte Carlo estimates of the Okun coefficient c_1 and we tested for $H_0 : c_1 = 0$ (i.e. emergence of an Okun curve - as long as $c_1 < 0$), see Table 6. Our economy allows for an Okun relationship in all settings, especially when technological progress is present. Again, this might be considered as a not-too-surprising result when $\sigma_Z = 0$, but it becomes a truly emergent property when $\sigma_Z > 0$.

	Setups			
	WA		ISE	
	$\sigma_Z = 0$	$\sigma_Z > 0$	$\sigma_Z = 0$	$\sigma_Z > 0$
MC Average of \hat{c}_1	-2.064 (0.042)	-2.196 (0.047)	-2.635 (0.068)	-3.072 (0.063)
R^2	0.939 (0.026)	0.925 (0.060)	0.928 (0.064)	0.936 (0.025)
Max of Tail Prob. Distrib. for $H_0 : c_1=0$	0.000	0.001	0.000	0.001
% of rejections ($H_0 : c_1=0$) at 5%	100%	99%	100%	99%

Table 6: Emergence of the Okun curve in alternative setups. WA = “Walrasian Archetype”. ISE= “Institutionally- Shaped Environment”. Estimation of $\Delta \log(Q_t) = c_0 + c_1 \Delta \log(u_t) + \epsilon_t$. Monte Carlo Standard Errors in parentheses. Monte Carlo sample size $M = 100$. Benchmark parametrization: $N/F = 5$, $\beta = 0.5$, $\sigma_Z = 0.1$ (when >0), $\sigma_v = 0.1$ (under ISE).

The absolute value of the Okun coefficient is larger than one (and indeed close to Attfield and Silverstone’s empirical estimates), implying some emergent aggregate dynamic increasing returns to labor. The effect becomes stronger when an ISE is assumed: Monte Carlo averages of the Okun coefficient range from -2.196 to -3.072 .

Notice that one did not assume any increasing returns regime at the individual firm level. In fact, firms produce using constant returns production functions; see (2). Moreover, no Phillips curve relationships is in place: our economy typically displays a negative relationship between unemployment rates and wage *levels*. This suggests that aggregation of imperfect and persistently heterogeneous behaviors leads to macro-economic dynamic properties that were not present at the individual level. Therefore, aggregate dynamic increasing returns emerge as the outcome of aggregation of dynamic, interdependent, microeconomic patterns (Forni and Lippi, 1997).

Some Comparative Dynamics Monte Carlo Exercises

We turn now to a comparative dynamics Monte Carlo investigation of the effect of system parameters on emergent aggregate regularities. We focus on the “institutionally-shaped” setup, wherein the economy robustly exhibits well-behaved Beveridge, Wage, and Okun curves, and we study what happens under alternative parameter settings. In particular, we compare parameter setups characterized by:

1. low vs. high N/F ratio (i.e. degrees of concentration of economic activity);
2. low vs. high σ_v (i.e. sensitivity to market signals in the way firms set their vacancies);
3. low vs. high β (i.e. firms’ bargaining strength in wage setting);
4. low vs. high σ_Z (technological opportunities).

We *first* ask whether a higher sensitivity to market signals in vacancy setting induce detectable shifts in aggregate regularities. As Table 7 shows, the smaller σ_v , the stronger the revealed increasing dynamic returns: GDP growth becomes more responsive to unemployment growth and the Okun curve becomes steeper. Notice that σ_v can also be interpreted as an inverse measure of path-dependence in firms’ vacancy setting. The smaller σ_v , the more firms tend to stick to last-period job openings. Therefore, a *smaller* path-dependence implies a steeper Okun relation.

Analogously, we investigate the impact on the BC of simultaneously increasing N/F (i.e. increasing N for a given F) and σ_v (i.e. firms’ “sensitivity to market signals”). Notice that a higher concentration allows firms, *ceteris paribus*, to more easily fill their vacancies. Similarly, the higher σ_v , the more firms are able to react to aggregate conditions and correspondingly adjust vacancies. Therefore, one might be tempted to interpret economies characterized by high values for both N/F and σ_v as “low friction” worlds, and expect the BC curve to lie closer to the axes. Notice, however, that, in our model, an “indirect” effect is also present. If labor demand is very low (e.g. because the economy is in a recession), then the unemployment rate might be high, irrespective of the value of N/F . Moreover, if σ_v is

	ISE Setup			
	$\sigma_v = 1.0$ (HSMS)		$\sigma_v = 0.2$ (LSMS)	
	$\sigma_Z = 0$	$\sigma_Z > 0$	$\sigma_Z = 0$	$\sigma_Z > 0$
MC Average of \hat{c}_1	-2.700 (0.082)	-2.960 (0.085)	-2.900 (0.064)	-3.270 (0.060)
R^2	0.928 (0.064)	0.936 (0.025)	0.939 (0.026)	0.925 (0.060)
Max of Tail Prob. Distrib. for $H_0 : c_1=0$	0.001	0.001	0.000	0.001
% of rejections ($H_0 : c_1=0$) at 5%	100%	99%	100%	99%

Table 7: Shifts in the Okun coefficient in an “Institutionally- Shaped Environment” under alternative parameter settings. HSMS: High Sensitivity to Market Signals. LSMS: Low Sensitivity to Market Signals. Estimation of $\Delta \log(Q_t) = c_0 + c_1 \Delta \log(u_t) + \epsilon_t$. Monte Carlo Standard Errors in parentheses. Monte Carlo sample size $M = 100$. Benchmark parametrization: $N/F = 5$, $\beta = 0.5$, $\sigma_Z = 0.1$.

high, firms will fire more workers during downswings, thus inducing a sort of “accelerator” effect on the recession. Thus, the consequences on the BC of assuming a larger market concentration and a higher sensitivity to market signals are *ex-ante* ambiguous: if “indirect” effects dominate, we should observe various combinations between shifts to the right and “business-cycle” movements along the curve.

Notwithstanding all that, Monte Carlo simulations show that the model is able to reproduce the predicted shifts in the BC. We observe (cf. Table 8) that, as N/F and σ_v both increase in an ISE economy, Monte Carlo averages of estimated intercepts stay constant, while the BC becomes, on average, steeper (and thus closer to the origin). A steeper BC implies that firms adaptively learn to open less vacancies and to adjust their filled-to-open vacancy ratios in response to market signals.

Second, we explore what happens to (within-simulation) average and standard deviation of GDP growth time-series²⁶ when both σ_v and firms’ bargaining strength β are allowed to vary. Recall that, the higher β , the less firms take into account workers satisficing wages when they decide their contractual wage. Figs. 9 and 10 show Monte Carlo means of average and standard deviation of GDP growth rates. We find that the *higher* firms’ bargaining strength, the *smaller* both average growth rates and their variability. Thus, allowing for some bargaining power on the workers’ side implies better aggregate performance, but also more fluctuations. Furthermore, if firms are *less* responsive to market signals (e.g. they employ a path-dependent vacancy setting rule), the economy enjoys persistently higher average growth rates and persistently smaller fluctuations.

Finally, we assess the consequences of “fueling” the economy with higher technological opportunities (i.e. higher σ_Z) for different levels of β (and setting σ_v to an intermediate level). While a higher σ_Z implies higher average growth rates in all parameter settings (Fig.

²⁶That is, we compute average and standard deviation of h_t of GDP growth rates within a simulation $\{h_t, t = 1, \dots, T\}$, $h_t = \Delta \log Q_t$.

	N/F σ_v	Parameter Settings			
		50	20	10	5
MC Mean of \widehat{b}_0		0.684 (0.018)	0.689 (0.024)	0.691 (0.043)	0.692 (0.043)
MC Mean of $\sigma(\widehat{b}_0)$		0.020 (0.002)	0.027 (0.002)	0.040 (0.004)	0.033 (0.004)
Max of MC Tail Prob. Distr. for $H_0: b_0 = 0$		0.001	0.000	0.001	0.001
% of Rejections for $H_0: b_0 = 0$		99%	100%	98%	99%
MC Mean of \widehat{b}_1		-0.679 (0.030)	-0.631 (0.043)	-0.535 (0.071)	-0.413 (0.077)
MC Mean of $\sigma(\widehat{b}_1)$		0.031 (0.003)	0.044 (0.004)	0.065 (0.006)	0.056 (0.007)
Max of MC Tail Prob. Distr. for $H_0: b_1 = 0$		0.000	0.001	0.002	0.001
% of Rejections for $H_0: b_1 = 0$		100%	99%	98%	99%
MC Mean of R^2		0.816 (0.038)	0.677 (0.045)	0.408 (0.064)	0.410 (0.062)

Table 8: Shifts in the Beveridge curve in an “Institutionally- Shaped Environment” under alternative parameter settings for: (i) concentration of economic activity N/F ; (ii) sensitivity to market signals σ_v . Estimation of $u_t = b_0 + b_1 v_t + \epsilon_t$. Monte Carlo Standard Errors in parentheses. Monte Carlo sample size $M = 100$. Benchmark parametrization: $\beta = 0.5$. No technical progress is assumed to focus on BC shifts for given resources.

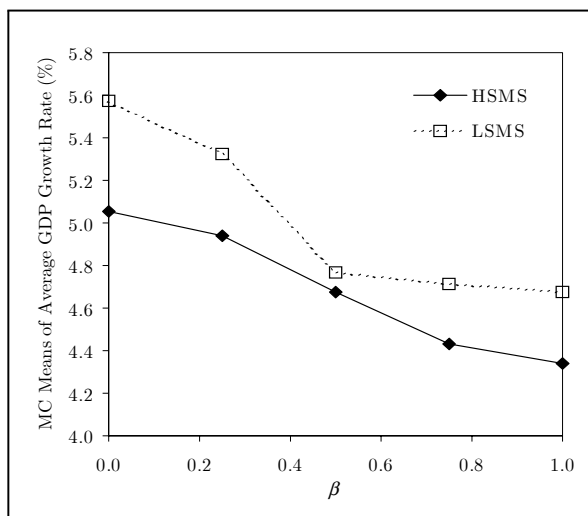


Figure 9: Monte Carlo Means of (within-simulation) Average Real GDP Growth Rates as a function of firms strength in wage bargaining (β). LSMS vs. HSMS: Low ($\sigma_v = 0.1$) vs. High ($\sigma_v = 1.0$) sensitivity to market signals in vacancy setting. “Institutionally-Shaped” Environment. Parameters: $N/F = 5$, $\sigma_Z = 0.1$.

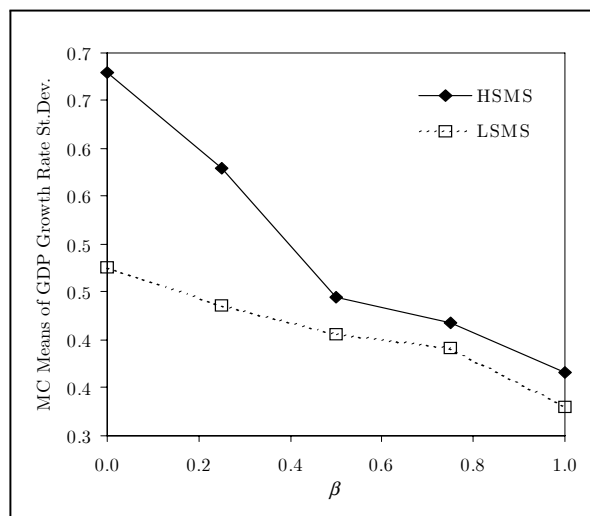


Figure 10: Monte Carlo Means of (within-simulation) Standard Deviation of Real GDP Growth Rates as a function of firms strength in wage bargaining (β). LSMS vs. HSMS: Low ($\sigma_v = 0.1$) vs. High ($\sigma_v = 1.0$) sensitivity to market signals in vacancy setting. “Institutionally-Shaped” Environment. Parameters: $N/F = 5$, $\sigma_Z = 0.1$.

11), a stronger bargaining power for workers still implies better aggregate performances. Together, more technological opportunities also entail a higher volatility in the growth process (see Fig. 12). Volatility can be weakened if one increases firm strength in wage bargaining.

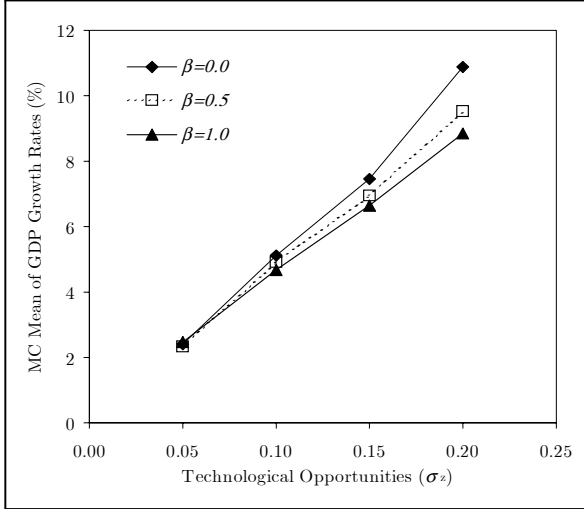


Figure 11: Monte Carlo Means of (within-simulation) Average Real GDP Growth Rates as a function of technological opportunities (σ_Z) and firms strength in wage bargaining (β). “Institutionally-Shaped” Environment. Parameters: $N/F = 5$, $\sigma_v = 0.1$.

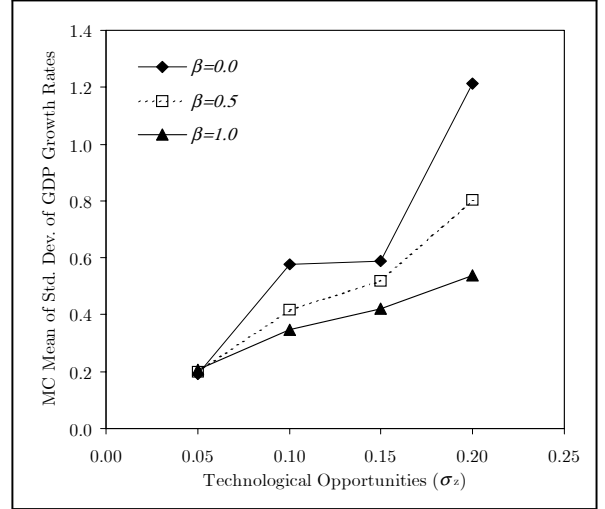


Figure 12: Monte Carlo Means of (within-simulation) Standard Deviation of Real GDP Growth Rates as a function of technological opportunities (σ_Z) and firms strength in wage bargaining (β). “Institutionally-Shaped” Environment. Parameters: $N/F = 5$, $\sigma_v = 0.1$.

5 Conclusions

As far as the properties of labor market dynamics and the business cycle are concerned, three well-known aggregate regularities (i.e. Beveridge, Wage, and Okun curves) seem to provide a quite complete picture. Nevertheless, the existing theoretical literature still lacks micro-founded models which are able *jointly* to account for these three crucial stylized facts.

In this paper, we presented a preliminary agent-based, evolutionary, model trying to formalize from the bottom up individual behaviors and interactions in both product and labor markets.

In the model, vacancy and wage setting, as well as matching and bargaining, demand, and price formation, are all endogenous processes. Firms enjoy labor productivity improvements thanks to technological progress and undergo a selection pressure acting on their revealed competitiveness (which is also affected by their hiring and wage-setting behaviors).

Simulations show that the model is able robustly to reproduce Beveridge, Wage and Okun curves under quite broad behavioral and institutional settings. Moreover, the system generates endogenously an Okun coefficient greater than one (i.e. aggregate dynamic increasing returns) even if individual firms employ production functions exhibiting constant returns to labor.

Monte Carlo simulations also indicate that statistically detectable shifts in Okun and Beveridge curves emerge as the result of changes in institutional, behavioral, and technological parameters. For example, a higher concentration of market activity (i.e. a higher number of workers per firm) and a higher sensitivity to market signals in firms' vacancy setting rules imply Beveridge curves which lie closer to the axes. Finally, the model generates quite sharp predictions about how the average aggregate performance (and volatility) of the system change in alternative behavioral, institutional, and technological setups.

Many issues remain to be explored. First, additional Monte Carlo simulation exercises could be performed to more finely map (e.g. within a given "system setup") parameters and aggregate behaviors.

Second, the issue as to whether (and how) heterogeneity is able to affect the emergence of aggregate regularities might be addressed. For instance, one could explore the effects to endow workers (resp. firms) with increasingly heterogeneous distributions of reservation wages (resp. desired ratios of filled to open vacancies). Third, one might investigate the consequences of assuming alternative matching and bargaining processes to allow for a richer institutional setting. Finally, the structure of the model might be complicated in order to investigate economies where jobs last more than one period and firms are able to transfer profits across time.

References

- Aghion, P. and P. Howitt (1994), "Growth and Unemployment", *Review of Economic Studies*, 61: 477–494.
- Aoki, M. (2003), "A New Model of Output Fluctuation: Ultrametrics, Beveridge Curve and Okun's Law", Economics online papers, no. 234, UCLA.
- Aoki, M. and H. Yoshikawa (2003), "A Simple Quantity Adjustment Model of Economic Fluctuations and Growth", in Cowan, R. and N. Jonard (eds.), *Heterogeneous Agents, Interaction and Economic Performance*. Berlin, Springer.
- Ashenfelter, O. and D. Card (eds.) (1999), *Handbook of Labor Economics. Volume 3*. Elsevier Science, Amsterdam.
- Ashenfelter, O. and R. Layard (eds.) (1986), *Handbook of Labor Economics. Volume 2*. Elsevier Science, Amsterdam.
- Attfield, C. and B. Silverstone (1997), "Okun's coefficient: A comment", *The Review of Economics and Statistics*, 79: 326–329.

- Belot, M. and J. V. Ours (2000), “Does the Recent Success of Some OECD Countries in Lowering their Unemployment Rates Lie in the Clever Design of their Labour Market Reforms?”, Discussion paper no. 147, IZA, Bonn, Germany.
- Blanchard, O. and P. Diamond (1989), “The Beveridge curve”, *Brooking Papers on Economic Activities*, 1: 1–76.
- Blanchard, O. and L. Katz (1997), “What We Know and Do Not Know about the Natural Rate of Unemployment”, *Journal of Economic Perspectives*, 11: 57–72.
- Blanchard, O. and N. Kiyotaki (1987), “Monopolistic Competition and the Effects of Aggregate Demand”, *American Economic Review*, 77: 647–666.
- Blanchard, O. and D. Quah (1989), “The Dynamic Effects of Aggregate Demand and Supply Disturbances”, *American Economic Review*, 79: 655–673.
- Blanchard, O. and J. Wolfers (2000), “The Role of Shocks and Institutions in the Rise of European Unemployment: The Aggregate Evidence”, *Economic Journal*, 110: C1–C33.
- Blanchflower, D. and A. Oswald (1994), *The Wage Curve*. Cambridge, Massachusetts, The MIT Press.
- Bleakley, H. and J. Fuhrer (1997), “Shifts in the Beveridge Curve, Job Matching, and Labor Market Dynamics”, *New England Economic Review*, pp. 3–19.
- Borsch-Supan, A. (1991), “Panel Data Analysis of the Beveridge Curve: Is There a Macroeconomic Relation between the Rate of Unemployment and Vacancy Rate?”, *Economica*, 58: 279–297.
- Burdett, K., S. Shi and R. Wright (2001), “Pricing and Matching with Frictions”, *Journal of Political Economy*, 109: 1060–1085.
- Cao, M. and S. Shi (2000), “Coordination, matching and wages”, *Canadian Journal of Economics*, 33: 1009–1033.
- Card, D. (1995), “The Wage Curve: A Review”, *Journal of Economic Literature*, 33: 785–799.
- Card, D. and D. Hyslop (1996), “Does Inflation Grease the Wheels of the Labor Market”, Working paper 5538, NBER.
- Dosi, G., L. Marengo and G. Fagiolo (2004), “Learning in Evolutionary Environment”, in Dopfer, K. (ed.), *Evolutionary Principles of Economics*. Cambridge, Cambridge University Press, Forthcoming.
- Dosi, G. and R. Nelson (1994), “An introduction to evolutionary theories in economics”, *Journal of Evolutionary Economics*, 4: 153–172.
- Dosi, G. and S. Winter (2002), “Interpreting Economic Change: Evolution, Structures and Games”, in Augier, M. and J. March (eds.), *The Economics of Choice, Change, and Organizations*. Cheltenham, Edward Elgar Publishers.

- Epstein, J. and R. Axtell (1996), *Growing Artificial Societies: Social Science from the Bottom-Up*. Washington D.C., MIT Press.
- Evans, G. (1989), “Output and Unemployment Dynamics in the U.S. 1950-1985”, *Journal of Applied Econometrics*, 4: 213–237.
- Fitoussi, J.-P., D. Jestaz, E. Phelps and G. Zoega (2000), “Roots of the Recent Recoveries: Labor Reforms or Private Sector Forces?”, *Brooking Papers on Economic Activities*, 1: 237–291.
- Flaschel, P., G. Kauermann and W. Semmler (2003), “Testing Wage and Price Phillips Curves for the United States”, Unpublished manuscript, Bielefeld University, Faculty of Economics.
- Forni, M. and M. Lippi (1997), *Aggregation and the Microfoundations of Dynamic Macroeconomics*. Oxford, Clarendon Press.
- Hahn, F. and R. Solow (1997), *A Critical Essay on Modern Macroeconomic Theory*. Cambridge, MA, The MIT Press.
- Julien, B., J. Kennes and I. King (2000), “Bidding for Labor”, *Review of Economic Dynamics*, 3: 619–649.
- Kirman, A. (1992), “Whom or what does the representative individual represent?”, *Journal of Economic Perspectives*, 6: 117–136.
- Laffond, G. and J. Lesourne (2000), “The genesis of expectations and of sunspot equilibria”, *Journal of Evolutionary Economics*, 2: 211–231.
- Lagos, R. (2000), “An Alternative Approach to Search Frictions”, *Journal of Political Economy*, 108: 851–872.
- Layard, R., R. Jackman and S. Nickell (1991), *Unemployment*. Oxford University Press, Oxford.
- Lesourne, J. (1992), *The Economics of Order and Disorder*. Oxford, Clarendon Press.
- Moosa, I. (1997), “A Cross-Country Comparison of Okun’s Coefficient”, *Journal of Comparative Economics*, 24: 335–356.
- Mortensen, D. (1986), “Job Search and Labor Market Analysis”, in Ashenfelter, O. and R. Layard (eds.), *Handbook of Labor Economics, Volume 2*. Amsterdam, Elsevier Science.
- Mortensen, D. and C. Pissarides (1994), “Job creation and job destruction in the theory of unemployment”, *Review of Economic Studies*, 61: 397–415.
- Nickell, S., L. Nunziata, W. Ochell and G. Quintini (2001), “The Beveridge curve, Unemployment and wages in the OECD from 1960s to 1990s”, Working paper no. 502, CEPR, London, U.K.
- Okun, A. (1962), “Potential GDP: Its Measurement and Significance”, *Proceedings of the Business and Economics Statistics*, pp. 98–103.

- Okun, A. (1970), *The political economy of prosperity*. The Brookings Institution, Washington D.C.
- Peters, M. (1991), “Ex ante price offers in matching games non-steady states”, *Econometrica*, 59: 1425–1454.
- Petrongolo, B. and C. Pissarides (2001), “Looking into the Black Box: A Survey on the Matching Function”, *Journal of Economic Literature*, 39: 390–431.
- Phelps, E. (1972), *Inflation Policy and Unemployment Theory*. MacMillan, London.
- Phelps, E. (1994), *Structural Slumps*. Cambridge, Harvard University Press.
- Phelps, E. and S. Winter (1970), “Optimal Price Policy Under Atomistic Competition”, in Phelps, E., G. Archibald and A. Alchian (eds.), *Microeconomic Foundations of Employment and Inflation Theory*. New York, Norton.
- Pissarides, C. (2000), *Equilibrium Unemployment Theory*. Oxford, Blackwell.
- Prachowny, M. (1993), “Okun’s Law: Theoretical Foundations and Revised Estimates”, *The Review of Economics and Statistics*, 75: 331–336.
- Schaik, A. V. and H. D. Groot (1998), “Unemployment and Endogenous Growth”, *Labour*, 12: 189–219.
- Shapiro, C. and J. Stiglitz (1984), “Equilibrium Unemployment as a Worker Discipline”, *American Economic Review*, 74: 1257–78.
- Smith, T. and Y. Zenou (2003), “A Discrete-Time Stochastic Model of Job Matching”, *Economic Dynamics*, 6: 54–79.
- Sögner, L. and A. Stiasny (2000), “A Cross-Country Study on Okun’s Law”, Working paper no. 13, Vienna University, Faculty of Economics and Business Administration.
- Solow, R. (1998), “What is Labour-Market Flexibility? What is it Good for?”, *Proceedings of the British Academy*, 97: 189–211.
- Tesfatsion, L. (1997), “How Economists Can Get ALife”, in Arthur, W., S. Durlauf and D. Lane (eds.), *The Economy as an Evolving Complex System II*. Santa Fe Institute, Santa Fe and Reading, MA, Addison-Wesley.
- Tesfatsion, L. (2001), “Structure, Behavior, and Market Power in an Evolutionary Labor Market with Adaptive Search”, *Journal of Economic Dynamics and Control*, 25: 419–457.
- Witt, U. (1985), “Coordination of Individual Economic Activities as an Evolving Process of Self-Organization”, *Economie appliquée*, 37: 569–595.