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Graphical Models for Structural Vector Autoregressions

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Graphical Models for Structural Vector Autoregressions

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Abstract

The identification of a VAR requires differentiating between correlation and causation. This paper presents a method to deal with this problem. Graphical models, which provide a rigorous language to analyze the statistical and logical properties of causal relations, associate a particular set of vanishing partial correlations to every possible causal structure. The structural form is described by a directed graph and from the analysis of the partial correlations of the residuals the set of acceptable causal structures is derived. This procedure is applied to an updated version of the King et al. (American Economic Review, 81, (1991), 819) data set and it yields an orthogonalization of the residuals consistent with the causal structure among contemporaneous variables and alternative to the standard one, based on a Choleski factorization of the covariance matrix of the residuals.

JEL classification: C32, C49, E32.

Keywords: Structural VARs, Identification, Directed Acyclic Graphs, Causality, Impulse Response Functions.

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1 Introduction

Vector autoregressive (VAR) models have been extensively used in applied macroeconomic research since the seminal work of Sims (1980). Sims's idea is to formulate unrestricted reduced forms and to make inferences from them without imposing the "incredible restrictions" used by the Cowles Commission approach. A zero-mean stationary VAR model can be written as:

$$Y_t = A_1 Y_{t-1} + \ldots + A_p Y_{t-p} + u_t, \tag{1}$$

where $Y_t = (y_{1t}, \ldots, y_{kt})'$, $u_t = (u_{1t}, \ldots, u_{kt})'$, and $A_1, \ldots, A_p$ are $(k \times k)$ matrices. The components of u_t are white noise innovation terms, $E(u_t) = 0$, and u_s and u_t are independent for $s \neq t$. The matrix $\Sigma_u = E(u_t u_t')$ is in general nondiagonal. The relations among the contemporaneous components of Y_t, instead of appearing in the functional form (as in simultaneous equation models), are embedded in the covariance matrix of the innovations. If one neglects, as I do for the scope of this paper, problems of overparameterization, estimation of (1) by OLS is straightforward and the estimates coincide with MLE (under normality of the errors) and the SURE method introduced by Zellner (1962).

Major problems arise when discussing how to transform equation (1) in order to orthogonalize the matrix of the innovations and to study the evolution of the system caused by a single innovation using impulse response functions or forecast error variance decomposition. A way to orthogonalize the matrix of the innovations is premultiplying each member of (1) by a matrix W such that $E[W u_t u_t' W']$ is diagonal. A typical practice is to decompose the matrix Σ_u according to the Choleski factorization, so that $\Sigma_u = PP'$, where P is lower-triangular, to define a diagonal matrix D with the same diagonal as P and to multiply both sides of (1) by $W = DP^{-1}$, so that the covariance matrix of the transformed residuals turns out to be equal to $\Lambda = DD'$, which is diagonal. A problem with this method is that W changes if the ordering on the variables of the system changes and, in general, there are infinite matrices W for which $E[W u_t u_t' W']$ is diagonal. The matrix W introduces relations among the contemporaneous components of Y_t in the functional form. Such relations should be consistent with the causal structure among the variables, although causal relations among contemporaneous economic variables have been sometimes considered a controversial issue (see e.g. Granger 1988). Nevertheless, the conventional approach has been criticized as arbitrary, since it "restricts attention to recursive models, which (roughly speaking) occupy a set of measure zero" within the set of linear models (Bernanke 1986, p. 55).

Thus the literature on structural VAR deals with an identification problem for many respects analogous to the one considered by standard simultaneous equation models: how to recover an economic model from a set of reduced form equations. The main difference is that restrictions are imposed in a second stage, after estimation. The structural equation considered is of the form:

$$\Gamma Y_t = B_1 Y_{t-1} + \ldots + B_p Y_{t-p} + C v_t, \tag{2}$$

where v_t is a $(k \times 1)$ vector of serially uncorrelated structural disturbances with mean zero and diagonal covariance matrix Σ_v. The identification problem consists in finding a way to infer the unobserved parameters in (2) from the estimated form (1), where $A_i = \Gamma^{-1} B_i$ for $i = 1, \ldots, p$, and $u_t = \Gamma^{-1} C v_t$. The problem is that at most $k(k+1)/2$ unique, non-zero elements can be obtained from $\hat{\Sigma}_u$. On the other hand, there are $k(k+1)$ parameters in Γ and Σ_v and k^2 parameters to be identified in C. Even if it is assumed $C = I$ and the diagonal elements of Γ are normalized to 1, as it is typically done in the literature, at least $k(k-1)/2$ restrictions are required to satisfy the order condition for identification.

In order to address this problem, this paper proposes a method which emphasizes the interpretation of structural relations as causal relations, which historically has been maintained by the Cowles Commission approach (see e.g. Simon 1953). A graph is associated with the causal structure of the model and the properties of a given causal structure are obtained by analyzing the properties of the graph. The rationale of using graphical models is to consider the statistical implications of causal relations jointly with their logical implications, in order to use data and background knowledge in an efficient way. The idea is that causal relations, under some general assumptions, are tied with particular sets of vanishing (partial) correlations among the variables that constitute them. Therefore, I use tests on vanishing (partial) correlations among the estimated residuals of a VAR to narrow the class of the possible causal structures among the contemporaneous variables. Each causal structure implies a set of overidentifying restrictions. This constitutes an advantage with respect to the standard recursive VAR models identified using the Choleski factorization mentioned above, which are just-identified, because overidentified models can be tested using a χ^2 test statistic.

Many ideas of this paper have been inspired by the method discussed in Swanson and Granger (1997). This paper is also in the spirit of Glymour and Spirtes (1988), Gilli (1992), Dahlhaus and Eichler (2000), Reale and Tunnicliffe Wilson (2001) and Hoover (2001, chapter 7), which present or discuss different graph-based approaches to econometrics. But there is a set of works, namely Bessler and Lee (2002), Awokuse and Bessler (2003), Bessler and Yang (2003), Demiralp and Hoover (2003), Haigh and Bessler (2004), with which this paper is directly concurrent. The works belonging to this group apply a graph-based search procedure — the PC algorithm — developed by Spirtes et al. (2000, 2nd edn), and embedded in the various versions of software Tetrad (see Scheines et al. 1994 and Spirtes et al. 1996), with the aim of addressing the problem of identification in a Structural VAR. I also use a graph-based search procedure derived from Spirtes et al. (2000), but this paper makes the following advances over the previous studies.

First, there is an important difference in the testing procedure. This paper, like the concurrent studies, bases the search procedure upon tests of vanishing partial correlations among residuals. The mentioned papers, in order to test vanishing partial correlations among the residuals, use Fisher's z statistic, suggested by Spirtes et al. (2000, p. 94) and embedded in the Tetrad program. The Tetrad testing procedure, however, is aimed to test vanishing partial correlations using population partial correlations, while these studies, in the empirical applications, use partial correlation among estimated residuals, rather than among the "true" residuals. In practice, these studies use estimated residuals, as if they were population residuals and the asymptotic distribution of the test statistic is left in a mystery. The test I develop in this paper (Appendix A), based on a Wald statistic, is, on the contrary, more appropriate when the correlations are among estimated residuals than the actual errors of the original variables. Indeed, the test I use is based on the asymptotic distribution of the partial correlations among the estimated residuals.

Second, there is an important difference in the search algorithm used. I make a modification of the PC algorithm to adapt it to the peculiarities of the VAR model. Spirtes, Glymour and Scheines, in developing the PC algorithm, were concerned with computational complexity issues, as witnessed by the discussion in Spirtes et al. (2000, pp. 85-86). In order to avoid a computationally unefficient search, they structure the algorithm so that the number of conditional independence tests is bounded by a certain polynomial, which is function of the number of variables object of investigations. The idea is that one does not need to test all the possible independence relations, because the number of such tests increases exponentially with the number of variables. Thus, with the PC algorithm, "it is possible to recover sparse graphs with as many as a hundred variables" (Spirtes et al. 2000, p. 87). But one should not be much concerned with such computational issues, when considers the case of VAR models. Indeed VAR models of macroeconomic time series, for well known reasons related to the number of parameters to be estimated, deals with a very limited number of variables. The typical VAR model, indeed, is constituted by a number of variables between 4 and 7. With such a number of variables, it is computationally feasible to perform even all the possible conditional independence tests. I modify the algorithm (section 3.3, Table 1) allowing a larger number of conditional independence tests than the original PC algorithm. In doing that, the algorithm gains stability, in the sense small errors of the algorithm input (conditionally independence tests) are likely to produce less large errors of the algorithm output (casual relationships), with respect to the original PC algorithm.

Third, I present some graph-based results (section 3.2 and 3.3), which are complementary with respect to the Swanson and Granger's (1997) analysis of how causally ordering the estimated residuals from the reduced-form VAR is equivalent to causally ordering the contemporaneous terms in the structural VAR.

As an illustration of the method, I present an example which uses an updated version of the King et al. (1991) data set. The results show that this method permits the orthogonalization of the residuals in a way consistent with the statistical properties of the data. The calculation of the impulse response functions confirm the conclusion of King et al. (1991) that US data do not support "the view that a single permanent shock is the dominant source of business cycle fluctuations." However, it should be emphasized that the solution of the identification problem cannot depend on statistical inference alone and that *a priori* knowledge is essential. The more background knowledge (in particular *causal* knowledge) is available, the more detailed is the causal structure one is able to identify, as it is intuitive. An advantage of this method is that *a priori* knowledge can be incorporated in an explicit and efficient way.

The rest of the paper is organized as follows. In the next section I introduce the recent literature on graphical models for causal inference and I contextualize the method in the macroeconometric framework. In Section 3 I present my method of identification for structural VAR models. In Section 4 I discuss the empirical application and in Section 5 I draw some conclusions and suggest further developments of the research.

2 Graphical models

Graphical models in econometrics have their sources in works developed in other scientific areas, like statistical physics (Gibbs 1902) and genetics (Wright 1921 and 1934). Wright founded the so-called path analysis in the 1920s for the study of inherited properties of natural species, but some of his ideas have inspired part of the econometric literature on structural equations and causality (see e.g. Wold 1954 and Blalock 1971). Graphical models have been used in multivariate statistics to describe and manipulate conditional independence relations (Whittaker 1990 and Lauritzen 1995). Gilli (1992) uses graphical techniques to explore the logical implications of large-scale simultaneous equation models. In the recent years, a particular class of graphical models — directed acyclic graphs — has been used for the identification of causal relationships from data and for the prediction of interventions in a given system (see Spirtes et al. 2000, Pearl 2000, Lauritzen 2001). These works refer in their applications to social sciences in general, clinical trials and expert systems. Swanson and Granger (1997) are, to the best of my knowledge, the first who apply such graph-based techniques to VAR models.

The idea of using graphical models in multivariate statistics is to represent random variables by means of vertices, and probabilistic dependence between the variables by means of edges. Under particular assumptions, the directed edges (represented by arrows) that constitute a DAG describe causal connections. It is necessary to introduce some graph terminology. I blend the terminology of Spirtes et al. (2000) with the terminology of Pearl (2000) and Lauritzen (2001).

A *graph* is an ordered pair $\mathcal{G} = (V, E)$, where V is a nonempty set of *vertices*, and E is a subset of the set $V \times V$ of ordered pairs of vertices, called the *edges* of $\mathcal{G}$. It is assumed that E consists of pair of distinct vertices, so that there are no self-loops. For example, in the graph in Figure 1 we have $V = \{V_1, V_2, V_3, V_4, V_5\}$ and $E = \{(V_1,V_2),(V_2,V_1),(V_2,V_3),(V_3,V_4),(V_4,V_3),(V_3,V_5)\}$.

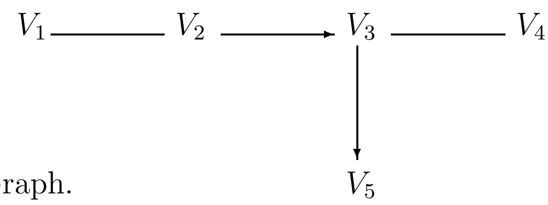


Figure 1: Graph.

A line between V_1 and V_2, called an *undirected* edge, is drawn if both (V_1, V_2) and (V_2, V_1) belong to E. On the contrary, if (V_1, V_2) belongs to E, but (V_2, V_1) does not belong to E, an arrow, called a *directed* edge, is drawn from V_1 to V_2. If there is an edge (directed or undirected) between a couple of vertices, these are said *adjacent*. A graph is called a *directed graph* if all its edges are directed. A graph is *complete* if every pair of its vertices is adjacent. A *directed acyclic graph* (DAG) is a directed graph which contains no cycles.[1]

DAGs are particularly useful to represent conditional independence relations. Given a DAG $\mathcal{G}$ and a joint probability distribution P on a set of variables $X = \{X_1, X_2, \ldots X_n\}$, $\mathcal{G}$ *represents* P if to each variable X_i in X is associated a vertex in $\mathcal{G}$ and the following condition is satisfied:

(**A1**) Markov Condition (Spirtes et al. 2000, p. 29).
Any vertex in $\mathcal{G}$ is conditionally independent of its nondescendants (excluded its parents), given its parents, under P.[2]

A graphical procedure introduced by Pearl (1988) and called *d-separation* permits to check if two variables (in a DAG representing a probability distribution according to the Markov Condition) are (conditionally) independent, simply looking at the paths that connect the two variables.

Let us define first a collider and active vertex in a path. In a DAG $\mathcal{G}$ a vertex X is a *collider* on a path α if and only if there are two distinct edges on α both containing X and

[1] The notion of "cycle" is very intuitive. A *path* of length n from V_0 to V_n is a sequence $\{V_0, \ldots, V_n\}$ of distinct vertices such that $(V_{i-1}, V_i) \in E$ for all $i = 1, \ldots, n$. A *directed path* is a path such that $(V_{i-1}, V_i) \in E$, but $(V_i, V_{i-1}) \notin E$ for all $i = 1, \ldots, n$. A *cycle* is a directed path with the modification that the first and the last vertex are identical, so that $V_0 = V_n$.

[2] The notion of "parent" and "ancestor" is also very intuitive. If there is a directe edge from a vertex V_1 to a vertex V_2, V_1 is called the *parent* of V_2. Given a directed graph, the set of vertices V_i such that there is a directed path from V_j to V_i is the set of the *descendants* of V_j.

both directed on X. For example, if in the graph it appears the configuration $X \longrightarrow Y \longleftarrow Z$, Y is said a collider on the path X, Y, Z. In a DAG $\mathcal{G}$ a vertex X is *active* on a path β relative to a set of vertices Z of $\mathcal{G}$ if and only if: (i) X is not a collider on β and $X \notin Z$; or (ii) X is a collider on β and X or a descendant of X is in Z. A path β is active relative to Z if and only if every vertex on β is active relative to Z.

The definition of d-separation is the following. In a DAG $\mathcal{G}$ two vertices X and Y are *d-separated* by Z if and only if there is no active path between X and Y relative to Z. X and Y are *d-connected* by Z if and only X and Y are not d-separated by Z.

Directed acyclic graphs and, more in general, graphical models form a rigorous language in which causal concepts can be discussed and analyzed. There have been applications of graphical models to causal inference both in experimental data and in observational data framework. The first type of application is concerned with the prediction of the effect of interventions in a given system (see e.g. Lauritzen 2001). The focus of this paper is on the second type of application, for the observational nature of economic data. In this framework a directed acyclic graph is interpreted as a causal structure which has generated the data V with a probability distribution $P(V)$. A directed acyclic graph interpreted as a causal structure is called *causal graph* (or causal DAG). In a causal graph a directed edge pointing from a vertex X to Y represents a direct cause from X to Y.

The search for causal structure is based on two assumptions. The first one is the *Causal Markov Condition*, which is the Markov condition stated above, with the difference that the DAG is given a causal interpretation. The second one is the following.

(**A2**) Faithfulness Condition (Spirtes et al. 2000, p. 31).
Let $\mathcal{G}$ be a causal graph with vertex set V and P be a probability distribution over the vertices in V such that $\mathcal{G}$ and P satisfy the Causal Markov Condition. $\mathcal{G}$ and P satisfy the Faithfulness Condition if and only if every conditional independence relation true in P is entailed by the Causal Markov Condition applied to $\mathcal{G}$.

Causal Markov and Faithfulness Condition together entail a reciprocal implication between the causal graph $\mathcal{G}$ that (it is assumed) has generated the data and the joint distribution P of a set X of random variable, whose realizations constitute the data. The constraint-based approach to *causal discovery* takes place in a framework in which the conditional independence relations among the variables are known, whereas the causal graph $\mathcal{G}$ is unknown. Both assumptions A1 and A2 should be taken with caution, because, although in general statistical models for social sciences with a causal significance satisfy these conditions (Spirtes et al. 2000, p. 29), there are still several environments where such conditions are violated.

A first issue that could be seen as controversial is whether such a causal structure that has generated the data exists, although there are many environments, as in macroeconomics, where the assumption that it exists can be taken at least as a good approximation. It may be useful to regard Causal Markov Condition as containing the two following claims (Hausman and Woodward, 1999, p. 524). Given a set $V = \{X_1, \ldots, X_n\}$ of random variables generated by a causal structure: (i) if X_i and X_j are probabilistically dependent, then either X_i causes X_j or X_j causes X_i or X_i and X_j are effects of some common cause X_h; (ii) for every variable X_i in V it holds that, conditional on its direct causes, X_i is independent of every other variable in V except its effects.

There are environments where one should expect these conditions to be violated. Causal Markov Condition does not hold if relevant variables to the causal structure are not included in V, if probabilistic dependencies are drawn from nonhomogenous populations, if variables are not properly distinct from one another or if one is in environments (for example in quantum mechanical experiments) where causality cannot assumed to be local in time and space. However, in all the environments where one can exclude "nonsense correlations" and assume temporally and spatially local causality, one can think the Causal Markov Condition to be satisfied. In macroeconomics Causal Markov Condition should be assumed with caution, for the use of time series data and the problem of aggregation (see Hoover 2001, pp. 167-168).

The Faithfulness Condition claims that $P(V)$ embodies only independencies that can be represented in a causal graph, excluding independencies that are sensitive to particular values of the parameters and vanish when such parameters are slightly modified. Pearl (2000, p.48 and p.63) calls this assumption *stability*, because it corresponds to assume that the relationships among variables generated by a causal structure remains invariant or stable when the system is subjected to external influence. In economics this concept recalls the characterization of causal relations as invariant under interventions by Simon (1953) and Frisch and Haavelmo's concept of "autonomy" or "structural invariance" (see Aldrich 1989).

Based on Causal Markov and Faithfulness Condition, Spirtes et al. (2000) provide some algorithms (operationalized in a computer program called Tetrad) that from tests on conditional independence relationships identify the causal graph, which usually is not a unique DAG, but a class of Markov equivalent DAGs, i.e. DAGs that have the same set of d-separation relations. Variants of these algorithms are given for environments where the possibility of latent variables is allowed (Spirtes et al. 2000, chapter 6). Richardson and Spirtes (1999) extend the procedure to situations involving cycles and feedbacks.

In this work, Causal Markov and Faithfulness Condition will be taken as working assumptions. In fact, before applying this method, specification issues such latent variables, aggregation and structural breaks should be emphasized. In other words, the statistical techniques are being presented work in a correct way, as long as they are based on sound background knowledge, besides the data.

3 Recovering the structural model

In this section I present a method to identify a VAR using graphical models. A causal graph is associated to the unobserved structural model and the problem of identification is studied as a problem of searching a directed acyclic graph from vanishing partial correlations. The next subsection shows how to associate a DAG to a structural model. Subsection 3.2 presents a result about the relations holding between vanishing partial correlations among residuals and vanishing partial correlations among contemporaneous variables in a VAR model. Subsection 3.3 applies this and more graph theory results to develop a search algorithm to derive a set of DAGs, which represents the acceptable causal structures among contemporaneous variables, from vanishing partial correlations among the VAR residuals. Subsection 3.4 summarizes the method. Vanishing partial correlations among residuals are tested according a Wald test procedure described in Appendix A.

3.1 Causal graph for the structural model

Following Bernanke (1986), let us suppose that a $(k \times 1)$ vector of macroeconomic variables $Y_t = (y_{1t}, \ldots, y_{kt})'$ is governed by a structural model:

$$Y_t = \sum_{i=0}^{p} B_i Y_{t-i} + C v_t, \tag{3}$$

where the vector of the "structural disturbances" $v_t = (v_{1t}, \ldots, v_{kt})'$ is serially uncorrelated and $E(v_t v_t') = \Sigma_v$ is a diagonal matrix. The B_i $(i = 0, \ldots, p)$ are $(k \times k)$ matrices. It is assumed that the equation (3) represents a causal structure which has generated the data. Such causal structure can be represented by a causal DAG, whose vertices are the elements of $Y_t, \ldots, Y_{t-p}$.

Notice that since it is assumed that the causal structure is representable by means of a DAG, feedbacks are excluded. This is the same as assuming that if the (i, j) element of B_0 is different from zero, then the (j, i) element of B_0 must be equal to zero. The extension to mixed graphs in which undirected edges are allowed among contemporaneous variables (while edges among lagged variables remain directed) is left to further research (see Moneta 2004).

I assume here that $C = I_k$, so that the relations among the contemporaneous components of Y_t are embedded only in the matrix B_0. It is possible to generalize by allowing $C \neq I_k$, and adapting the algorithm given here to a more complex pattern. However, assuming $C = I_k$ does not impede a structural shock v_{it} to affect simultaneously components of Y_t besides y_{it}. This assumption means only that, for example, v_{it} affects y_{jt} through the effect of y_{it} on y_{jt} and not directly. In many contexts the two situations are observationally equivalent.

Although the entire structural model can be represented by a DAG, the focus here is the subgraph[3] induced on the contemporaneous variables $y_{1t}, \dots, y_{kt}$. This subgraph is tied to the matrix B_0, in the sense that there is a directed edge pointing from y_{it} to y_{jt} if and only if the element corresponding to the j^{th} row and the i^{th} column of B_0 is different from zero. Recovering the matrix B_0 is sufficient to recover the structural model because it permits to impose the right transformation on the estimated reduced form.

The method proposed here is consistent with the structural VAR approach and starts by estimating the reduced form:

$$Y_t = \sum_{i=1}^{p} A_i Y_{t-i} + u_t, \tag{4}$$

where $A_i = (I - B_0)^{-1} B_i$, for $i = 1, \dots, p$. The vector $u_t = (I - B_0)^{-1} v_t$ is a serially uncorrelated vector of disturbances. It holds that:

$$u_t = B_0 u_t + v_t \tag{5}$$

Then, from the estimate of the covariance matrix of u_t ($\hat{\Sigma}_u$), it is possible to test all the possible vanishing partial correlations among the elements of u_t. Such tests are used to constrain the possible causal relationships among the contemporaneous variables. The next subsections illustrate this procedure. For convenience, I assume the vector of the error terms u_t to be normally distributed. However, the testing and search procedure can be extended to non-Gaussian processes (see footnote 5 below).

3.2 Partial correlations among residuals

The correlation coefficient and the partial correlation coefficient are measures of dependence between variates. For a clear definition of partial correlation see Anderson (1958, p. 34). Let $X = (x_1, \dots, x_p)'$ be a vector of random variables and let us denote by $\rho(x_i, x_j | x_{q+1}, \dots, x_p)$ or by $\rho_{ij.q+1,\dots,p}$ the *partial correlation* between x_i and x_j given $x_{q+1}, \dots, x_p$. Then it holds that:

$$\rho_{ij.q+1,\dots,p} = \frac{\rho_{ij.q+2,\dots,p} - \rho_{i(q+1).q+2,\dots,p}\ \rho_{j(q+1).q+2,\dots,p}}{\sqrt{1 - \rho^2_{i(q+1).q+2,\dots,p}}\sqrt{1 - \rho^2_{j(q+1).q+2,\dots,p}}} \tag{6}$$

(see Anderson 1958, p. 35).

I want to show that partial correlations among the residuals u_t in (4) are tied to partial correlations among the contemporaneous components of Y_t.

Proposition 3.1. *Let $u_{1t}, \dots, u_{kt}$ be the residuals of k OLS regressions of $y_{1t}, \dots, y_{kt}$ on*

[3]The graph $\mathcal{G}_A = (A, E_A)$ is called a *subgraph* of $\mathcal{G} = (V, E)$ if $A \subseteq V$ and $E_A \subseteq E \cap (A \times A)$. Besides, if $E_A = E \cap (A \times A)$, G_A is called the subgraph of G *induced* on the vertex set A.

the same vector $J_{t-1} = (y_{1(t-1)}, \ldots, y_{k(t-1)}, \ \cdots \ , y_{1(t-p)}, \ldots, y_{k(t-p)})$. *Let* u_{it} *and* u_{jt} $(i \neq j)$ *be any two distinct elements of* $\{u_{1t}, \ldots, u_{kt}\}$, $\mathcal{U}_t$ *any subset of* $\{u_{1t}, \ldots, u_{kt}\} \setminus \{u_{it}, u_{jt}\}$ *and* $\mathcal{Y}_t$ *the corresponding subset of* $\{y_{1t}, \ldots, y_{kt}\} \setminus \{y_{it}, y_{jt}\}$, *so that* u_{gt} *is in* $\mathcal{U}_t$ *iff* y_{gt} *is in* $\mathcal{Y}_t$, *for* $g = 1, \ldots, k$. *Then it holds that:*

$$\rho(u_{it}, u_{jt}|\mathcal{U}_t) = \rho(y_{it}, y_{jt}|\mathcal{Y}_t, J_{t-1}).$$

Proof of Proposition 3.1. It follows by well known orthogonal properties of linear least squares residuals (see e.g. Whittaker 1990, pp. 125-132).[4]

To test vanishing partial correlations among residuals I apply a procedure illustrated in Appendix A.

If one considers only multivariate normal distributions, vanishing partial correlations and conditional independence relationships are equivalent. Therefore, if one considers a DAG with set of vertices $\mathbf{X} = \{X_1, \ldots, X_n\}$ and a normal probability distribution $P(\mathbf{X})$ that satisfy Markov and Faithfulness condition, it holds that: $\rho(X_i, X_j|X^{(h)}) = 0$ if and only if X_i *is independent from* X_j *given* $X^{(h)}$ if and only if X_i *and* X_j *are d-separated by* $X^{(h)}$, where $X^{(h)}$ is any subset of $\mathbf{X}\setminus\{X_i, X_j\}$ and $i \neq j$.[5]

3.3 Searching for the causal graph among contemporaneous variables

In this subsection an algorithm to identify the causal graph among the contemporaneous variables is presented. In real applications the output of the algorithm is an unique DAG only in rare cases. The algorithm allows to narrow significantly the set of possible DAGs and the output obtained is usually a pattern of DAGs. Therefore, some background knowledge may be necessary to select the appropriate DAG from this pattern.

Proposition 3.1 implies that testing a vanishing partial correlation coefficient between u_{it} and u_{jt} given some other components $u_{qt}, \ldots, u_{pt}$ is equivalent to test a vanishing partial correlation coefficient between y_{it} and y_{jt} given some other components $y_{qt}, \ldots, y_{pt}$ and J_{t-1}. Therefore, from tests on partial correlations among the components of u_t it is possible to obtain d-separation relations for the graphical causal model representing the structural equation (3). The next proposition proves that the d-separation relations that obtained correspond to all the possible d-separation relations among the contemporaneous variables for the graph

[4]The proof is available from the author on request.

[5]However, some results of Spirtes et al. (2000, p. 47) show that assuming the Faithfulness Condition for linear systems is equivalent to assume that in a graph G the vertices A and B are d-separated given a subset C of the vertices of G if and only if $\rho(A, B|C) = 0$, without any normality assumption.

induced on the contemporaneous variables $y_{1t}, \ldots, y_{kt}$ alone.

Proposition 3.2. *Let us call $\mathcal{G}$ the causal DAG representing equation (3) and $\mathcal{G}_{Y_t}$ the subgraph of $\mathcal{G}$ induced on $y_{1t}, \ldots, y_{kt}$. Let J_{t-1} and $\mathcal{Y}_t$ be the same as in Proposition 3.1. y_{it} and y_{jt} are d-separated by $\mathcal{Y}_t$ and J_{t-1} in $\mathcal{G}$, if and only if y_{it} and y_{jt} are d-separated by $\mathcal{Y}_t$ in $\mathcal{G}_{Y_t}$.*

Proof of Proposition 3.2. See Appendix B.

The next proposition shows that d-connection (d-separation) relations entail some restrictions on the graph in terms of adjacencies among the vertices and directions of the edges. The aim is to justify the procedures given by the search algorithm below.

Proposition 3.3. *$\mathcal{G}_{Y_t}$ is defined as in Proposition 3.2. Let us assume $P(\boldsymbol{X})$ to be a probability distribution over the variables $\boldsymbol{X}$ that form $\mathcal{G}_{Y_t}$, such that $< \mathcal{G}_{Y_t}, P(\boldsymbol{X}) >$ satisfies Markov and Faithfulness Condition. Then: (i) for all distinct vertices y_{it} and y_{jt} of $\mathcal{G}_{Y_t}$, y_{it} and y_{jt} are adjacent in $\mathcal{G}$ if and only if y_{it} and y_{jt} are d-connected in $\mathcal{G}_{Y_t}$ conditional on every set of vertices of $\mathcal{G}_{Y_t}$ that does not include y_{it} and y_{jt}; and (ii) for all vertices y_{ht}, y_{it} and y_{jt} such that y_{ht} is adjacent to y_{it} and y_{it} is adjacent to y_{jt}, but y_{ht} and y_{jt} are not adjacent, $y_{ht} \longrightarrow y_{it} \longleftarrow y_{jt}$ is a subgraph of $\mathcal{G}_{Y_t}$ if and only if y_{ht}, y_{jt} are d-connected in $\mathcal{G}_{Y_t}$ conditional on every set of vertices of $\mathcal{G}_{Y_t}$ containing y_{it} but not y_{ht} or y_{jt}.*

Proof of Proposition 3.3. This proposition is a particular case of a theorem proved in Spirtes et al. (2000, theorem 3.4, p. 47) and in Verma and Pearl (1990).

The goal of the algorithm described in Table 1 is to obtain a (possibly narrow) class of DAGs, which contains the causal structure among the contemporaneous variables $\mathcal{G}_{Y_t}$. The algorithm starts from a complete undirected graph $\mathcal{C}$ among the k components of Y_t (in which every vertex is connected with everything else) and uses d-separation relations to eliminate and direct as many edges as it is possible.

The modifications, anticipated in the Introduction, I made to the PC algorithm of Spirtes *et. al.* (2000, pp. 84-85) are the following. The first important difference is the definition of *Sepset* (step A of the algorithm). I define *Sepset* (y_{ht}, y_{it}) at the beginning, and once for all, as the set of sets of vertices S so that y_{ht} and y_{it} are d-separated by S. On the contrary, in Spirtes et al. (2000, p. 84) *Sepset* is defined in the step B of the algorithm and contains only one set of vertices S so that y_{ht} and y_{it} are d-separated by S.

Indeed, if I were using the original formulation of the PC algorithm the middle part of step B would have been written as: "...and if y_{ht} and y_{it} are d-separated by S in $\mathcal{G}_{Y_t}$ delete edge

Table 1: Search algorithm (adapted from the PC Algorithm of Spirtes et al. 2000, pp. 84-85: in bold character the modifications).

A.)

Form the complete undirected graph $\mathcal{C}$ on the vertex set $y_{1t}, \ldots, y_{kt}$. Let *Adjacencies*$(\mathcal{C}, y_{it})$ be the set of vertices adjacent to y_{it} in $\mathcal{C}$ and let *Sepset* (y_{ht}, y_{it}) be **the set of sets of vertices** S so that y_{ht} and y_{it} are d-separated given S;

B.)

$n = 0$

repeat :

  repeat :

    select an ordered pairs of variables y_{ht} and y_{it} that are adjacent in $\mathcal{C}$ such that *Adjacencies*$(\mathcal{C}, y_{ht})\backslash\{y_{it}\}$ has cardinality greater than or equal to n, and a subset S of *Adjacencies*$(\mathcal{C}, y_{ht})\backslash\{y_{it}\}$ of cardinality n, and if y_{ht} and y_{it} are d-separated given S in $\mathcal{G}_{Y_t}$ delete edge y_{ht} — y_{it} from $\mathcal{C}$;

  until all ordered pairs of adjacent variables y_{ht} and y_{it} such that *Adjacencies*$(\mathcal{C}, y_{ht})\backslash\{y_{it}\}$ has cardinality greater than or equal to n and all subsets S of *Adjacencies*$(\mathcal{C}, y_{ht})$ $\backslash\{y_{it}\}$ of cardinality n have been tested for d-separation;

  $n = n + 1$;

until for each ordered pair of adjacent variables y_{ht}, y_{it}, *Adjacencies*$(\mathcal{C}, y_{ht})$ $\backslash\{y_{it}\}$ is of cardinality less than n;

C.)

for each triple of vertices y_{ht}, y_{it}, y_{jt} such that the pair y_{ht}, y_{it} and the pair y_{it}, y_{jt} are each adjacent in $\mathcal{C}$ but the pair y_{ht}, y_{jt} is not adjacent in $\mathcal{C}$, orient y_{ht} — y_{it} — y_{jt} as $y_{ht} \longrightarrow y_{it} \longleftarrow y_{jt}$ if and only if y_{it} **does not belong to any set** of *Sepset*(y_{ht}, y_{jt});

D.)

repeat :

  if $y_{at} \longrightarrow y_{bt}$, y_{bt} and y_{ct} are adjacent, y_{at} and y_{ct} are not adjacent and y_{bt} belongs to **every set of** *Sepset*(y_{at}, y_{ct}), then orient y_{bt} — y_{ct} as $y_{bt} \longrightarrow y_{ct}$;

  if there is a directed path from y_{at} to y_{bt}, and an edge between y_{at} and y_{bt}, then orient y_{at} — y_{bt} as $y_{at} \longrightarrow y_{bt}$;

until no more edges can be oriented.

y_{ht} — y_{it} from $\mathcal{C}$ and record S in $Sepset(y_{ht},\, y_{it})$ and $Sepset(y_{it},\, y_{ht})$."

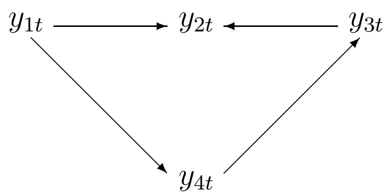
The second change I made with respect to the PC algorithm is at the end of step C. My formulation: "... orient y_{ht} — y_{it} — y_{jt} as $y_{ht} \longrightarrow y_{it} \longleftarrow y_{jt}$ if and only if y_{it} does not belong to any set of $Sepset(y_{ht}, y_{jt})$." Following the original PC algorithm I would have written: "... orient y_{ht} — y_{it} — y_{jt} as $y_{ht} \longrightarrow y_{it} \longleftarrow y_{jt}$ if and only if y_{it} is not in $Sepset(y_{ht}, y_{jt})$."

The third change is in step D. My formulation: "...and y_{bt} belongs to every set of $Sepset$ (y_{at}, y_{ct}), then orient y_{bt} — y_{ct} as $y_{bt} \longrightarrow y_{ct}$." The original PC algorithm formulation would be: "...and there is no arrowhead at y_{bt}, then orient y_{bt} — y_{ct} as $y_{bt} \longrightarrow y_{ct}$."

These modifications of the original formulation of the PC algorithm have simply one goal: providing more stability to the algorithm task of directing edges. The original PC algorithm is very efficient from a computational point of view, since it minimizes the number of conditional indipendence relations to be tested, but it is quite unstable, in the sense that small errors of input can produce large errors of output (wrong direction of edges). It works very well when the number of variables is high and the vanishing partial correlations are "faithful," that is generated by the causal structure. But, as the empirical application will show, in the case of contemporaneous causal structure in a VAR model, it is likely to have a small number of vanishing partial correlations which are "unfaithful," that is unrelated to the causal structure. This may be due to the problem of temporal aggregation, latent variables or feedbacks. In this case one has to be very cautious in the task of directing edges.

Suppose, for example, that the unobserved causal structure is described by the DAG in Figure 2.

Figure 2



Suppose also that the results of the tests on vanishing partial correlation say that all the d-separation relations are the following: y_{1t} and y_{3t} are d-separated by y_{2t}; y_{1t} and y_{3t} are d-separated by y_{4t}; y_{2t} and y_{4t} are d-separated by $\{y_{1t}, y_{3t}\}$. Then, the d-separation between y_{1t} and y_{3t} given y_{2t} is wrong, due to an error in the vanishing partial correlation test, or to the presence of an unfaithful vanishing partial correlation. Suppose that one uses the original PC algorithm to infer the causal DAG and that the algorithm selects in the step B the pair of variables y_{1t} and y_{3t} and $S = \{y_{2t}\}$. Then, the algorithm would correctly delete the edge between y_{1t} and y_{3t} and record S in $Sepset(y_{1t},\, y_{3t})$. But in the step C it would wrongly orient y_{1t} — y_{4t} — y_{3t} as $y_{1t} \longrightarrow y_{4t} \longleftarrow y_{3t}$, since y_{4t} was not recorded in $Sepset(y_{1t},\, y_{3t})$. In step D the algorithm would not produce any orientation.

In this example, my version of the algorithm would not produce any orientation in step C and D, leaving this task to background knowledge or to simply rules of thumbs such as: in y_{1t} and y_{3t} cannot be any collider, so it has to be either in y_{2t} or y_{4t}, but looking at *all* the d-separation relations, it seems to be more likely that the collider is in y_{2t}, etc.

Thus, the ultimate reason in changing the algorithm is that in VAR models there is no computational constraint in testing a large set of vanishing partial correlation. In the case of six time series variables, for example, one may look even at all the possible vanishing partial correlation tests. The criterion of orienting edges is more severe in the version of the algorithm I propose, because errors in conditional independence tests are always possible.

3.4 Summary of the search procedure

The search procedure for identifying the graph of the structural model can be summarized as follows:

Step 1: Estimate a VAR and perform the usual diagnostic checking of the Box-Jenkins methodology. Testing hypothesis on structural change is particularly important to assume the Faithfulness Condition.

Step 2: Estimate the covariance matrix of the residuals from the reduced form.

Step 3: Test all the possible vanishing partial correlations among the residuals (according to the procedure described in Appendix A) and list the consequent d-separation sets among the contemporaneous variables.

Step 4: Apply the Search Algorithm (plus background knowledge) described in Table 1 to such d-separation sets in order to determine the causal structure among the contemporaneous variables.

4 Empirical application

The procedure to recover the structural model, which is represented by a causal graph, from a VAR has been developed so far for stationary data. In this section I show how this procedure can be extended to nonstationary time series, for the particular case in which data are cointegrated, i.e. there are some linear combinations of the time series which are stationary. Finally, an empirical example with macroeconomic data is discussed.

4.1 The case of cointegrated data

Suppose Y_t is a Gaussian k-dimensional VAR(p) process, whose components $y_{1t}, \ldots, y_{kt}$ are $I(1)$, and suppose there are r linearly independent $(k \times 1)$ vectors c_i such that $c_i' Y_t \sim I(0)$, for $i = 1, \ldots, r$. In this case, it is well known that it is possible to reparameterize the model in level

$$Y_t = A_1 Y_{t-1} + \ldots + A_p Y_{t-p} + u_t \tag{7}$$

as

$$\Delta Y_t = D_1 \Delta Y_{t-1} + \ldots + D_{p-1} \Delta Y_{t-p+1} - \Pi Y_{t-p} + u_t, \tag{8}$$

where $D_i = -(I_k - A_1 - \ldots - A_i)$, for $i = 1, \ldots, p-1$ and $\Pi = I_k - A_1 - \ldots - A_p$. The $(k \times k)$ matrix Π has rank r and thus Π can be written as HC with H and C' of dimension $(k \times r)$ and of rank r. $C \equiv [c_1, \ldots, c_r]'$ is called the *cointegrating matrix*.

Is is also well known (see Lütkepohl 1991, pp. 356-358) that, if $\tilde{C}, \tilde{H}$ and $\tilde{D}$ are the maximum likelihood estimator of C, H, according to Johansen's (1988, 1991) approach, then the asymptotic distribution of $\tilde{\Sigma}_u$, that is the maximum likelihood estimator of the covariance matrix of u_t, is:

$$\sqrt{T} vech(\tilde{\Sigma}_u - \Sigma_u) \xrightarrow{d} N(\mathbf{0},\ 2\mathbf{D}_k^{+}(\Sigma_u \otimes \Sigma_u)\mathbf{D}_k^{+'}), \tag{9}$$

where $\mathbf{D}_k^{+'} \equiv (\mathbf{D}_k'\mathbf{D}_k)^{-1}\mathbf{D}_k'$ and $\mathbf{D}_k$ is the duplication matrix. Confronting equation (9) with equation (12) in Appendix A, it turns out that the asymptotic distribution of $\tilde{\Sigma}_u$ is the same as in the case of a stationary VAR model.

Thus, the application of the method described sofar to cointegrated data is straightforward. The model can, in this case, be estimated as an error correction model using Johansen's (1988, 1991) approach, and then, since the asymptotic distribution of $\tilde{\Sigma}_u$ is the same as in the stationary case, one can apply the testing procedure described in Appendix A to obtain the set of vanishing partial correlations among the residuals.

The results obtained in the last section hold also for nonstationary time series. Thus, vanishing partial correlations among residuals are equivalent to d-separation relations among contemporaneous variables and the search algorithm of Table 1 is applicable.

4.2 Results

The method discussed is applied to an updated version of the data set used by King et al. (1991). The data are six quarterly U.S. macro variables for the period 1947:2 to 1994:1 (188 observations): C denotes the real 1987 per capita consumption expenditures (in logarithms); I denotes the real 1987 per capita investment (in logarithms); M denotes the real balances, the logarithm of per capita M2 minus the logarithm of the implicit price deflator; Y denotes the real 1987 per capita "private" gross national product (total GNP less real total government purchases of goods and services, in logarithms); R denotes the nominal interest rate, 3-month U.S. Treasury bill rate; ΔP denotes the price inflation, log of the implicit price deflator at the time t minus log of the implicit price deflator at the time t-1.

The model is estimated in the ECM formulation of equation (8), where Y_t =(C_t, I_t, M_t, Y_t, R_t, ΔP_t), with the addition of an intercept term ν. In accordance with the model and estimation of King et al. (1991), eight lags of the first differences are used and three cointegrating relationships are imposed. The cointegrating relationships are between C_t and Y_t, between I_t and Y_t and among M_t, Y_t and R_t. The maximum likelihood estimation of the matrix of variance and covariance among the error terms turns out to be:

$$\tilde{\Sigma}_u = \begin{bmatrix} 322 & 557 & 103 & 298 & 8418 & -663 \\ 557 & 2942 & 416 & 958 & 37101 & 5368 \\ 103 & 416 & 4896 & 11 & -5152 & -77904 \\ 298 & 958 & 11 & 631 & 16688 & 18496 \\ 8418 & 37101 & -5152 & 16688 & 3156879 & 84176 \\ -663 & 5368 & -77904 & 18496 & 84176 & 26282024 \end{bmatrix} \times 10^{-7}.$$

Using the test procedure described in Appendix A, all the possible (partial) correlations among the error terms $u_{C_t}, u_{I_t}, u_{M_t}, u_{Y_t}, u_{R_t}, u_{\Delta P_t}$, which determine a class of d-separation relations among contemporaneous variables, are estimated. In Table 2 d-separation relations between each couple of contemporaneous variables are shown.

Applying the search algorithm described in Table 1 to d-separation relations among the error terms tested at 0.05 level of significance, the pattern of DAGs shown in Figure 3 is obtained, where $C, I, M, Y, R, \Delta P$ correspond to $y_{1t}, y_{2t}, y_{3t}, y_{4t}, y_{5t}, y_{6t}$ respectively.

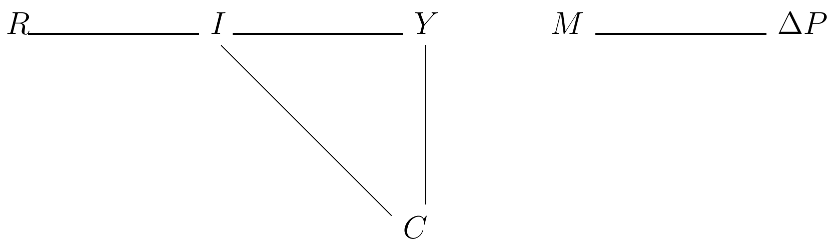


Figure 3: Output of the search algorithm.

The set of DAGs for this pattern consists of 24 elements, which are all testable, because they imply overidentifying constraints. I exclude from this pattern the DAGs which contain one or both of the following configurations: $R \rightarrow I \leftarrow Y$ and $R \rightarrow I \leftarrow C$. This is motivated by the fact that I is contained in several d-separation sets of both $< C \quad R >$ and $< Y \quad R >$ (see Table 2). The number of DAGs ruled out is 8, thus there are 16 DAGs left. This exclusion is also supported by likelihood ratio tests on the overidentifying constraints that this set of DAGs implies. Indeed the 8 models excluded are rejected, while the other 16 models are not

Table 2: d-separation relations.

	$Sepset(1)$	$Sepset(2)$	$Sepset(3)$	$Sepset(4)$
$C\ I$				
$C\ M$	$\{\emptyset\}$ $\{Y\}$ $\{R\}$ $\{\Delta P\}$ $\{I\}$	$\{I,Y\}\{I,R\}$ $\{I,\Delta P\}$ $\{Y,R\}$ $\{Y,\Delta P\}$ $\{R,\Delta P\}$	$\{I,Y,R\}$ $\{I,R,\Delta P\}$ $\{I,Y,\Delta P\}$ $\{Y,R,\Delta P\}$	$\{I,Y,R,\Delta P\}$
$C\ Y$				
$C\ R$	$\{I\}$ $\{Y\}$	$\{I,M\}$ $\{I,Y\}$ $\{I,\Delta P\}$ $\{M,Y\}$ $\{Y,\Delta P\}$	$\{I,M,Y\}$ $\{I,M,\Delta P\}$ $\{I,Y,\Delta P\}$ $\{M,Y,\Delta P\}$	$\{I,M,Y,\Delta P\}$
$C\ \Delta P$	$\{\emptyset\}$ $\{I\}$ $\{M\}$ $\{Y\}$ $\{R\}$	$\{I,M\}$ $\{I,Y\}$ $\{I,R\}$ $\{M,Y\}$ $\{M,R\}$ $\{Y,R\}$	$\{I,M,Y\}$ $\{I,M,R\}$ $\{I,Y,R\}$ $\{M,Y,R\}$	$\{I,M,Y,R\}$
$I\ M$	$\{\emptyset\}$ $\{C\}$ $\{Y\}$ $\{R\}$ $\{\Delta P\}$	$\{C,Y\}$ $\{C,R\}$ $\{C,\Delta P\}$ $\{Y,\Delta P\}$ $\{R,\Delta P\}$	$\{C,Y,R\}$ $\{C,Y,\Delta P\}$ $\{C,R,\Delta P\}$ $\{Y,R,\Delta P\}$	$\{C,Y,R,\Delta P\}$
$I\ Y$				
$I\ R$				
$I\ \Delta P$	$\{\emptyset\}$ $\{C\}$ $\{M\}$ $\{Y\}$ $\{R\}$	$\{C,M\}$ $\{C,Y\}$ $\{C,R\}$ $\{M,Y\}$ $\{M,R\}$ $\{Y,R\}$	$\{C,M,Y\}$ $\{C,M,R\}$ $\{M,Y,R\}$ $\{C,Y,R\}$	$\{C,M,Y,R\}$
$M\ Y$	$\{\emptyset\}$ $\{C\}$ $\{I\}$ $\{R\}$ $\{\Delta P\}$	$\{C,I\}$ $\{C,R\}$ $\{C,\Delta P\}$ $\{I,R\}$ $\{I,\Delta P\}$ $\{R,\Delta P\}$	$\{C,I,R\}$ $\{C,I,\Delta P\}$ $\{C,R,\Delta P\}$ $\{I,R,\Delta P\}$	$\{C,Y,R,\Delta P\}$
$M\ R$	$\{\emptyset\}$ $\{C\}$ $\{I\}$ $\{Y\}$ $\{\Delta P\}$	$\{C,I\}$ $\{C,Y\}$ $\{C,\Delta P\}$ $\{I,Y\}$ $\{I,\Delta P\}$ $\{Y,\Delta P\}$	$\{C,I,Y\}$ $\{C,I,\Delta P\}$ $\{C,Y,\Delta P\}$ $\{I,Y,\Delta P\}$	$\{C,I,Y,\Delta P\}$
$M\ \Delta P$				
$Y\ R$		$\{C,I\}$ $\{I,M\}$	$\{C,I,M\}$ $\{C,I,\Delta P\}$ $\{I,M,\Delta P\}$	$\{C,I,M,\Delta P\}$
$Y\ \Delta P$	$\{\emptyset\}$ $\{M\}$ $\{R\}$			
$R\ \Delta P$	$\{\emptyset\}$ $\{C\}$ $\{I\}\{M\}$ $\{Y\}$	$\{C,I\}$ $\{C,M\}$ $\{C,Y\}$ $\{I,M\}$ $\{I,Y\}$ $\{M,Y\}$	$\{C,I,M\}$ $\{C,I,Y\}$ $\{C,M,Y\}$ $\{I,M,Y\}$	$\{C,I,M,Y\}$

Notes: $C,I,M,Y,R,\Delta P$ correspond to $y_{1t},y_{2t},y_{3t},y_{4t},y_{5t},y_{6t}$. For each couple of error terms, the Table shows the separation sets of cardinality 1,2,3,4. D-separation relations are derived by Wald tests on vanishing (partial) correlations at 0.05 level of significance (for the testing procedure see Appendix A).

rejected[6]. Among these models, two are consistent with the conjecture that interest rate and investment are leading indicator for output, and money is a leading indicator for inflation, which correspond to the graphs shown in Figure 4.

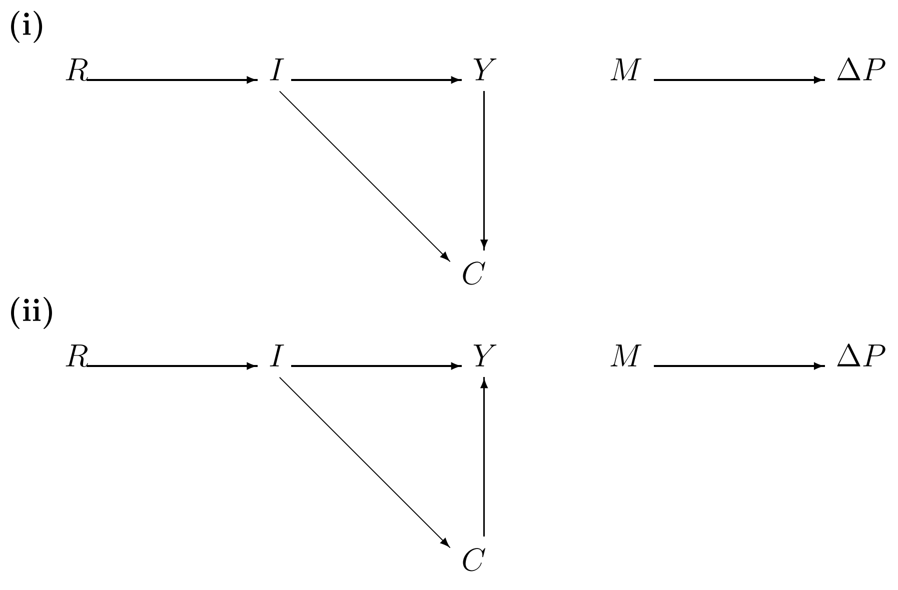


Figure 4. (i) Causal graph for model 1. (ii) Causal graph for model 3.

I proceed to estimate the model associated with graph (i) of Figure 4, which I call model 1, and to calculate the impulse response functions associated with it. Then I explore the sensitivity of the results to changes in the specifications of the model. It should be noted, however, that not all the d-separation relations that were found to hold in the data, are implied by the 16 DAGs output of the search procedure. In particular, C and R were found to be d-separated by the sets $\{Y\}$, $\{Y, M\}$, $\{Y, \Delta P\}$, $\{Y, M, \Delta P\}$ according to a Wald test of 0.05 level of significance (see Table 2). I interpret this deficiency as deriving by the presence of some "unfaithful" partial correlations, i.e. vanishing partial correlations which are not tied to the causal structure generating the data and could be connected with some misspecification of the model.

From each of the two graphical causal model among the error terms it is possible to derive the zeros in the matrix B_0 of equations (5). The matrix B_0 corresponding to model 1 of Figure 4 is:

[6]These results are available from the author on request.

Table 3: Estimation of model 1

Log Likelihood 3371.3585
Log Likelihood Unrestricted 3380.5104
Chi-Squared(10) 18.3038
Significance Level 0.0500

Coefficient	Estimate	Standard Error	T-Statistic	Significance level
b_1	0.0706	0.0266	-2.6491	0.0080
b_2	0.3650	0.0589	-6.1897	0.0000
b_3	0.0117	0.0021	-5.3460	0.0000
b_4	0.3257	0.0259	-12.5531	0.0000
b_5	-15.9090	5.4233	2.9334	0.0033

Notes: The header displays the log likelihood of the estimated model 1, and the log likelihood of an unrestricted model. The likelihood ratio test for the overidentifying restrictions is based on a χ^2 with degrees of freedom equal to the number of overidentifying restrictions. The estimation is performed using the BFGS method in RATS (for details see Doan (2000)).

$$B_0 = \begin{pmatrix} 0 & b_1 & 0 & b_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & b_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_5 & 0 & 0 & 0 \end{pmatrix}.$$

The results of the maximum likelihood estimates of the nonzero coefficients of B_0, using the RATS procedure illustrated in Doan (2000, p. 295), are shown in Table 3.

The impulse response functions are calculated considering the system in levels. The forecast error of the h-step forecast of Y_t is:

$$Y_{t+h} - Y_t(h) = u_{t+h} + \Phi_1 u_{t+h-1} + \ldots + \Phi_{h-1} u_{t+1}. \tag{10}$$

The Φ_i are obtained by the A_i recursively by:

$$\Phi_i = \sum_{j=1}^{i} \Phi_{i-j} A_j, \qquad i = 1, 2, \ldots$$

with $\Phi_0 = I_k$. Since $v_t = (I - B_0)u_t$, equation (10) can be rewritten as

$$Y_{t+h} - Y_t(h) = \Theta_0 v_{t+h} + \Theta_1 v_{t+h-1} + \ldots + \Theta_{h-1} v_{t+1}, \tag{11}$$

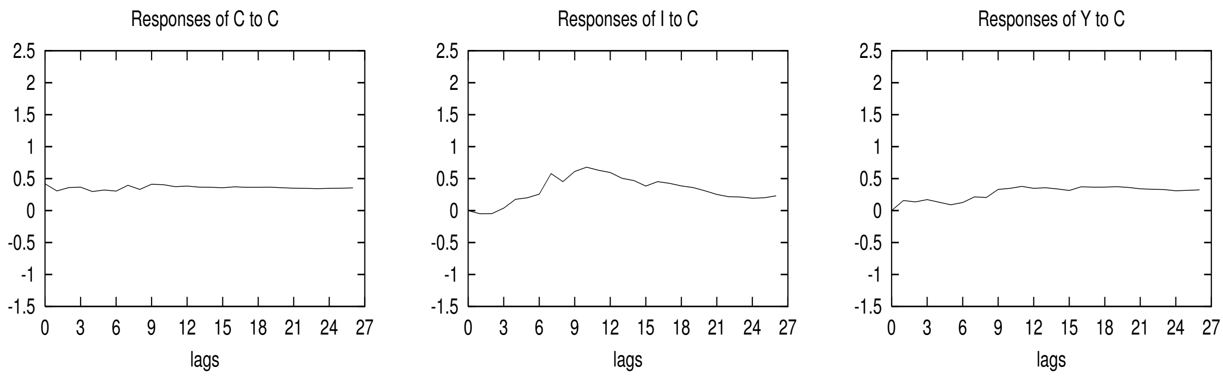


Figure 5: Responses of consumption, investment and output to one-standard-deviation shock in consumption

where $\Theta_i = \Phi_i(I - B_0)^{-1}$. The element (j, k) of Θ_i represents the response of the variable y_j to a unit shock in the variable y_k, i periods ago. The response to one standard deviation innovation is obtained by multiplying the element (j, k) of Θ_i by the standard deviation of the k-th element of v_t. Since the variables are $I(1)$, as i goes to infinity the responses do not necessarily taper off as in a stable system. Figures 5-10 describe the responses of the three real flow variables (C,I,Y) for lags $0 - 26$, calculated using model 1.

Figure 5 shows the responses to one-standard-deviation percent impulse in the consumption shock. The estimated standard deviation of consumption shock is 0.0042 per quarter. The response of consumption to consumption shock is constantly positive. Investment responds slightly negatively over the first few quarters, then increases and ends up having a slightly positive permanent response. The response of output is slightly positive initially, then it ends up being permanently positive in a similar way to the response of consumption.

Figure 6 shows the responses of the variables to one-standard-deviation percent impulse in the investment shock. The estimated standard deviation of investment shock is 0.0159. The response of consumption is positive over the first 6-9 quarters, then turns out to be negligible. Investment, on the other hand, shows a large positive response for the first 6-9 quarters, then turns negative after the 12^{th} quarter and eventually shows a positive response. The response of output is considerably positive over the first 10 quarters, then is negligible.

Figure 7 is the most relevant for the study of the effects of monetary shocks. The figure shows the response of the real flow variables to one-standard-deviation percent impulse in the real balance shock. The estimated standard deviation of this shock is 0.0222. The real balance shock has largely positive and permanent effects on all flow real variables, but over the first three years the effects are smaller than in the long-run. Consumption has a negligible positive response in the first three years, then the response increases. Investment has also a slightly positive response in the first 5 quarters. Then, for the next 7 quarters, the response turns

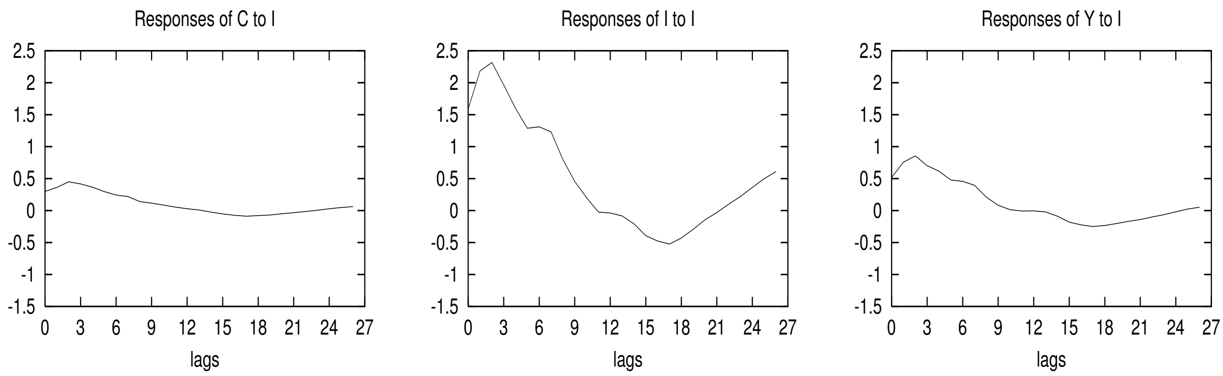


Figure 6: Responses of consumption, investment, and output to one-standard-deviation shock in investment

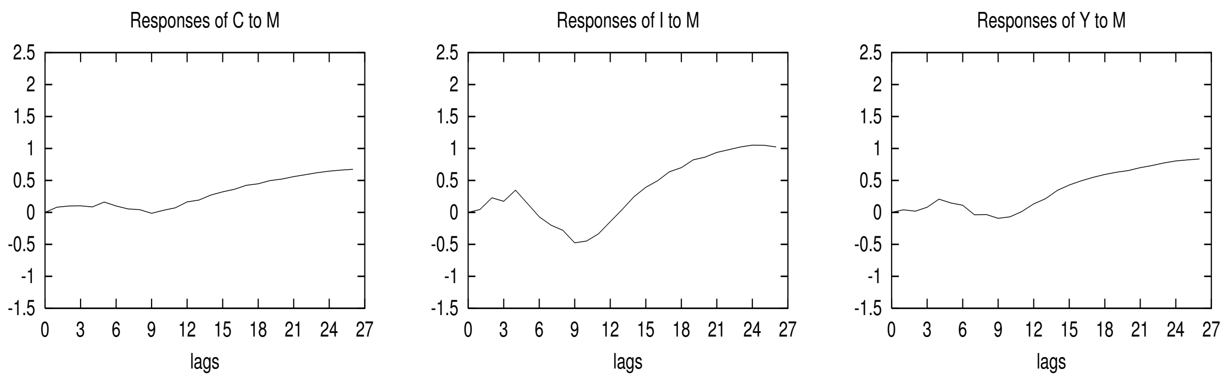


Figure 7: Responses of consumption, investment, and output to one-standard-deviation shock in real balances

out to be negative. Eventually the response is largely positive. The response of output is very similar to the consumption one: negligible for the first three years, then increasing and eventually largely positive.

Figure 8 shows the responses of the variables to one-standard-deviation percent impulse in the output shock. The estimated standard deviation of output shock is 0.0057. The response of consumption is not very large and is quite constant over time. Investment responds considerably around the fourth quarter, but around the 10^{th} quarter the response is negative. Eventually the response is positive. Output has a quite large response in the short-run, then the response decreases and is eventually slightly positive.

Figure 9 shows the responses to one-standard-deviation percent impulse in the interest rate shock. The estimated standard deviation of interest rate shock is 0.5634. The responses of consumption, investment and output are similar: positive in the first quarters, negative in the second and third year, eventually positive. The response of investment is particularly large in

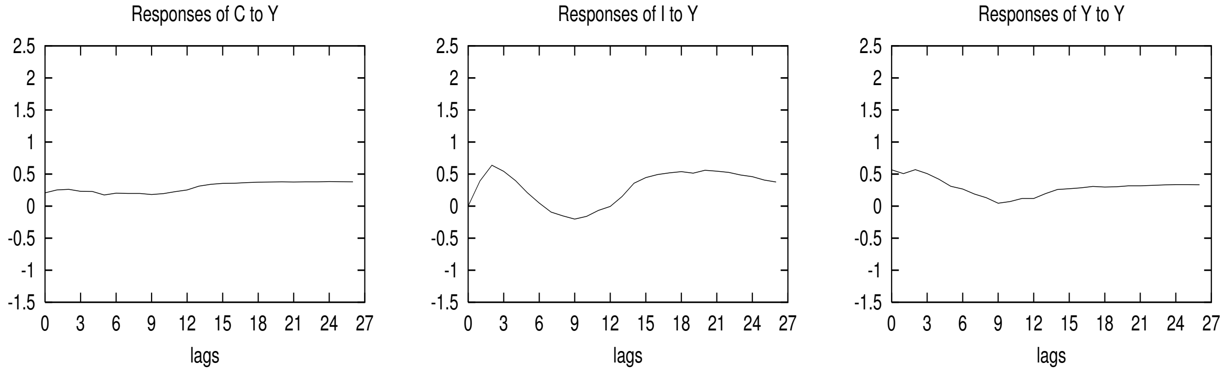


Figure 8: Responses of consumption, investment, and output to one-standard-deviation shock in output

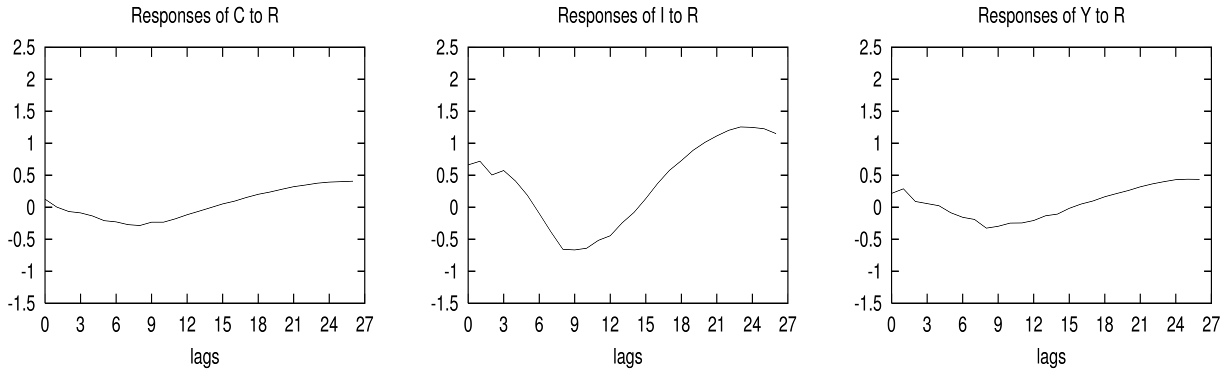


Figure 9: Responses of consumption, investment, and output, to one-standard-deviation shock in interest rate

the long-run.

Figure 10 shows the responses to one-standard-deviation percent impulse in the inflation shock. The estimated standard deviation of interest rate shock is 1.5869. The eventual effect of an inflationary shock to consumption, investment and output is negligibly negative. Consumption is moving down in the second year after the shock. The response of investment is particularly negative in the second and third year, but the shock does not have permanent effects. The response of output is slightly positive in the first year, but it ends up having an almost negligible negative effect.

Some qualitative features of the impulse response functions carry over into all the other 15 specifications output of the search procedure. I focus the sensitivity analysis on the models in which interest rate precedes investment and output. I call model 2 the model which is equal to model 1, except that the relation between real balances and inflation is inverted (we have $M \leftarrow \Delta P$), I call model 3 the model, which corresponds to graph (ii) of Figure 4 and I call

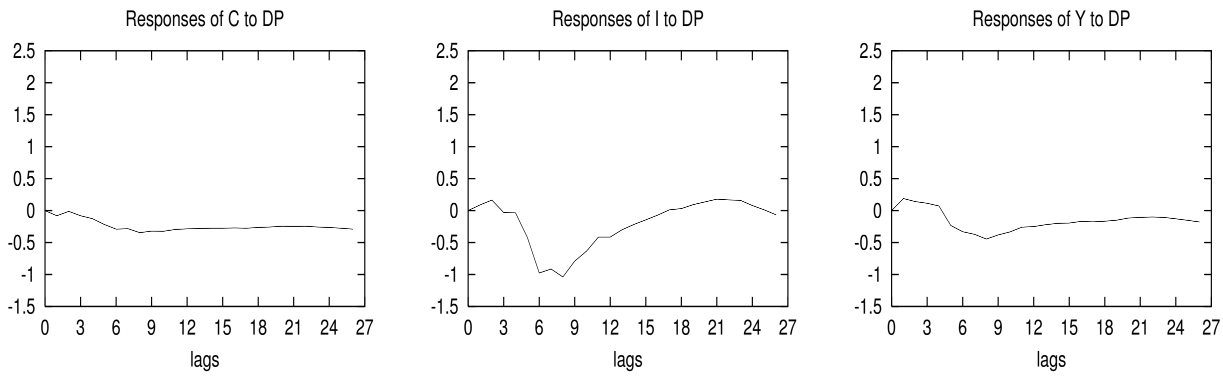


Figure 10: Responses of consumption, investment, and output to one-standard-deviation shock in inflation

model 4 which is equal to model 3, except that the relation between real balances and inflation is $M \leftarrow \Delta P$. Figure 11 shows the impulse response functions calculated for the four models. There are no relevant differences between the impulse response functions derived by model 1 and 2, except for minor differences in the response of I to the real balance shock and in the responses of C, I and Y to the inflation shock.

The impulse response functions calculated using model 3 present some relevant differences with respect to the response functions calculated using the other models. The responses of C to I using model 3 have a shape similar to the responses using 1, except that the former are much lower than the latter: model 3 yields responses of C to I negative in the long-run. The same evidence holds for the responses of Y to I. These differences make model 1 more consistent with broadly accepted stylized facts. There are also quantitative differences as far as responses of I to Y and R are concerned. In particular the responses of I to R are negative for the first three years.

Model 4 yields responses which also present some important differences with respect to the other models. The shape of the responses of I and Y to consumption shocks are quite different from the shape of the responses derived from the other models. The responses of C to output shock are almost null (while in the other models result slightly positive), the responses of I and Y to output shock are also mostly below the other responses. In the other cases, the responses of model 4 are very similar to the responses of model 3.

The results presented here confirm somewhat the analysis of King et al. (1991), according to which postwar US macroeconomic data do not support the key implication of the standard real business cycle model, that permanent productivity shocks are the dominant source of economic fluctuations. Indeed monetary shocks and interest rate shocks seem to play a role not inferior to the one played by shocks associated with consumption, investment and output. The present analysis is different from the one of King et al. (1991), because these authors

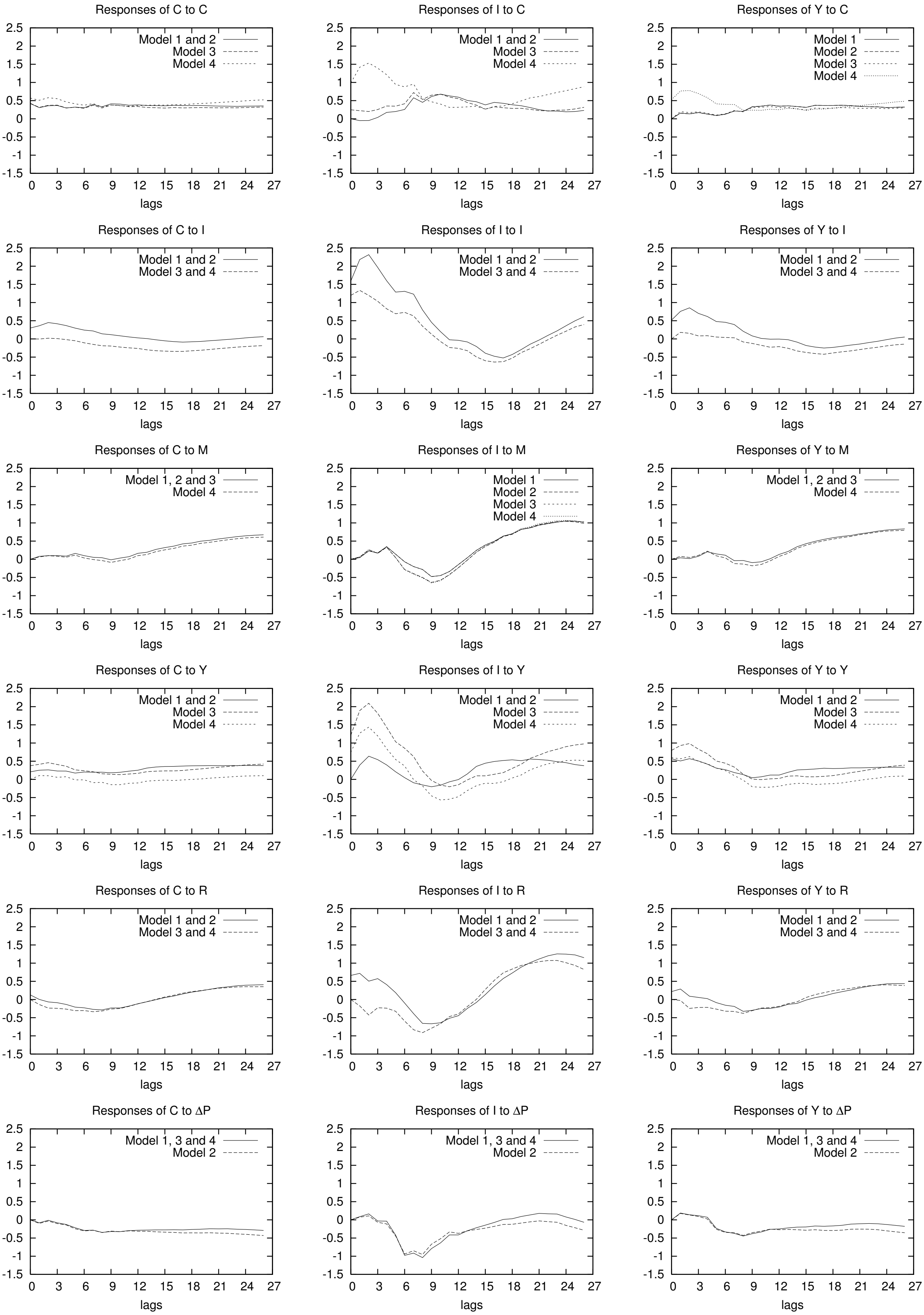


Figure 11: Sensitivity Analysis

impose long-run restrictions in order to obtain three permanent shocks (associated with the common stochastic trends) and three transitory shocks (associated with the cointegrating relationships)[7]. In my analysis each of the six shocks (each of them associated with a particular variable) has, at least theoretically, permanent effects. Thus, it is possible to distinguish among the three real flow variables shocks. Among C, I and Y shocks, the shock associated with investment seems to play the largest role in the short-run. In the long-run a larger role is played by shocks associated with consumption and output. An interesting result of the present analysis is the major role played by the monetary shock, as I interpret the shock associated with M. In the medium-run the effect is non-monotonic, but the permanent effect is largely positive. This result is consistent with the the claim that monetary shocks, not only productivity shocks, are the sources of macroeconomic fluctuations. An important role is also played by the shock associated with the interest rate. Here the responses are much more fluctuating than the case of M shock: positive in the short-run, considerably negative in the medium-run and positive in the long-run. The effect of this shock on investment is particularly large in the short and in the long-run. Thus an important source of economic fluctuations is associated with this shock, in accordance with the results of King et al. (1991). But, as these authors point out, it is somewhat difficult to interpret this shock with standard macroeconomic models. It is also confirmed the small role played by inflation on output in the long-run. Although it has a larger role in explaining investment movements, this result seems at odds with a monetarist perspective.[8]

The sensitivity analysis can be straightforwardly extended to the other 12 models in which interest rate does not causally precede investment and output. I do not report these results here, which do not change the substance of the main conclusions[9]. However, let us emphasize that the main advantage of using this method consists in dealing with a reasonably limited

[7]For a criticism of the use of long-run restrictions to identify a VAR, see Faust and Leeper (1997).

[8]It may also be useful to compare the impulse responses functions obtained in this analysis with the impulse responses functions obtained by King et al. (1991). (See Figure 4 in King et al. (1991, p. 834)). In the six variables model, these authors study the effect of three permanent shock: balanced-growth shock, inflation shock, and real interest rate shock. The shape of the responses of C, I and Y to the balanced growth shock does not present significant similarities with the responses to the consumption, investment or output shock of the present analysis, except for the fact that the responses tend to be positive in both analyses. Perhaps in my analysis it emerges even with more evidence the fact that shocks related to real variables are not significantly more important than shocks related to nominal variables. The responses of Y and C to the inflation shock in the analysis of King et al. are very similar to the responses obtained in my analysis, while the response of I to the same shock is very different: mostly positive in the analysis of King et al, mostly negative in my analysis. There also some similarities in the shape of the responses of C, I and Y to the real (nominal in my analysis) interest shock between the analysis of King et al. and my analysis, but the responses are quite different in quantitative terms.

[9]Results on the other specifications are available from the author on request.

number of models in the sensitivity analysis. Furthermore, the class of models output of my search procedure is a class of overidentified models that can be tested.

5 Conclusions

In this paper a method to identify the causal structure related to a VAR has been proposed. Particular emphasis has been posed on the causal structure among contemporaneous variables, which explains the correlations and the partial correlations among the residuals. The identification of the causal structure among the contemporaneous variables has permitted an orthogonalization of the residuals, which is alternative to the common practice, which uses the Choleski factorization and has been often criticized as arbitrary. The method of identification proposed is based on a graphical search algorithm, which has as inputs tests on vanishing partial correlations among the residuals. Although the method is apparently data-driven, the more background knowledge is incorporated, the more detailed is the causal structure identified. Also the reliability of the latter depends on the reliability of the former. One of the claimed advantage of this method is to give to background knowledge an explicit causal form. In the empirical example considered, prior economic knowledge was essential to select the appropriate model, but since the number of acceptable models was reasonably low, it was possible to assess the robustness of the results to different causal restrictions.

This method will result much improved if the possibility of latent variables and the possibility of feedbacks and cycles (among contemporaneous variables) will be taken into consideration. Such possibilities should be addressed jointly with the problem of aggregation. Directions for further research may consider these issues.

Appendix A: Testing vanishing partial correlations among residuals

In this appendix I provide a procedure to test the null hypotheses of vanishing correlations and vanishing partial correlations among the residuals. Tests are based on asymptotic results.

Let us write the VAR which is estimated in a more compact form, denoting $X_t' = [Y_{t-1}'$, ...,$Y_{t-p}']$, which has dimension $(1 \times kp)$ and $\Pi' = [A_1, \ldots, A_p]$, which has dimension $(k \times kp)$. It is possible to write: $Y_t = \Pi' X_t + u_t$. The maximum likelihood estimate of Π turns out to be given by

$$\hat{\Pi}' = \left[\sum_{t=1}^{T} Y_t X_t'\right] \left[\sum_{t=1}^{T} X_t X_t'\right]^{-1}.$$

Moreover, the ith row of $\hat{\Pi}'$ is

$$\hat{\pi}_i' = \left[\sum_{t=1}^{T} y_{it} X_t'\right] \left[\sum_{t=1}^{T} X_t X_t'\right]^{-1},$$

which coincides with the estimated coefficient vector from an OLS regression of y_{it} on X_t (Hamilton 1994, p. 293). The maximum likelihood estimate of the matrix of variance and covariance among the error terms Σ_u turns out to be $\hat{\Sigma}_u = (1/T)\sum_{t=1}^{T} \hat{u}_t \hat{u}_t'$, where $\hat{u}_t = Y_t - \hat{\Pi}' X_t$. Therefore the maximum likelihood estimate of the covariance between u_{it} and u_{jt} is given by the (i,j) element of $\hat{\Sigma}_u$: $\hat{\sigma}_{ij} = (1/T)\sum_{t=1}^{T} \hat{u}_{it}\hat{u}_{jt}$.

Proposition A.1 (Hamilton 1994, p. 301). *Let* $Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \ldots + A_p Y_{t-p} + u_t$ *where* $u_t \sim$ *i.i.d.* $N(\boldsymbol{0}, \Sigma_u)$ *and where roots of* $|I_k - A_1 z - A_2 z^2 - \ldots - A_p z^p| = 0$ *lie outside the unit circle. Let* $\hat{\Sigma}_u$ *be the maximum likelihood estimate of* Σ_u*. Then*

$$\sqrt{T}[vech(\hat{\Sigma}_u) - vech(\Sigma_u)] \xrightarrow{d} N(\boldsymbol{0},\ \Omega), \tag{12}$$

where $\Omega = 2\boldsymbol{D}_k^{+}(\Sigma_u \otimes \Sigma_u)(\boldsymbol{D}_k^{+})'$, $\boldsymbol{D}_k^{+'} \equiv (\boldsymbol{D}_k'\boldsymbol{D}_k)^{-1}\boldsymbol{D}_k'$ *and* $\boldsymbol{D}_k$ *is the duplication matrix.*

Therefore, to test the null hypothesis that $\rho(u_{it}, u_{jt}) = 0$, I use the Wald statistic:

$$\frac{T\ (\hat{\sigma}_{ij})^2}{\hat{\sigma}_{ii}\hat{\sigma}_{jj}\ +\ \hat{\sigma}_{ij}^2}\ \approx\ \chi^2(1).$$

The Wald statistic for testing vanishing partial correlations is obtained by means of the delta method.

Proposition A.2 (*delta method*, see e.g. Lehmann-Casella, 1998, p. 61). *Let* X_T *be a* $(r \times 1)$ *sequence of vector-valued random-variables (indexed by the sample size* T*). If* $[\sqrt{T}(X_{1T} - \theta_1), \ldots, \sqrt{T}(X_{rT} - \theta_r)] \xrightarrow{d} N(\mathbf{0}, \Sigma)$ *and* $h_1, \ldots, h_r$ *are* r *real-valued functions of* $\theta = (\theta_1, \ldots, \theta_r)$, $h_i : \mathbf{R}^r \rightarrow \mathbf{R}$, *defined and continuously differentiable in a neighborhood* ω *of the parameter point* θ *and such that the matrix* $B = ||\partial h_i / \partial \theta_j||$ *of partial derivatives is nonsingular in* ω*, then:*

$$[\sqrt{T}[h_1(X_T) - h_1(\theta)], \ldots, \sqrt{T}[h_r(X_T) - h_r(\theta)]] \xrightarrow{d} N(\mathbf{0}, B\Sigma B').$$

For example, for $k = 4$, suppose one wants to test $\rho(u_1, u_3|u_2) = 0$. First, notice that $\rho(u_1, u_3|u_2) = 0$ if and only if $\sigma_{22}\sigma_{13} - \sigma_{12}\sigma_{23} = 0$ (see definition of partial correlation in Anderson 1958, p. 34), where σ_{ij} is the (i,j) element of Σ_u. Let us define a function $g : \mathbf{R}^{k(k+1)/2} \rightarrow \mathbf{R}$, such that $g(vech(\Sigma_u)) = \sigma_{22}\sigma_{13} - \sigma_{12}\sigma_{23}$. Thus,

$$\nabla g' = (0,\ -\sigma_{23},\ \sigma_{22},\ 0,\ \sigma_{13},\ -\sigma_{12},\ 0,\ 0,\ 0,\ 0).$$

Proposition A.2 implies that:

$$\sqrt{T}[(\hat{\sigma}_{22}\hat{\sigma}_{13} - \hat{\sigma}_{12}\hat{\sigma}_{23}) \ - \ (\sigma_{22}\sigma_{13} - \sigma_{12}\sigma_{23})] \xrightarrow{d} N(0, \nabla g'\Omega\nabla g).$$

The Wald test of the null hypothesis $\rho(u_1, u_3|u_2) = 0$ is given by:

$$\frac{T(\hat{\sigma}_{22}\hat{\sigma}_{13} - \hat{\sigma}_{12}\hat{\sigma}_{23})^2}{\nabla g'\Omega\nabla g} \approx \chi^2(1).$$

Suppose now I want to test the null hypothesis $\rho(u_1, u_4|u_2, u_3) = 0$, which implies $\sigma_{22}\sigma_{33}\sigma_{14} - \sigma_{33}\sigma_{12}\sigma_{24} - \sigma_{14}\sigma_{23}^2 - \sigma_{22}\sigma_{13}\sigma_{34} + \sigma_{13}\sigma_{23}\sigma_{24} + \sigma_{12}\sigma_{23}\sigma_{34} = 0$. I define $g(vech(\Sigma_u)) = \sigma_{22}\sigma_{33}\sigma_{14} - \sigma_{33}\sigma_{12}\sigma_{24} - \sigma_{14}\sigma_{23}^2 - \sigma_{22}\sigma_{13}\sigma_{34} + \sigma_{13}\sigma_{23}\sigma_{24} + \sigma_{12}\sigma_{23}\sigma_{34}$. Thus,

$$\nabla g = \begin{pmatrix} 0 \\ -\sigma_{33}\sigma_{24} + \sigma_{23}\sigma_{34} \\ -\sigma_{22}\sigma_{34} + \sigma_{23}\sigma_{24} \\ \sigma_{22}\sigma_{33} - \sigma_{23}^2 \\ \sigma_{33}\sigma_{14} - \sigma_{13}\sigma_{34} \\ -2\sigma_{14}\sigma_{23} + \sigma_{13}\sigma_{24} + \sigma_{12}\sigma_{34} \\ -\sigma_{12}\sigma_{33} + \sigma_{13}\sigma_{23} \\ \sigma_{22}\sigma_{14} - \sigma_{12}\sigma_{24} \\ -\sigma_{22}\sigma_{13} + \sigma_{12}\sigma_{23} \\ 0 \end{pmatrix}.$$

Let us call $\tau_{14.23}$=$(\sigma_{22}\sigma_{33}\sigma_{14} - \sigma_{33}\sigma_{12}\sigma_{24} - \sigma_{14}\sigma_{23}^2 - \sigma_{22}\sigma_{13}\sigma_{34} + \sigma_{13}\sigma_{23}\sigma_{24} + \sigma_{12}\sigma_{23}\sigma_{34})$ and $\hat{\tau}_{14.23} = (\hat{\sigma}_{22}\hat{\sigma}_{33}\hat{\sigma}_{14} - \hat{\sigma}_{33}\hat{\sigma}_{12}\hat{\sigma}_{24} - \hat{\sigma}_{14}\hat{\sigma}_{23}^2 - \hat{\sigma}_{22}\hat{\sigma}_{13}\hat{\sigma}_{34} + \hat{\sigma}_{13}\hat{\sigma}_{23}\hat{\sigma}_{24} + \hat{\sigma}_{12}\hat{\sigma}_{23}\hat{\sigma}_{34})$. Proposition A.2 implies that:

$$\sqrt{T}[\hat{\tau}_{14.23} - \tau_{14.23}] \xrightarrow{d} N(0, \nabla g'\Omega\nabla g).$$

The Wald test of the null hypothesis $\rho(u_1, u_4|u_2, u_3) = 0$ is given by:

$$\frac{T(\hat{\tau}_{14.23})^2}{\nabla g'\Omega\nabla g} \approx \chi^2(1).$$

Tests for higher order correlations follow analogously.

Appendix B: Proof of Proposition 3.2

(i) Suppose y_{it} and y_{jt} are d-separated by J_{t-1} and $\mathcal{Y}_t$ in $\mathcal{G}$. If there is a path in $\mathcal{G}$ between y_{it} and y_{jt} that contains only components of $\mathcal{Y}_t$ (and possibly of u_t), such path is not active. Then any path in $\mathcal{G}_{Y_t}$ between y_{it} and y_{jt} is not active. Then y_{it} and y_{jt} are d-separated by $\mathcal{Y}_t$ in $\mathcal{G}_{Y_t}$.

(ii) Suppose y_{it} and y_{jt} are d-separated by $\mathcal{Y}_t$ in $\mathcal{G}_{Y_t}$. Then, if there is an active path between

y_{it} and y_{jt} in $\mathcal{G}$, such path must contain a component of J_{t-1} which is not a collider, since there are no directed edge from any component of Y_t pointing to any component of J_{t-1}. Therefore such path is not active relative to J_{t-1} in $\mathcal{G}$ and y_{it} and y_{jt} are d-separated by J_{t-1} and $\mathcal{Y}_t$ in $\mathcal{G}$.

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