Cumulative Causation and Evolutionary Micro-Founded Technical Change: A Growth Model with Integrated Economies

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Abstract

We propose to develop in this paper an alternative approach to the New Growth Theory to analyse growth rate divergence among integrated economies. The model presented here considers economic growth as a disequilibrium process. It introduces in a cumulative causation framework, micro-founded process of technical change taking into account elements rooted in evolutionary and Neo-Austrian literature. We then attempt to open the ‘Kaldor-Verdoorn law black-box’ using a micro-level modelling of industrial dynamics.

We use this framework to study the nature and sources of growth rate divergence, focusing on the effect of some macro-economic parameters (income elasticities) and of some technological parameters (technological opportunities and absorptive capacities). If the results remain broadly in Kaldorian lines, this framework allows for more subtle considerations of growth rate divergence.

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1 Introduction.

The literature on economic growth is dominated since the 90s by the developments of the New Growth Theory (NGT), also called “Endogenous growth theory”. It is at least a good indicator of the relevance of two propositions:

- to explain why economies growth is again a relevant and still an open question in economics;

- the factors of growth should be “endogenous” to be acceptable; that economist should go beyond the simplified version of the Solow canonical model, with exogenous technical changes.

But at the same time, this literature did hide a long tradition of research which could certainly give some alternative explanations to the persistence of phenomena such as growth rate divergence among countries or regions. Among these potential alternatives there are at least three which are worth mentioning in the context of our paper: the Kaldorian approaches of a cumulative causation, the evolutionary perspectives of diversity and selection and the Austrian view of the decision sequences and path dependency. Even if it is usually considered that the alternative approaches are too heterogeneous to be built into an integrated and coherent framework; they have at least some common features which could justify a comprehensive complementarity. Contrary to the NGT, all three approaches economise in terms of degree of rationality of economic agents, escape from technologies being dealt with as information, and introduce innovation as a process of knowledge creation, and finally consider that ‘history matters’ i.e. the focus should be rather in the ‘out of equilibrium’ processes than on the equilibrium characteristics and existence.

The purpose of this paper is not to propose a ‘re-constructed’ and ‘integrated’ alternative theory of growth, but to build a simple model, integrating some of the main features of these alternative theories in order to show their complementarity in explaining some classical ‘stylised phenomena’. Our focus will be in this paper the growth rate divergence among integrated economies.

The main aspect of Kaldorian approaches (Kaldor (1972, 1981); Dixon and Thirlwall (1975); Verspagen (2002)) is essentially based on two principles: A demand-driven growth and a cumulative causation. In this framework, Kaldor’s explanation of growth rate is the result of two “interrelated mechanisms”: First output growth is driven by the growth of aggregate demand, so that growth and technological progress are demand-driven processes. In
Kaldor’s mind this aggregate demand factor driving growth is concretely repre-
represented by the growth of exports that are driven by the country’s degree
of international competitiveness. Second, productivity is a “by-product of
output”; this is due to the existence of dynamic increasing returns through
the Verdoorn law and the mechanisms underlying it. The interrelation be-
tween these two mechanisms can be described as follows. The rule for prices
is a mark-up over unit labour cost. The growth of productivity based on the
growth of output would reduce this unit labour cost, then prices, and thus in-
crease the country’s competitiveness. This increasing competitiveness would
lead to increasing exports, themselves leading to a higher growth rate. Thus
for a given initial competitiveness advantage, growth rates will tend, through
the circular and cumulative mechanisms, to be maintained or increased over
time, by themselves. This implies also that initial conditions strongly dic-
tate the growth process, providing for virtuous cumulative mechanisms (as
described previously) rather than a vicious one where growth could never be
self-sustained and so cumulative processes could never start.

The major mechanisms driving the Kaldorian cumulative growth pro-
cess can be summarised as follows: the Verdoorn law allows self-sustained
growth, dynamic increasing returns allow cumulativeness of the growth pro-
cess, and finally initial conditions define this process as a path-dependent
process, where initial competitiveness differences tend to increase rather than
decrease.

One of the main drawbacks of the approach is the “Kaldor-Verdoorn black
box”. Our paper is to substitute it with a micro-founded technical change,
using a evolutionary model of industrial dynamics à la Nelson and Winter
(1982). The main task is here to model the innovation process (through R&D
expenditure by firms, innovation and integration into new investments), in
order to endogenise the evolution of productivity and so to close the model
with a micro-founded alternative to the Kaldor-Verdoorn law.

Finally we have some Austrian flavour in our model, because we explicitly
constraint the decision process at the firm level to a given sequence: invest-
ment and R&D expenditure are financially constrained by previous profits.
The liquidity constraint is essential as a device to structure both the ongoing
processes: selection and innovation.

Only few attempts exist in the literature to merge these approaches, the
main one being by Verspagen (1993, 2002). Our contribution is principally to
add a fully specified model, as a first step for further developments. In partic-
ular we wish to differentiate the impact of macro diversity from technological
diversity among countries in terms of divergence-convergence of growth rate.
The main point of our model is that, even if it is a combination of different approaches, is to preserve one of their major feature: unlike new growth theories, it never assumes full employment, but above all never considers a general equilibrium framework for analysing growth, so it never assumes the existence of a natural rate of growth along a given balanced growth path. The growth process is cumulative in this analysis because “growth creates the necessary resources for growth itself”\textsuperscript{1}. This cumulative process allows an endogeneity of growth through growth itself as a self-reinforcing process.

The next section is devoted to a presentation of the model, followed, in section 3, by the development both of the main results and of their interpretations.

2 A model of cumulative causation growth with evolutionary micro-founded industrial dynamics.

In order to consider the co-evolution of these components, we assume that aggregate demand is defined at the macro-economic level, through the balance of payment constraint. First, demand provides the necessary resources for firms to finance their activities and development (through both R&D and investments). Second, selection among firms takes place at the macro-economic level, as resulting from international competition. Firms located in a given country compete among themselves and with foreign firms on an integrated market\textsuperscript{2}. Hence the macro-dynamics can be considered as a constraint on firm micro-dynamics.

On the other hand technical change, a necessary engine for growth, is rooted in firms’ dynamics. The competitiveness of the entire economy relies on the firm’s ability to generate technological progress. In other words, firms contains the essence of macro-dynamics.

As a consequence, micro and macro-dynamics are strongly interrelated. In this section we first present the macro-frame, then the micro-dynamics of firms.

\textsuperscript{1}León-Ledesma (2000)

\textsuperscript{2}Assuming then neither trade limitations nor barriers to access foreign markets
2.1 Aggregate demand and the balance of payment constraint: the macro-frame.

We suppose that the economies under consideration are part of an integrated system constrained by the balance of payment with fixed exchange rates (or a common monetary system). Moreover, assuming that the member countries of the integrated system external debt with other members is restricted. Given the monetary integration, the balance of payment adjustments through monetary mechanisms (exchange rates) are excluded and the balance of payment constraint corresponds then to a clearing of countries trade balance. In other words imports have to match exactly exports, for each integrated economy.

The macro-economic framework we develop here is directly rooted in the formal interpretation of Kaldor’s cumulative causation approach of economic growth. These formal representations can be found among others in Dixon and Thirlwall (1975), or more recently Amable (1992), Verspagen (1993) and León-Ledesma (2000).

Economic growth is driven by demand. Aggregate demand is a function of an autonomous component, represented by external demand, i.e., countries’ exports. For each economy, exports are given as a function of the income of the rest of the world and of the market share of the economy.

Formally, exports for a given economy can be computed as follows:

\[ X_{j,t} = (Y_{w,t})^{\alpha_j} z_{j,t} \]  

where \( Y_{w,t} \) represents the GDP of the rest of the world, computed as the sum of GDP levels of all foreign economies, \( z_{j,t} \) represents the market share of the economy, on the international markets and \( \alpha_j \) income elasticity to exports for the rest of the world.

The market share of the economy is a function of the price competitiveness of the country. In other words if the first component of the export function represents the income determinant of exports, the market share then represents the price component of external demand. The economy’s market share is given by the sum of the market shares of the domestic firms (denoted \( z_{i,j,t} \)):

\[ z_{j,t} = \sum_i z_{i,j,t} \]

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3 We assume here that no external debt among member countries is allowed.

4 Note that the subscript \( j \) always refers to an economy, while the subscript \( i \) refers to a firm. We suppose that the model is composed of \( J \) economies, each of them counts \( I \) firms. Hence a variable with the subscripts “\( j, i \)” concerns the firm \( i \) based in the country \( j \).
Each firm’s market share is defined through a replicator dynamics\(^5\), a function of a firm’s relative competitiveness. Hence the market share of each firm will be computed as follows:

\[
    z_{i,j,t} = z_{i,j,t-1} \left(1 + \phi \left(\frac{E_{i,j,t}}{\bar{E}_t} - 1\right)\right)
\]

(2)

where \(z_{i,j,t}\) represents the market share of firm \(i\), \(p_{i,j,t}\) the price of its product, \(E_{i,j,t}\) stands for firm \(i\)’s level of competitiveness:

\[
    E_{i,j,t} = \frac{1}{p_{i,j,t}}\]

\(\bar{E}_t\) the average competitiveness on the international market, given by:

\[
    \bar{E}_t = \sum_{j,i} z_{j,i,t-1}E_{j,i,t}
\]

The parameter \(\phi \in [0; 1]\) represents the degree of reactivity of demand to price competitiveness.

To complete the formal definition of the macro-economic framework, we have to define the economy’s imports. They are basically defined following exports’ scheme, as a function of the domestic economy income and of the rest of the world’s market share. Formally imports will be represented as follows:

\[
    M_{j,t} = (Y_{j,t})^{\beta_j}(1 - z_{j,t})
\]

(3)

The parameter \(\beta_j\) represents the income elasticity to import. \(Y_{j,t}\) represents the economy GDP equal to the sum of firms production.

The growth rate of exports and imports for each sector can be deduced from these previous expressions as:

\[
    x_{j,t} = \alpha_j y_{w,t} + \ln(z_{j,t}) - \ln(z_{j,t-1})
\]

(4)

\[
    m_{j,t} = \beta_j y_{j,t} + \ln(1 - z_{j,t}) - \ln(1 - z_{j,t-1})
\]

(5)

Where \(x_{j,t}\), \(m_{j,t}\), \(y_{l,t}\) and \(y_{w,t}\) stand for the growth rates\(^6\) of the previously defined corresponding variables.

We assume that each economy has to satisfy the balance of payment constraint. In our model this corresponds to an equilibrated trade balance.

\(^5\)For a comprehensive view on the use of the replicator dynamics in evolutionary economics see Metcalfe (1998)
\(^6\)Approximated through difference in logarithms.
An economy $j$’s external expenditures have to match exactly its external resources. In other words each economy is subject to an external financial constraint. Formally, exports have to equal imports:

$$M_{j,t} = X_{j,t}$$

Dynamically, the growth rate of imports is constrained by the growth rate of exports:

$$m_{j,t} = x_{j,t}$$

The introduction of the balance of payment constraint allows us to express the GDP growth rate as function of the growth rate of GDP of the rest of the world and of the growth rate of market share. GDP growth rate for country $j$ is computed as follows:

$$y_{j,t} = \alpha_j \beta_j y_{w,t} + \frac{1}{\beta_j} \left[ \ln \left( \frac{z_{j,t}}{z_{j,t-1}} \right) - \ln \left( \frac{1 - z_{j,t}}{1 - z_{j,t-1}} \right) \right]$$

The first component of the right end side of the equation captures in fact Harrod’s trade multiplier. Hence GDP growth rate in our model will be defined through the trade multiplier and through a second component linked to the competitiveness of the economy. This second component is rooted in firms behaviours and characteristics. Hence, we can distinguish explicitly the growth effect linked to the trade multiplier and the one emerging from firm dynamics.

We can deduce from the expression for GDP growth rate the GDP level at time $t$. It equals the domestic aggregated demand. GDP is given by:

$$Y_{j,t} = Y_{j,t-1} \left( \frac{Y_{w,t}}{Y_{w,t-1}} \right)^{\alpha_j} \left( \frac{z_{j,t}}{z_{j,t-1}} \frac{1 - z_{j,t}}{1 - z_{j,t-1}} \right)^{\beta_j}$$

This expression also represents the gross production of all firms at time $t$. In our model, the time dimension allows aggregate supply to match entirely aggregate demand. We do not consider here explicitly the process of coordination of demand and supply in the market for goods.

Aggregate (economy wide) demand is then distributed among the firms in the economy given their market shares on the integrated markets. This first component then constitutes the first macro-economic constraint the firms have to face.

The second macro-economic constraint imposed on firms concerns the labour market. Wages are in fact set at the macro-economic level. We assume here that wages are strictly correlated to the average labour productivity in
the economy. We suppose here that labour supply is perfectly elastic to firms labour demand: only productivity is considered in the process of wage determination.

Wages are set at period $t$ for the next period, given the average labour productivity in the economy at period $t$. As we will see below, firms produce at time $t$ with a technology developed during the previous period, through the exploitation of the outcome of R&D activity. Hence wages used in period $t+1$ and bargained in $t$ are indexed on the productivity levels resulting from the technologies developed in $t-1$. In other words wages will be set as follows:

$$w_{j,t} = w_{j,t-1}\left(\frac{A_{j,t}}{A_{j,t-1}}\right)$$

where $A_{j,t-1}$ represents the average labour productivity level of the economy $j$ at time $t$. Given the average labour productivity as follows, given that the productivity level at time $t$ for each firm depends on the technology developed at time $t-1$, represented by $A_{i,j,t-1}$:

$$A_{j,t} = \frac{\sum_i z_{i,j,t} A_{i,j,t-1}}{\sum_i z_{i,j,t}}$$

This second macro-economic constraint implies that at the firm level competitiveness will increase at time $t$ as a function of the difference in productivity growth rate at time $t-1$ with respect to the average increase in productivity of the economy at time $t-2$. It explains why the competitiveness of the entire economy will increase at time $t$ as a function of the difference in average productivity growth rate at time $t-1$ with its growth rate at time $t-2$.

### 2.2 Evolutionary micro-foundations of technical change.

This second level of the model concerns firms and industrial dynamics. We explain here firms’ behaviour and characteristics. This part is largely inspired by evolutionary literature on the modelling of industrial dynamics.\(^7\) We assume here that firms are heterogeneous in their characteristics, i.e. in terms of productivity level, and in their investment behaviours in terms of R&D and capital goods. Moreover, firms mutate, by learning about the production process, improving their productivity, and by learning about their decisions, adapting the latter to their situation compared to their competitors. Hence firms are bounded rational agents applying adaptive behaviour.

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\(^7\)See Kwasnicki (2001) for a comprehensive survey of evolutionary models of industrial dynamics, Silverberg and Verspagen (1995) for a comprehensive survey of evolutionary growth models based on “industrial dynamics”. 

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Firms will have two distinct but complementary roles in our model. First they will produce the necessary resources to sustain economic growth, by responding to the demand needs. Second they will increase the competitiveness of the economy by trying to improve their productivity level to survive the selection process. This second process will be broken down into two stages:

- Exploration or R&D. Firms first search for new production facilities, through innovation or adaptation of existing production facilities. The outcome of the R&D process is uncertain, and defines efficiency (in terms of productivity) of the new generation of capital goods.

- Exploitation of R&D outcome. This second stage requires that firms invest to incorporate the outcome of research in the production process. This second stage is financed by profits, and then directly subject to the success of previous investments.

More formally firms are modelled below.

Firms’ production processes are represented by Leontiev production functions with labour as a unique production factor. Capital enters indirectly in the production function by influencing labour productivity. Investment in the different generations of capital goods will increase labour productivity. The production function will then be represented as follows:

\[
Y_{i,j,t} = A_{i,j,t-1}L_{i,j,t}^P
\]  

where \( Y_{i,j,t} \) is the output of firm \( i \), producing in country \( j \) at time \( t \). \( A_{i,j,t-1} \) represents labour productivity and \( L_{i,j,t}^P \) the labour force employed in the production process. The output is constrained by the demand directed at the firms and defined at the macro-economic level. The level of production of each firm is computed as a share of GDP given by their relative market shares such as:

\[
Y_{i,j,t} = \frac{z_{i,j,t}}{\sum_i z_{i,j,t}} Y_{j,t}
\]

Labour productivity is a function of the firms’ accumulated generations of capital goods through investment:

\[
A_{i,j,t} = \frac{I_{i,j,t}a_{i,j,t-1}}{\sum_{\tau=1}^{t-1} I_{i,j,\tau}} + \frac{\sum_{\tau=1}^{t-1} I_{i,j,\tau}}{\sum_{\tau=1}^{t} I_{i,j,\tau}} A_{i,j,t-1}(1 - \delta)
\]

where \( a_{i,j,t-1} \) represents the labour productivity embodied in the capital good developed by \( i \) during period \( t - 1 \). The parameter \( \delta \) represents the depreciation rate of capital goods. \( I_{i,j,t} \) represents the level of investment in capital goods of the firm. This component will be explained later.
Firms set prices through a mark-up process. This mark-up is applied to the production costs (i.e. labour cost). To simplify the model, labour costs linked to R&D activity are financed by profits. Thus prices can be represented as follows:

\[ p_{i,j,t} = (1 + \mu_j) \frac{w_{j,t-1}}{A_{i,j,t-1}} \]  

where \( p_{i,j,t} \) represents the price set by firm \( i \) at time \( t \), \( \mu_j \) the mark-up coefficient and \( w_{j,t-1} \) the nominal wage set at the macro level as defined above. It should be noted that we assume here that the mark-up coefficients are fixed for each firms in a given economy. First this insures that the share of profits in GDP is constant over time, which corresponds to one of Kaldor’s stylised facts. Second, it does not really affect the results in terms of macro-dynamics of the economy, as shown by Dosi et al (1994) that consider the mark-up coefficient as an endogenous variable. This can be explained by the fact that a drastic reduction in the mark-up coefficient to increase the competitiveness of the firm, implies as well a drastic reduction in the profits and thus in resources that can be used to finance R&D and then reduces potential productivity increases of the firm.

The firm’s profit level will then be computed as follows:

\[ \Pi_{i,j,t} = \mu_j \frac{w_{j,t-1}}{A_{i,j,t-1}} Y_{i,j,t} \]  

In the model profits constitutes the only financial resource for firms’ investments.

To improve their competitiveness and thus gain some market shares firms have to improve their production processes (i.e. to increase labour productivity). The process of technical improvement can be divided into two distinct phases. Firms explore new technological possibilities, through local search (innovation) and/or by capturing external technological possibilities (through spill-overs). This process leads to a production design (or capital good design) that can be exploited by firms in their production process. The second stage consists then in the exploitation of the design by incorporating it as a new generation of capital goods. The exploitation process is related to investment in capital goods and the exploration is related to investments in research. Given the process of obsolescence of capital goods we assume priority is given to investments exploiting already discovered technological opportunities. Formally the latter will be represented as explained below: Investment in capital goods is financed by the profits of the firm, using a share \( \iota_{ij,t} \) of sales. This share is defined through a decision rule given below:
firms adapt their investment behaviours to their relative (to average) competitiveness, as a direct response to the selection mechanisms. The capital goods investment decision rule will be the following:

\[ \iota_{i,j,t} = \begin{cases} \iota_{i,j,t-1} & \text{if } E_{i,j,t} \geq \bar{E}_t \\ \iota_{i,j,t-1} \left(1 + \xi \left(\frac{E_{i,j,t}}{\bar{E}_t} - 1\right)\right) & \text{otherwise} \end{cases} \]

where the parameter \( \xi \in [0; 1] \) represents the degree of adaptation of the firm decision rule to its relative competitiveness gap and \( \bar{E}_t \) the average competitiveness. This formulation of the investment decision in capital goods induces some aggregate structural effects according to the number of firms lagging behind relatively to the average competitiveness.\(^8\)

Investment is subject to a financial constraint. Hence, as investments are completely financed by profits, they cannot exceed the period’s profit level. Formally this constraint will be represented as follows:

\[
I_{i,j,t} = \min \left\{ \iota_{i,j,t} Y_{i,j,t}; \mu_j \frac{w_{j,t-1}}{A_{i,j,t-1}} - A_{i,j,t-1} Y_{i,j,t} \right\}
\] (13)

Investment in R&D is decided according to the following decision rule: Firms then adapt their R&D investments to their relative technological gap. These investments are a share \( \rho_{i,j,t} \) of their sales.

\[ \rho_{i,j,t} = \begin{cases} \rho_{i,j,t-1} & \text{if } A_{i,j,t} \geq \bar{A}_t \\ \rho_{i,j,t-1} \left(1 + \psi \left(\frac{A_{i,j,t}}{\bar{A}_t} - 1\right)\right) & \text{otherwise} \end{cases} \]

with

\[ \bar{A}_t = \sum_{j,i} z_{i,j,t-1} A_{i,j,t-1} \]

R&D investment will correspond to the hiring of workers assigned to the research activity:

\[
L_{i,j,t}^r = \frac{1}{w_{j,t-1}} \min \{\Pi_{i,j,t} - I_{i,j,t}; \rho_{i,j,t} Y_{i,j,t}\}
\] (14)

The model takes into account the sequential nature of the decision process and the existence of a financial constraint linked to the success (or failure) of previous decisions. The investment decision sequence is such that a priority is given to exploitation (investment in capital goods). Then exploration investment (R&D) depends on the remaining resources. Resources available for investment depend on firms’ profits and on the outcome of their previous

\(^8\)see Gaffard (1978), p.73 and following.
decisions. This gives the “industrial dynamics” part of the model an additional “Austrian flavour”\textsuperscript{9} to our Kaldorian and evolutionary approach.

The formalisation of the R&D process is explicitly inspired by evolutionary modelling of technical change. Hence following Nelson and Winter (1982) we consider that the probability of success of the R&D activity will increase with the ratio $L_{i,j,t} / Y_{i,j,t}$. The outcome of R&D is itself uncertain and can be formally represented as follows:

$$a_{i,j,t} = \max\{a_{i,j,t-1}, \hat{a}_{i,j,t}\} \quad (15)$$

$$\hat{a}_{i,j,t} \sim N(a_{i,j,t-1}, \sigma_{i,j,t}) \quad (16)$$

Hence, the potential increase in labour productivity embodied in the generation of capital developed during the R&D process, is a random variable, the mean of which equals the previous period value. The variance is a function of the spill-overs absorbed by the firm. The variance given by $\sigma_{i,j,t}$ will formally be defined as follows:

$$\sigma_{i,j,t} = \bar{\sigma}_j + \chi_j (\tilde{a}_{t-1} - a_{i,j,t-1})$$

where the parameter $\bar{\sigma}_j$, is constant and related to the technology used in $j^{th}$ economy. It represents the range of technological opportunities. The variable $\tilde{a}_{t-1}$ represents the worldwide maximal value of labour productivity embodied in already discovered generation of capital, i.e. it represents the technological frontier at time $t - 1$. Thus $\tilde{a}_{t-1} - a_{i,j,t-1}$ will represent the distance between firm’s technological level and the frontier level, in other words, the technological gap. And finally, the parameter $\chi_j$ defines absorptive capacity\textsuperscript{10}.

3 Growth rate divergence among integrated economies.

The model as developed in the previous section aims to consider the determinants of possible divergence in GDP growth rates among integrated economies. Traditionally, mainstream economics considers that the integration of economies and openness to trade imply convergence due to the diffusion of knowledge and/or technologies.

\textsuperscript{9}see Amendola and Gaffard (1998), p.126)

\textsuperscript{10}We consider here exogenous and fixed values for the absorptive capacities
For Neo-Schumpeterian evolutionary economics, growth rates divergence depends on the balance between two effects:

- Innovation, heterogeneous among economies both in its timing and in the outcome, that increases differences in GDP growth rates, and
- Imitation that reduces this difference.

Hence in this framework growth rate divergence directly depends on the accessibility of technologies, innovation and imitation capabilities, and on the decision processes linked to R&D investment.

For the Kaldorian approach growth rate divergence is structural depending on both demand and technological parameters, and cumulative, due to the emergence of vicious and virtuous circles.

As for most of the models incorporating evolutionary features we need to resort to numerical simulations. Simulations are set through the following scheme. We consider 5 economies, each of which counts 20 firms. All the firms of a given economy are equally defined (same initial conditions and parameters). The details of the parameters values used can be found in appendix.

We focus our analysis on the effect of four key parameters on growth rate divergence. Two of them concern the macro-economic constraint. More precisely, we choose to concentrate on the effect of heterogeneous income elasticity of exports and imports, respectively $\alpha_j$ and $\beta_j$. The other two concern the process of technological change itself and are usually considered in evolutionary literature. The first one concerns the range of technological opportunities represented by $\bar{\sigma}_j$ while the second concerns the absorptive capacity $\chi_j$.

To isolate the effect of each parameter we consider each of them separately, the other being then homogeneously set among economies. Moreover, when considering heterogeneity in the parameter setting, we use the following procedure: Over the five considered economies, three of them keep the initial settings (also denoted reference setting) and the two remaining (denoted Country 1 and Country 2) parameters are set in such a way that the first one gets the most favourable ones while the second gets the worth ones. Heterogeneous parameters are set so that the average value of the parameters among economies remains unmodified. We only increase the variance.

One of the characteristic of the model is to generate two distinct types of growth rate divergence:

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11We used LSD (Laboratory for Simulation Development) environment to implement the simulations. The source code for the model can be available on request to the authors.
- A sustained growth rate divergence. In this case variance in growth rates stabilises itself over time. All economies continue to grow but at different rates.

- A destructive growth rate divergence. In this case growth rate divergence increases over time. Divergence here leads to the collapse of some lagging economies coupled with the ongoing domination of others. This situation leads at the end to the survival of only one economy, the most competitive one.

The key results of the simulations are detailed below.

3.1 Macro-constraint and growth rate divergence.

The first set of parameters concerns the macro-economic constraint due to the trade balance restriction. We characterise the nature of the growth rate divergence in this case.

**Proposition 1** Increases in the heterogeneity of income elasticity of export and imports respectively \( \alpha_j \) and \( \beta_j \) increases the variance in growth rates essentially by affecting the trade multiplier.

The effect of income elasticities (\( \alpha_j \) and \( \beta_j \)) heterogeneity on growth rate divergence is one of the main and maybe most obvious results one can think about when considering a Kaldorian flavoured macro-economic framework. The increase in the divergence in growth rate due to this demand elasticities effect is in fact directly linked to the modifications induced on the trade multiplier in the definition of the GDP growth rate (see equation (8)).

Moreover, when considering the parameter \( \beta_j \), the latter does not only affect the trade multiplier but also modifies the weight of the market share dynamics on GDP growth rates. There are then two opposite effects, one directly related to the trade multiplier. It increases the weight of a diverging process as when considering heterogeneous \( \alpha_j \). The second effect is linked to the increasing weight of a converging process, which is itself linked to the market share dynamics, affected by technical change. When no spill-overs are absorbable, productivity growth rates (with homogenous parameters) are convergent over time. This explains directly why the range of the variance in growth rates while modifying \( \beta_j \) is lower then when modifying \( \alpha_j \) (see Figure 1 and Figure 2).\(^{12}\)

\(^{12}\)All the figures presented, unless otherwise mentioned, show the average values (at each time step) over 50 simulation runs. Each simulation run lasts 500 time steps.
**Proposition 2** Growth rate divergence generated by heterogeneity in demand parameters ($\alpha_j$ and $\beta_j$) generates sustained growth rate divergence without generating vicious circles.

As depicted in Figure 3 and Figure 4, the divergence in GDP growth rates stands among times. The sustainability of this divergence without generating vicious circles driving to the collapse of the least competitive economies differs from the usual Kaldorian results. It is principally due to the effect on the trade multiplier, of the exports and imports elasticities, and the relative neutrality of this effect on technological change. To generate a vicious circle growth rate divergence of GDP should directly affect competitiveness.

Figure 3 and Figure 4 about here

The differences in resources induced by the differences in aggregate demand generated through time by heterogeneous demand parameters are not sufficient to observe significant differences in technology levels (see Figure 5 and Figure 6).

Figure 5 and Figure 6 about here

In other words the macro-economic constraint is not sufficient to generate significant heterogeneity in technologies at the macro-level among economies. In particular this type of constraints does not generate destructive growth rate divergence usually induced by vicious circles.

### 3.2 Technology and growth rate divergence.

The second set of parameters considered in this analysis concerns the technological characteristics of the economy. We concentrate on two of them, namely the range of technological opportunities ($\bar{\sigma}_j$) and absorptive capacities ($\chi_j$), assuming that the economies are initially identically set (concerning the other parameters). It should be noted that heterogeneity in initial settings of productivity levels does not generate significant or remarkable effects on growth rate divergence among economies. Unless one considers extreme cases of heterogeneity, that will almost directly push the less favoured economies to enter a vicious circle.

**Proposition 3** Increasing heterogeneity in technological opportunity parameter ($\bar{\sigma}_j$), increases growth rate divergence among economies during a transitory phase.
As depicted in Figure 7 and Figure 8 considering heterogeneous settings for the parameter defining the range of technological opportunity, increases the variance in GDP growth rates only during a transitory phase. These phases emerge when innovation is successful (which corresponds to a random and rare event) and last for few time steps.

Figure 7 and Figure 8 about here

Moreover heterogeneity of technological opportunities also affects transitorily the productivity levels (see Figure 9), even if the effect seems to last for longer periods.

Figure 9 about here

The relative neutrality of uneven technological opportunities is directly due to the macro-economic mechanisms of wage determination. The latter follows the dynamics of productivity with a time lag. The gain in competitiveness then depends mainly on the growth rate of productivity rather than on the productivity levels themselves. Hence a higher increases in productivity linked to wider technological opportunities will increase the GDP growth rate by increasing competitiveness until the complete adaptation of wages.

**Proposition 4** Increasing access to spillovers (increasing $\chi_j$) increases GDP growth rate divergence among economies.

Figure 10 about here

Precisely, for high values of $\chi_j$ (0.5, 0.75 as a share of appropriable spillovers), as depicted in Figure 10, we can observe higher variance in GDP growth rates among economies. This result seems to contradict the classical results: the more the knowledge diffuses among firms and countries the less technologies differ and the more countries converge. While focusing on GDP growth rates and productivity levels dynamics (see Figure 11 and Figure 12), the higher variance of growth rates among economies appears to be generated by the emergence of vicious circles. The latter drives the less favoured economies to collapse.

Figure 11 and Figure 12 about here

If the emergence of vicious circles is almost certain with high values of $\chi_j$, the model cannot exhibit clear explanations of this process, and needs some additional research. The fact that increased access to others’ technologies increases growth rate divergence among countries might be due to the
additional effects of the macro-constraint combined with the absence of an integrated outcomes of the R&D in the production process. Hence, the firms which gain higher market shares, might also benefit more from spillovers; they have more resources to imitate (capture others technologies) and to exploit others more efficient technologies. This puzzling result of the model will imply further analysis and development on absorption of spill-overs and imitation processes along the line of Llerena and Oltra (2002).

When considering heterogeneous access to spill-overs (see Figure 13), the higher the heterogeneity, the higher the variance in GDP growth rate will be.

Figure 13 about here

But here again growth rate divergence is driven by the emergence of vicious circles. Moreover the economy with the best access to spillovers does not significantly emerge as a leading economy. The frequency nevertheless appears to increase with highly heterogeneous settings. On the contrary, with heterogeneous settings of $\chi_j$, the less favoured in terms of absorptive capacity almost always enters a vicious circle, and disappear.

Contrary to the macro-constraint based divergence presented in the previous section, the existence of a higher technological heterogeneity among firms entails a destructive divergence of growth rate. It means in particular that the substitution of an explicit industrial dynamics, for the relatively mechanical “Kaldor-Verdoorn law” allows a more subtle analysis of the growth rate divergence and leaves rooms for sustained growth rate divergence without necessarily vicious circles for the less favoured countries.

4 Concluding remarks.

We attempt in this paper to open the ‘Kaldor-Verdoorn law black-box’ by introducing a micro-founded dynamics of technological change inspired by evolutionary modelling of industrial dynamics in the cumulative causation framework. We focus our analysis on the effect of four key parameters:

- Income elasticities of exports and imports, on the one hand, considering then the effect of the macro-constraint on growth rate dynamics.

- Technological opportunities and absorptive capacity, on the other hand, considering the influence of technological change on growth rate dynamics.
The simulations results allow us to sort out two distinctive types of divergence in growth rates among economies.

On the one side, the model generates ‘sustained’ growth rate divergence while considering heterogeneous parameters for the macro-constraint. This result seems to show that the macro-constraint might not directly influence competitiveness.

On the other, the model leads ‘destructive’ growth rate divergence, generating vicious circles, if considering different settings of absorptive capacity. This result might be due to the reinforcement effect of the combination of the macro-constraint, the technical change sequentiality constraint by resources, and access to more efficient technologies, on randomly emerged competitive advantages. This last result opens the way for further development of the imitation process and absorptive capacity mechanisms.

Moreover, heterogeneity in technological opportunities seems to affect transitorily growth rate divergence. Its effect on growth dynamics appears counter-balanced by the wage determination process.

Hence, the introduction of evolutionary micro-foundations of technical change in a Kaldorian framework, allows for more subtle considerations in understanding growth rate divergence among integrated economies. However, this model might constitute the starting point for further analysis. Hence, the way technical change is considered remains sketchy, and some mechanisms such as imitation, diffusion of technologies and its access for firms might be reconsidered.

Finally, another aspect of Kaldorian literature concerns specialisation patterns and their effect on growth dynamics. This concern, due to the one-sector specification of the model, is not considered here and might be the object of further studies.

References


Appendix

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Table 1: Simulation settings\(^{13}\)

\(^{13}\)Note that $Y_{w,t-1}$ corresponds to the sum of the others’ economy GDP plus an exogeneous component. The latter represents the ‘rest of the world’ component. It is set initially to 100 and grows at a given rate of 0.1.
Figure 1: Variance in GDP growth rates with heterogeneous $\alpha_j$. 
Figure 2: Variance in GDP growth rates with heterogeneous $\beta_j$. 
Figure 3: GDP growth rates with heterogeneous $\alpha_j$ (Var = 0.00625).
Figure 4: GDP growth rates with heterogeneous $\beta_j$ (Var = 0.009).
Figure 5: Productivity levels with heterogeneous $\alpha_j$ (Var = 0.00025).
Figure 6: Productivity levels with heterogeneous $\beta_i$ (Var = 0.009).
Figure 7: Variance in GDP growth rates with heterogeneous $\sigma_j$. 
Figure 8: GDP growth rates with heterogeneous $\sigma_i$ (Var = 6.4E-4).
Figure 9: Productivity levels with heterogeneous $\sigma_j$(Var = 6.4E-4).
Figure 10: Variance in GDP growth rates with different settings of $\chi_j$. 
Figure 11: GDP growth rates with $\chi_j = 0.75$ (example of simulation run).
Figure 12: Productivity levels with $\chi_j = 0.75$ (example of simulation run).
Figure 13: Variance in GDP growth rates with heterogeneous $\chi_j$. 