Mapping Sectoral Patterns of Technological Accumulation into the Geography of Corporate Locations. A Simple Model and Some Promising Evidence

Giulio Bottazzi*
Giorgio Fagiolo*
and
Giovanni Dosi*

* Sant’Anna School of Advanced Studies, Pisa, Italy.
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Giulio Bottazzi∗ Giorgio Fagiolo† Giovanni Dosi‡

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Abstract

Economies of agglomeration are central to the understanding of the emergence of industrial clustering. However, existing models that incorporate such agglomeration economies have been largely neglecting the vast amount of empirical evidence on inter-sectoral differences in the patterns of industrial concentration. In this paper, we propose a baseline model of firm location in presence of dynamic increasing returns. The model is able to deliver testable implications about the long-run distribution of the size of spatial clusters which we test against data on geographical location of Italian firms belonging to different sectors. We show that accordance of theoretical predictions with data is quite high. Moreover, we find statistically significant differences in the strength of economies of agglomeration, not only across geographical locations but also across industrial sectors. We argue that geographical clustering is highly affected by intersectoral differences in innovation patterns and learning regimes that map into different drivers of sector- and location-specific dynamic increasing returns to agglomeration.

Keywords: Industrial Clustering, Economies of Agglomeration, Firm Locational Choice, Dynamics.

JEL Classification: F12, R10.

∗ Sant’Anna School of Advanced Studies, Pisa, Italy. Email: bottazzi@sssup.it
† Corresponding Author. Sant’Anna School of Advanced Studies, Laboratory of Economics and Management (LEM), Piazza Martiri della Libertà, 33, I-56127 PISA (Italy). Email: fagiolo@sssup.it. Tel: +39-050-883341. Fax: +39-050-883344.
‡ Sant’Anna School of Advanced Studies, Pisa, Italy. Email: g dosi@sssup.it
1 Introduction

Over the last two decades, there has been a widespread resurgence of studies sharing the notion that “space matters in economic activity”. In particular, much effort has been devoted to a theoretical exploration of the mechanisms underlying the process of industrial clustering both within and across countries.

Patterns of spatial concentration of firms have often been interpreted in a comparative advantage framework as the outcome of a static, well-defined, trade-off between agglomeration and dispersion forces. In this view, spatial locations display ex-ante, well identifiable, differences in initial endowments, transport costs and market interactions which uniquely determine the observed industrial concentration as a predictable equilibrium outcome.

However, the vast amount of empirical and appreciative studies about firm locational patterns in the U.S., Asia and Europe, seems to suggest that industries are more highly clustered than any standard theory of comparative advantage might predict (cf. Krugman (1991) and Fujita, Krugman & Venables (1999)).

Many interpretations have consequently assumed that the primary engine of concentration lies instead in some form of economies of agglomeration (i.e. positive market externalities). Long-run concentration patterns would therefore arise because of a self-reinforcing process in which the decision of a firm to locate in a given area induces a net increase in profits enjoyed by firms deciding to follow her thereafter. As a result, clustering processes might display multiple equilibria and path-dependence. Historical accidents could then have long-run cumulative consequences, possibly leading to agglomeration patterns that would not have been selected on the basis of initial conditions only.

Within a such an expanding literature, distinct families of models subscribe to quite different assumptions, on both system-level drivers of agglomeration and microeconomic behaviors. On the one hand, the ‘New Economic Geography’ (NEG) perspective (see Fu-
jita, Krugman & Venables (1999)) primarily focuses on locational choices undertaken by fully informed ‘rational’ firms who live within static environments and interact in monopolistically competitive markets. On the other hand, a second class of formalizations is based on quite distinct assumptions, including sequential, irreversible, decisions made by adaptive firms who interact in explicitly dynamic environments (cf. Arthur (1994) and Rauch (1993)).

Notwithstanding their respective merits and weaknesses (cf. Martin (1999) for a critical overview), it is rather remarkable that both approaches largely share the neglect for a parallel, massive, literature from innovation studies concerning sector-specific processes of technological learning, bearing obvious effects upon the locational stickiness of productive knowledge; the different nature and importance of technological externalities and spillovers; the abilities of incumbents to internalize and ‘carry within themselves’ knowledge complementarities. In brief, one still witnesses a dramatic lack of dialogue between economic geography and ‘spatial’ economics, on the one hand, and the economics of technological change, on the other.

This work is meant as a preliminary contribution to fill this large gap. We explore the basic drivers of spatial agglomeration processes of economic activities and their specificities, both across industries and across spatial locations. More specifically, we ask the following questions: How can one explain the huge, empirically observed, differences in agglomeration patterns across industrial sectors? Are there systematic agglomeration drivers that are entirely sector-specific and operate besides local agglomeration forces (possibly generated by some widespread form of dynamic increasing returns or spatial externalities) inducing concentration independent of technological and learning characteristics of each firm?

In order to address these issues, we propose a simple stochastic model of industrial clustering in which myopic firms make locational choices in presence of dynamic agglomeration economies. The latter stem from both standard comparative advantage arguments (making some locations inherently more attractive than others) and dynamic increasing returns in locating close to other firms. In turn, dynamic increasing returns may be characterized
by both location-specific and technology-specific drivers. Therefore, heterogeneous concentration patterns among geographical sites and industrial sectors are likely to arise. The model yields empirically testable predictions on the equilibrium distribution of the size of spatial clusters and on the diverse relevance of agglomeration forces across industrial sectors. We compare the predictions of the model with some evidence on the geographical distribution of Italian firms across a set of industries which might be considered archetypes of distinct regimes of technological learning (see Pavitt (1984), Dosi (1988), Malerba & Orsenigo (1996) and Marsili (2001)). The evidence strongly supports the view that intersectoral differences in economies of agglomeration might be (at least partly) explained by differences in innovation patterns and learning regimes displayed by firms belonging to diverse industrial sectors.

The rest of the paper is organized as follows. In Section 2 we briefly discuss the state-of-the-art on both theoretical and empirical studies of spatial clustering of economic activities. Section 3 describes the model. In Section 4 we present testing procedures and econometric results with reference to some benchmark industries (i.e. leather products, transport equipment, electronics, financial intermediation services). Finally, in Section 5, we suggest some extensions of the basic model and directions for future work.

2 Economies of Agglomeration and Industrial Concentration: Theory vs. Empirical Evidence

In a nutshell, one might identify four main questions that scholars concerned about the ‘spatial dimension’ of economic interactions have been all trying to address, albeit from different perspectives, for more than a century, namely: (i) Could one neatly identify agglomeration (centripetal) and dispersion (centrifugal) forces lying at the heart of the processes generating sustained spatial concentration (and possibly its destabilization)? (ii) Why and when could one observe persistent spatial patterns that cannot be explained
by resorting to pre-existing heterogeneity in agents and locations (i.e. by some kind of “comparative advantage theory” alone)? (iii) What is the role of mere “chance” in the observed spatial concentration of economic activities? And: (iv) How and when emerging spatial structures of production and innovation tend to become self-sustained over time? (And, conversely, what make them wither away?)

As well known, Von Thünen (1826) and Marshall (1920) have been among the pioneers in the investigation of economic forces driving geographical differentiation and agglomeration. For instance, Von Thunen’s simple analysis of land use - by stressing the importance of space constraints in decentralized economies - began to uncover the relationships between micro decisions and macro geographical outcomes. Even more importantly, Marshall’s discussion of the ‘localization externalities triad’\(^2\) became a cornerstone in the theory of economic agglomeration. From then on, however, diverse trajectories of theoretically-grounded exploration emerged.

A first family of models has been hinging upon the basic idea that many different spatial agglomeration patterns (from concentration of economic activities in few locations to hierarchical structures) can be explained as the solution of a static, well-defined, trade-off between identifiable agglomeration and dispersion forces. This intuition, rooted once again in Von Thunen’s work, has become the core of the analyses provided by ‘central-place’ theory developed by Christaller (1933) and Lösch (1940), of ‘regional science’ models building on Isard (1956) and of the treatment of urban systems by Henderson (1974). More recently, it has inspired models with non-market externalities such as Papageorgiou & Smith (1983), Fujita (1988) and Fujita (1989).

A second class of models that has become prominent in the last few years, known under the heading of ‘New Economic Geography’ (NEG)\(^3\), acknowledges instead some form of increasing returns (or indivisibilities) as both the incentive triggering agglomeration

\(^2\)That is: (i) backward/forward linkages associated to the trade-off between market-size and market-access; (ii) informational spillovers and (iii) advantages of thick markets for specialized local providers of inputs.

\(^3\)See Fujita, Krugman & Venables (1999) and references therein.
and the force able to sustain concentration (once the latter has emerged). One of the achievements of this stream of research has been to provide a treatment of some form of increasing returns *cum* monopolistic competition in a static equilibrium framework with fully rational agents. By bridging monopolistic competition models (cf. Dixit & Stiglitz (1977)) and Samuelson “iceberg-like” trade costs (cf. Samuelson (1952)), such models have been able to account for agglomeration patterns and inter-locational specialization by positing a self-reinforcement process - stemming from some form of market externality - which finds its counterpart in dispersion forces caused either by agglomeration itself or by the immobility of some factors (e.g. labor).

*Third,* building on inspiring works by Brian W. Arthur and Paul David, a few scholars have been attempting to analyze the nature of economies/diseconomies of agglomeration in explicitly dynamic frameworks where persistent spatio-temporal patterns emerge out of the very sequence of interactions among heterogeneous economic agents. By acknowledging the history- (or path-) dependent nature of the observed uneven spatial distribution of economic activities, the basic argument stresses the importance of dynamic increasing returns implied by some form of agglomeration economies/diseconomies (cf. Arthur (1994, Chs. 4 and 6)) and/or local network externalities (cf. David, Foray & Dalle (1998) and Cowan & Cowan (1998)). Notwithstanding their high level of abstraction, these models are able to account for a rather wide array of spatial outcomes and to shed some light on the ability of economies/diseconomies of agglomeration to shape long-run concentration patterns. Together, they highlight how early, small, mainly non-predictable, events might dynamically interact with more systematic forces in conveying persistent spatial structures.

Conversely, from a more inductive perspective, many scholars have been offering a wealth of qualitative analyses, building on several pieces of evidence on urban/regional development and industrial agglomeration phenomena. Furthermore, a long stream of

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4Extensions of the baseline NEG model range from urban systems and city formation (Fujita, Krugman & Mori (1999)), industrial specialization in an array of imperfectly competitive sectors (Venables (1998)) and growth (Fujita, Krugman & Venables (1999, Ch.4)).

5We refer here to the ‘economic geography’ literature (see Lee & Willis (1997) for a survey and the references in Martin (1999)), which includes studies on industrial districts, cf. Antonelli (1990), Sforzi.
literature on multinational investment - from the pioneering works by Vernon (1966), all the way to the recent contributions by Cantwell and colleagues (cf. e.g. Cantwell (1989) and Cantwell & Iammarino (1998)) - are rich of insights on the interaction between technologies, corporate strategies and locational features.

A survey of the evidence discussed in this enormous literature is well beyond the scope of this paper (cf. Bottazzi, Dosi & Fagiolo (2002) for a more detailed discussion and some taxonomic attempts). Here, let us just mention two sets of empirical regularities which are of particular interest in what follows.

First, agglomeration phenomena typically yield quite different ‘types’ of local structures. Examples range from: (i) ‘horizontally diversified agglomerations’ (whereby activities previously vertically integrated within individual firms undergo a sort of ‘Smithian’ process of division of labor cum branching out of different firms); to: (ii) ‘hierarchical spatially localized clusters’ (which generally involve an “oligopolistic core” together with subcontracting networks); and: (iii) ‘Silicon Valley’ districts (where agglomeration phenomena are driven by knowledge complementarities - at least partly fueled by ‘exogenous science’).

A high sectoral variability in agglomeration structures and in the nature of agglomeration drivers clearly hints at the existence of large underlying sectoral and geographical specificities permeating agglomeration processes. In this perspective, a second, related, set of robust empirical evidence concerns the huge intersectoral differences in the revealed spatial agglomeration outcomes. As a suggestive illustration, Fig. 1 plots the distributions of some statistics computed on the frequency profiles of Italian firms belonging to different manufacturing sectors and located in each geographical location6. It is easy to see the

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6Original data refers to geographical location (Year: 1996) of a sample of more than half a million business units (BUs) disaggregated with respect to the ATECO 91 classification (which coincides with the 2-digit ISIC, Rev. 3, classification). Each geographical location represents a ‘local system of labor mobility’ (LSLM), that is a geographical area characterized by relatively high inward commuters’ flows. LSLMs are periodically updated by multivariate cluster analyses employing census data about social, demographic and economic variables (see Sforzi (2000) for details). Frequency plots in Fig.1 are computed as follows. Consider, for each LSLM, the frequency profile of manufacturing BUs present in that location and belonging to each 2-digit manufacturing sector (weighted by the relative size of each sector). For each frequency profile (location) we compute MIN, MAX, RANGE and Standard Deviation statistics and we plot their frequency distribution. Among all LSLM in the dataset (784), we consider only those hosting at
high variability in the distribution of manufacturing sectors across geographical sites. For instance, there exists a high number of locations where firms belonging to almost all sectors are equally represented. On the contrary, for a quite large frequency of sites, agglomeration occurs only for firms belonging to a small number of sectors (in some cases 1 or 2). More generally, in more than 50% of locations, a quite large fraction of sectors are not even represented.

Taken together, these two pieces of evidence suggest a picture where different drivers of agglomeration, which might be economy-wide, location-specific and/or sector-specific, interact over time (at possibly different time- and space-scales) leading to patterns of concentration exhibiting high variability both across locations and across industries. In turn, different types of drivers of agglomeration seem to be often nested in the nature of sector-specific patterns of knowledge accumulation.

Indeed, the main conjecture that we want to explore in this paper is that cross-sectoral differences in agglomeration forces ought to be - at least partly - explained on the grounds of underlying differences in the processes of technological and organizational learning. The latter are in fact likely to affect the relative importance of phenomena such as localized knowledge spillovers; inter- vs. intra-organizational learning; knowledge complementarities fueled by localized labor-mobility; innovative explorations undertaken through spin-offs and, more generally, the birth of new firms.

With this task in mind, let us start by presenting a baseline, empirically testable, model of firm locational choice.

### 3 The Model

Consider an economy with one industry and a potentially infinite number of identical firms. In the economy there are $M \geq 2$ locations, labeled by $j = 1, ..., M$, which can be thought as ‘production sites’ or ‘industrial districts’. Each location $j$ is characterized by an

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least 10 BUs (about 99% of the entire sample).
intrinsic ‘geographical attractiveness’ \( a_j > 0 \) and by an ‘agglomeration’ parameter \( b_j > 0 \).

We suppose that both vectors \( \underline{a} = (a_1, ..., a_M) \) and \( \underline{b} = (b_1, ..., b_M) \) are common knowledge. The coefficients \( a_j \) capture the gain from choosing to locate in \( j \) net of any agglomeration effects. On the contrary, \( b_j \) measure the strength of agglomeration economies in location \( j \): a larger \( b_j \) implies a higher incentive to a firm from locating in \( j \) given the number of firms that have already settled their activities in that location.

Time is discrete. Let \( n^t_j \) be the number of firms present in location \( j \) at time \( t = 0, 1, 2, ... \). Suppose that at time \( t = 0 \) the size of the economy is \( N \gg M \) and assume an initial distribution \( n^0 = (n^0_1, ..., n^0_M) \), \( n^0_j \geq 0 \), \( \sum_{j=1}^{M} n^0_j = N \).

The dynamics of the economy is governed by the following simple rules. At the beginning of each time period \( t \geq 1 \), a firm is chosen at random among all incumbent firms to ‘die’ (i.e. disappear from the location where she operates). Next, a new firm enters and chooses the site where to locate her production facilities. In line with Arthur (1994, Ch.4), we model firms’ locational choices in a stochastic fashion. More precisely, we posit that a firm entering the industry at time \( t \) chooses site \( j \) with a probability proportional to:

\[
a_j + b_j \tilde{n}^t_j,
\]

where \( a_j \) is the ‘intrinsic attractiveness’ of site \( j \), \( b_j \) is the ‘agglomeration’ parameter of site \( j \) and \( \tilde{n}^t_j \) is the actual number of firms present at location \( j \) after exit has occurred (i.e. \( \tilde{n}^t_j = n^t_j - 1 \) if exit occurred in \( j \) and \( \tilde{n}^t_j = n^t_j \) otherwise).

The state of the system is completely defined, at each \( t \geq 0 \), by the ‘occupation’ vector \( \underline{n}^t = (n^t_1, ..., n^t_M) \). Since the state of the system at time \( t + 1 \) only depends on \( \underline{n}^t \), the dynamics of the economy is described by a finite Markov chain with state space \( S = \{ (n_1, ..., n_M) : n_j \geq 0, \sum_{j=1}^{M} n_j = N \} \).

Results about the existence of a stationary (invariant) distribution for \( \underline{n}^t \) (and its characterization) are provided in the following lemma.

**Lemma 1** Define \( p(\underline{n}^t; \underline{a}, \underline{b}) \) as the probability that the system is in the state \( \underline{n}^t \) at time
\[ t \geq 1 \text{ and } \]
\[ P(n_t' | \nu; a, b) = \Pr \{ n_{t+1} = n_t' | n_t = \nu; a, b \} \]

as the generic element of the transition probability matrix of the associated Markov chain, where \( \nu \in S \), \( \nu' \in S \). Then:

1. Let \( \Delta_h = (0, \ldots, 0, 1, 0, \ldots) \) the unitary \( M \)-vector with \( h \)-th component equal to 1. If \( \nu_{t+1} \neq \nu_t + \Delta_k - \Delta_j \) for all \( k, j = 1, \ldots, M \) then \( P(\nu_t' | \nu; a, b) = 0 \). Otherwise, if there exist \( k, j = 1, \ldots, M \) such that \( \nu_t = \nu \) and \( \nu_{t+1} = \nu + \Delta_k - \Delta_j \), then:

\[
P(\nu + \Delta_k - \Delta_j | \nu; a, b) = \begin{cases} 
\frac{n_j}{N} \frac{a_k + b_k n_k}{A \cdot (1 - N - 1)b / n} & k \neq j \\
\frac{n_j}{N} \frac{a_k + b_k (n_k - 1)}{A \cdot (1 - N - 1)b / n} & k = j
\end{cases} \tag{2}
\]

where \( \nu = (n_1, \ldots, n_M) \in S \), \( A = \sum_{m=1}^{M} a_m \), \( b \cdot \nu = \sum_{m=1}^{M} b_m n_m \).

2. The Markov chain governing the evolution of \( \nu_t \) is irreducible and therefore admits a unique stationary distribution \( \pi(\nu; a, b) \) which reads:

\[
\pi(\nu; a, b) = \frac{1}{Z(a, b; N)} \frac{N!}{n_1! \cdots n_M!} \prod_{j=1}^{M} \vartheta(n_j; a_j, b_j), \tag{3}
\]

where:

\[
\vartheta(n_j; a_j, b_j) = \begin{cases} 
\prod_{h=1}^{a_j} [a_j + b_j (h - 1)] & 1 \leq n_j \leq N \\
1 & n_j = 0
\end{cases}
\]

and

\[
Z(a, b; N) = \left( \frac{d}{ds} \right)^N \prod_{j=0}^{M} \left( 1 - s b_j \right)^{a_j / b_j} |_{s=0}.
\]

**Proof.** See Appendix A. \( \blacksquare \)

An interesting case arises when agglomeration coefficients are homogeneous across locations, i.e. \( b_j = b > 0 \), for all \( j \). In such circumstances, one can assume w.l.o.g. that \( b = 1 \)
so that any entrant firm will choose location \( j \) with probability proportional to:

\[ a_j + \tilde{n}_j, \tag{4} \]

where now the ‘intrinsic attractiveness’ \( a_j \) might be interpreted as a relative measure of agglomeration economies. In this simplified setting, a smaller \( a_j \) implies stronger economies of agglomeration. If (4) holds, the Markov chain governing the evolution of \( \underline{n}^t \) is still irreducible (since \( a_j > 0 \), all \( j \)) and the invariant distribution simplifies to:

\[ \pi(n;a) = \frac{N!}{A[N]} \prod_{j=1}^{M} \frac{a_j^{[n_j]}}{n_j!}, \tag{5} \]

where \( a_j^{[n_j]} = a_j(a_j + 1) \cdots (a_j + n_j - 1) \) is the Pochammer’s symbol (see Appendix B).7

Coefficients \( a_j > 0 \) determine the nature of the distribution. As the values of \( a_j \)’s get bigger, the effects of agglomeration economies wither away. In the limit, when \( a_j \to +\infty \) and \( a_j/a_{j'} \to 1 \) for any \( j \) and \( j' \), agglomeration economies disappear and the expression in (5) reduces to a multinomial distribution. On the contrary, when \( a_j = 1, \forall j \), (5) becomes the Bose-Einstein distribution.8

Some remarks are in order. First, we assume that entry rates (i.e. birth rates) are positive, constant and equal to exit rates (i.e. death rates). The idea behind this assumption comes from the observation that, at least in Italy9, the share of firms belonging to a given sector who enter and/or leave a given location in a relatively short period of time (e.g. a year) is typically much larger than the net growth of industry size, so that the time-scale at which spatial reallocations occur is generally very short. Therefore, the invariant (or

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7The model with homogeneous \( b_j \)'s is a variation of the Ehrenfest-Brillouin urn-scheme. See Garibaldi & Penco (2000) and Garibaldi, Penco & Viarengo (2002) for the case with 2 locations. A similar simplified version is in Kirman (1993).

8Cf. e.g. Wio (1994) and Johnson, Kotz & Balakrishnan (1997). Notice also that values \( a_j \leq 0 \), for some \( j \), can be in principle considered in order to allow for negative ex-ante geographical benefits. However, since a negative \( a_j \) would require the empirically questionable notion of upper bounds on the number of firms that can be hosted in a location, we prefer to stick to the assumption of non-negative \( a_j \)'s.

9See e.g. quarterly reports by Unioncamere, “Movimprese: Dati Trimestrali sulla Nati-Mortalità delle Imprese”, Uffici Studi e Statistica Camere di Commercio, Italy, various years, available on line at the url: http://www.starnet.unioncamere.it.
equilibrium) distributions in (3) and (5) does not necessarily depict a long-run state associated to some ‘old’ or ‘mature’ industry. Since each entry/exit decision made by any one firm constitutes one time-step in the model, our invariant distributions describe the state of the system after a sufficient large number of spatial reallocation events have taken place (which may well imply a relatively short real-time horizon). Invariant distributions can then be directly compared with cross-section empirical data because they describe a system which, for short real-time horizons, always appears in its equilibrium state\(^{10}\).

Second, and relatedly, we suppose that any firm remains in her location until she eventually exits from the industry. Individual locational choices might then be interpreted as being irreversible. However, one-step transition probabilities computed in (2) are also consistent with an alternative locational process involving reversible choices wherein: (i) there is a constant population of $N$ firms (no entry/exit); (ii) in each time period a randomly drawn firm is allowed to switch location with probabilities proportional to (1), with $a_j > 0$ for all $j$. In both cases, the size of the industry is constant (equal to $N$) throughout the whole process because the net growth rate is zero. Therefore, the impact of the noise introduced in the system by any single additional decision (either due entry/exit or between-location switches), albeit quite small, does not become negligible as $t$ becomes large. Thus, the equilibrium behavior of the system can be described, unlike models based on Polya-urn schemes, by a non degenerate stationary distribution.

In the next Section, we will test the predictions of the model against data on geographical distribution of firms across Italian industrial districts. As a preliminary exercise, we will focus on the case of homogeneous agglomeration coefficients ($b_j = b > 0$), and employ (5) to test for the existence of persistence differences in the strength of agglomeration economies among industrial sectors.

\(^{10}\)Cf. also Appendix C for an interpretation of this property in terms of Polya-urn schemes. Long-term modifications in the industrial structure might be instead captured by allowing $a$ and $b$ coefficients to change across subsequent phases of industry evolution, albeit in a time-scale much longer than the one related to spatial reallocation decisions (i.e. indexed by $t$).
4 Agglomeration Economies and Industrial Sectors: An Application to Italian Data

In this Section we shall attempt to address the following questions. *First:* Do theoretical distributions (derived from the model presented above) adequately replicate, for each given sector, the observed frequency distributions of firms across locations? *Second:* What is the statistical impact of intesectoral differences on the dynamics of spatial concentration?

Note that in order to start answering the latter question, one ought to disentangle two basic factors jointly contributing to the observed sector-specificities in agglomeration patterns, namely: (i) agglomeration drivers which, for any given sector, are location-specific and generate agglomeration benefits due to dynamic increasing returns to concentration (e.g. *ex-ante* differences across geographical locations, economy-wide agglomeration spillovers which cumulatively act upon the existing concentration patterns, etc.); (ii) agglomeration drivers that are entirely *sector-specific* and promote concentration across all geographical locations (e.g. thanks to economies of agglomeration forces that are intrinsically related to the way knowledge is accumulated, innovations are generated, etc.).

In this perspective, we present here a preliminary study focusing on four sectors: (a) leather products; (b) transport equipment; (c) electronics; (d) financial intermediation.

The choice is motivated by the observation that these industries display a large intersectoral variation as to their patterns of innovation and learning regimes, as well as the average sizes of their BUs and their competition patterns. More precisely, according to the descriptive taxonomy of industrial sectors firstly proposed by Pavitt (1984) and subsequently developed in Malerba & Orsenigo (1996) and Marsili (2001), these industries belong to four distinct groups (cf. also Table 1).

In Pavitt’s terminology, the leather industry - with the partial exception of ‘fashion products’ - might be classified as a ‘supplier dominated’ (SD) sector, characterized by relatively small firms whose innovative opportunities largely stem from external loci of innovation (e.g. intermediate and capital inputs produced elsewhere). SD industries usu-
ally involve high product differentiation and include most of the so-called “made-in-Italy” activities (e.g. textiles, clothing, furniture, toys, etc.).

Transport equipment is a standard ‘scale-intensive’ (SI) sector, wherein large firms generate (both internally and thanks to ‘specialized suppliers’) innovation in production processes and, together, master the design and production of quite complex artifacts.

Electronics typically belongs to the class of ‘science-based’ (SB) sectors. Here innovation in both products and processes is largely generated in R&D departments of firms which often maintain strong links with universities and research centers.

Finally, financial intermediation activities are ‘information intensive’ (II) sectors, which share with science-based industry the locus of innovation (R&D departments) and the sources of innovative opportunities (universities and research centers). However, II sectors typically differ from SB ones as to the means of appropriating the economic rents from their innovations. While science-based industries typically appropriate innovations through patents and lead times of innovators vis-à-vis would-be imitators, information intensive ones comparatively take more advantage of the tacitness of their knowledge bases (cf. Malerba & Orsenigo (1996)).

The conjecture that we preliminary test in this work is that intersectoral differences in the patterns of innovation creation, innovation flows and learning regimes, as proxied by Pavitt’s categorization, map into different degrees of local agglomeration economies, once differences due to location-specific agglomeration effects have been factored out. It is indeed likely that firms locational choices are affected in quite different ways by different appropriability means, distinct sources of innovation sources/types, as well as different channels through which technological information locally spills over. We suggest that such spatially local, sector-specific, drivers might be able to account for the observed differences of agglomeration patterns across industries.

Data and Methodology

The exercise employs a database provided by the Italian Statistical Office (ISTAT) from
the Census of Manufacturers and Services. Data contain observations about more than half a million business units (BUs), i.e. local plants. Each observation identifies the location of the BUs at a given point of time (end of 1996), as well as the industrial sector where it operates. Observations refer to \( L = 31 \) industrial sectors\(^{11} \) while locations correspond to \( M = 784 \) “local systems of labor mobility” (LSLM) (see footnote 6).

Let \( n_{i,l} \) be the number of BUs in LSLM \( i \) operating in sector \( l \). Denote with \( n_{.,l} \) the number of BUs operating in sector \( l \) and with \( n_{i,.} \) the total number of BUs belonging to \( i \)-th LSLM. Since a standard maximum likelihood procedure is not viable\(^{12} \), we shall estimate coefficients \( a_{i,l} \), for any given sector \( l \), in two benchmark cases:

1. \( a_{i,l} \) are homogeneous across locations, i.e. \( a_{i,l} = \alpha_l \), where \( \alpha_l > 0 \) is a sector-specific parameter;

2. \( a_{i,l} \) are heterogeneous across locations and \( a_{i,l} = \gamma_l \cdot \theta_{i|l} \), where \( \gamma_l \) is a sector-specific parameter and \( \theta_{i|l} \), for any given sector, is a location-specific parameter.

Notice that in the case 1. one is assuming that location-specific agglomeration drivers are homogeneous across LSLM. Under this hypothesis, BUs belonging to any given sector \( l \) would choose any given geographical site with equal probability. On the other hand, in the case 2. one assumes that the geographical attractiveness of any site \( i \) can be decomposed into a factor that accounts for location- (and possibly sector-) specific (i.e. \( \theta_{i|l} \)) local attractiveness and a strictly sector-specific factor accounting for activity-specific increasing returns to agglomeration (i.e. \( \gamma_l \)).

We estimate \( \theta_{i|l} \) by using data about all sectors different from \( l \), which are assumed to be exogenous with respect to the data generation process postulated in the single-sector model. Hence, sector distributions, in both case 1. and 2., will depend on a single parameter (\( \alpha_l \) or \( \gamma_l \)) that can be in turn estimated by a standard best-fit procedure (e.g. minimization of chi-square test between theoretical and empirical distributions).

\(^{11}\)Data about industrial sectors are disaggregated according to the Italian ATECO 91 classification which corresponds to the 2-digit ISIC (Rev. 3) classification.

\(^{12}\)Unfortunately, data about sufficiently long time series of homogeneous observations is still not available at the appropriate disaggregated level.
In order to compare theoretical predictions with empirical data, let us define the marginal, site-occupancy, stationary probability distribution \( \phi(h|a_{il}, A, N) \) as the probability that a site with “intrinsic attractiveness” \( a_{il} > 0 \) would host in the long-run exactly \( h = 0, 1, ..., N \) firms. From (5), one obtains (cf. Appendix C):

\[
\phi(h|a_{il}, A, N) = \frac{N!}{A^{[N]}} \frac{a_{il}^h (A - a_{il})^{[N-h]}}{h! (N-h)!} \]  

(6)

For each sector under analysis, we may therefore compare the theoretical distribution (6) with the corresponding observed frequency with which a LSLM hosting \( n_{i,l} = h \) business units appears in sector \( l \):

\[
f_l(h) = \frac{1}{M} \sum_{i=1}^{M} \delta(n_{i,l}, h) \]

(7)

where \( \delta(n_{i,l}, h) = 1 \) if and only if \( n_{i,l} = h \).

**Results**

Let us begin by assuming that all locations are homogeneous as to their intrinsic geographical attractiveness. In this case, the process is driven only by economies of agglomeration which are themselves homogeneous across locations. More formally, for any single sector \( l \), let \( a_{il} = \alpha_l \), \( i = 1, ..., M \) and \( A_l = M\alpha_l \), \( \alpha_l > 0 \). Theoretical frequencies (6), become:

\[
\varphi_l(h; \alpha_l) = \phi(h|\alpha_l, M\alpha_l, N) = \frac{N!}{(M\alpha_l)^{[N]}} \frac{\alpha_l^h (M\alpha_l - 1)^{[N-h]}}{h! (N-h)!}. \]

(8)

Notice that \( \phi \) will now depend, for any \( l \), on a single parameter \( \alpha_l \) measuring the strength of the agglomeration effect (recall that a low \( \alpha_l \) means strong agglomeration economies). For each sector under study, the agglomeration parameter will then be estimated as:

\[
\alpha_l^* = \arg \min_{\alpha_l \in \Lambda} \chi^2(f_l, \varphi_l), \]

(9)

where \( \chi^2 \) is the standard goodness-of-fit test between two binned (theoretical and empirical)
frequency distributions and $\Lambda$ is an evenly-spaced grid of values for $a_{i} > 0$.

Interestingly, tests of this model yield very bad agreement with data, with ‘predicted’ theoretical distributions $\varphi_{l}(\mathbf{h}; \alpha_{i}^{*})$ always underestimating observed distribution tails. In particular, $\chi^{2}$ tests reject the hypothesis that data come from the distribution in (8) for any value of $\alpha_{i} > 0$, in all four sectors under analysis.

The reason why this is the case becomes evident if one plots, for any given sector $l$, the number of BUs located in the LSLM $i$ ($n_{i,l}$) against the total number of BUs belonging to all sectors but $l$ (i.e. $n_{i,.} - n_{i,l}$). Under the assumption of homogeneous intrinsic geographical attractiveness, any two BUs belonging to different sectors should choose the same location with equal probability. Therefore, no statistically significant correlation should appear between $n_{i,l}$ and $n_{i,.} - n_{i,l}$ for any $l$. Conversely, as Fig. 2 shows, for any of the four chosen sectors a statistically significant positive correlation between the two variables appears, contradicting the conjecture that all LSLM have the same ex-ante attractiveness.

Suppose instead that the degrees of intrinsic geographical attractiveness are heterogeneous across locations and let $a_{i,l} = \gamma_{l} \cdot \theta_{i|l}$. Here $\theta_{i|l}$ represents the strength of agglomeration economies of location $i$ in sector $l$. Given the high correlation between $n_{i,l}$ and $n_{i,.} - n_{i,l}$ exhibited by the data for all sectors under study, we will assume that for any location $i$ and sector $l$:

$$a_{i,l} = \gamma_{l} \frac{n_{i,.} - n_{i,l}}{\sum_{i=1}^{M} (n_{i,.} - n_{i,l})} = \gamma_{l} \frac{n_{i,.} - n_{i,l}}{N - n_{i,l}},$$

where $\gamma_{l} > 0$ measures industry-specific effects due to economies of agglomeration. Coefficients $\theta_{i|l}$ capture here the effect of local agglomeration drivers which, for each given sector, are location-specific (as compared to agglomeration forces which on the contrary act at an economy-wide level). The latter include all factors which make a location intrinsically preferable compared to others, in terms of e.g. better industrial infrastructures, sheer overall size, etc., all the way to local spillovers that generate dynamic increasing returns to agglomeration for all sectors.

Since our data-generation process refers to a single sector, we can proxy $\theta_{i|l}$ by using
exogenous information about the behavior of all firms belonging to all sectors different from the one under consideration. If (10) holds, the theoretical frequency of finding a LSLM hosting exactly $h$ BUs in sector $l$ can be easily computed by averaging marginal probabilities in (6) over all LSLM, after having controlled for the size of each sector. The theoretical (weighted) frequency distribution for sector $l$ then reads:

$$
\psi_l(h; \gamma_l) = \frac{1}{M} \sum_{i=1}^{M} \phi(h|a_{i,l}, A_l, n_{.l}),
$$

(11)

where $\phi$ is the probability distribution in (6), $a_{i,l}$ are defined as in (10), $A_l = \sum_{i=1}^{M} a_{i,l}$ and $n_{.l}$ is the number of BUs in sector $l$. Since $\psi_l$ depends, for any sector $l$, only on $\gamma_l$, we can use the same fitting procedure we employed in the homogenous coefficients case. ‘Predicted’ values for $\gamma_l$ are therefore computed as:

$$
\gamma_l^* = \arg \min_{\gamma_l \in G} \chi^2(f_l, \psi_l),
$$

(12)

where $\chi^2$ is defined as above and $G$ is an evenly-spaced grid of values for $\gamma_l > 0$.

Table 2 reports ‘predicted’ values for sectoral agglomeration parameters, their 5% confidence intervals$^{13}$, together with $\chi^2$ test values and tail probabilities for the difference between $\psi_l^* = \psi_l(h; \gamma_l^*)$ and $f_l$. In all four sectors, ‘predicted’ theoretical distributions $\psi_l^*$ fit very well empirical frequencies. Indeed, one cannot reject the hypothesis that $\psi_l^*$ are different from empirical distributions $f_l(h)$. As to the magnitudes of the predicted parameters, notice that ‘leather’, ‘transport equipment’ and ‘electronics’ sectors seem to display higher agglomeration economies (i.e. comparably small $\gamma$’s) as compared to ‘financial intermediation’. This conjecture seems conﬁrmed by Figs. 3 through 6, where predicted ($\psi_l^*$) and empirical ($f_l$) frequencies are plotted. While the first three sectors studied exhibit the standard skewed shape associated to high agglomeration forces, financial intermediation is characterized by a more dispersed distribution of BUs across locations.

$^{13}$Confidence intervals contain all values of $\gamma_l$ such that Prob{$\chi^2_D > \chi^2(f_l, \psi_l)$} < 0.05, where $\chi^2_D$ is a r.v. distributed as a $\chi^2(D)$ and $D$ are the degrees of freedom of the test.
In order to test whether estimated coefficients statistically differ across sectors, we performed $\chi^2$ tests for the difference between any two distributions. We first test whether any two ‘predicted’ distributions are different. Results reported in Table 3 show that theoretical distributions $\psi_i^*$ are all statistically different from each other. Notice, however, that confidence intervals for $\gamma_i^*$ partly overlap (see Table 2), especially as far as ‘leather’ and ‘transport equipment’ sectors are concerned. Therefore, to further explore if estimated $\gamma$’s really differ between sectors, we compared any two distributions $\psi_{l_1}$ and $\psi_{l_2}(h; \gamma_{l_1}^*)$, i.e. the distribution of sector $l_2 \neq l_1$ computed employing the ‘predicted’ parameter value for sector $l_1$. If they statistically differ, then one might reasonably conclude that $\gamma_{l_1}^* \neq \gamma_{l_2}^*$.

Results reported in Table 4 confirm that ‘financial intermediation’ exhibits agglomeration economies statistically lower than the other three sectors. Furthermore, ‘electronics’ appears to display intermediate values of $\gamma$’s, while ‘leather’ and ‘transport equipment’ are characterized by high (but not statistically different) agglomeration strength.

These results are in line with qualitative analyses on the relationships between the intersectoral patterns of innovation/learning regimes and geographical concentration of economic activities (cf. inter alia Antonelli (1994)). Agglomeration economies may be expected to be relevant in $SI$ and $SD$ sectors (represented here indeed by ‘leather’ and ‘transport equipment’), albeit for different reasons. ‘Scale intensive’ sectors are likely to involve hierarchical relations among firms, leading to geographical clustering characterized by an “oligopolistic core” together with subcontracting networks. Conversely, different drivers might lead to observationally similar statistical effects in those sectors which Pavitt (1984) calls ‘supplier dominated’. The forces fueling many Italian industrial districts, mostly featuring in this category, point at processes of inter-firm division of labor, at knowledge complementarities, and at various district-specific institutional arrangements as factors underlying agglomeration (cf. for instance Brusco (1982) and Piore & Sabel (1984)). Agglomeration economies should also appear significant in $SB$ sectors, due to ‘Silicon Valley’ effects based on knowledge complementarities and on particular institutions fueling “exogenous science”. However, as already noted in Bottazzi et al. (2002), science-based
sectors do not display in Italy striking agglomeration effects, probably due to the underlying weakness of ‘fueling’ research institutions, with nothing even vaguely comparable to Stanford, UC Berkeley or MIT. Finally, firms belonging to II industries (e.g. banks and insurance companies) do not appear to enjoy important agglomeration economies. Therefore, at least in our benchmark II sector, agglomeration economies implied by the existence of large fixed costs associated to local provision of specialized intermediate goods (e.g. related to extensive adoption of information and communication technology goods and services) seem to be overcome by ‘monopolistically competitive’ strategies of branch location near the customers (see Fujita & Hamaguchi (2001)).

5 Conclusions

Economies of agglomeration have been shown to play a key role in the emergence of relatively stable patterns of industrial clustering. However, existing theoretical endeavours attempting to explain industrial concentration on the basis of some form of dynamic increasing returns to agglomeration have been largely neglecting the vast amount of empirical evidence about inter-sectoral differences in the patterns of spatial activity.

In this paper, we argued that cross-sectional differences in agglomeration forces might be (at least partly) explained by underlying differences in the processes of firms’ technological and organizational learning. We presented a very simple model of industrial clustering in which adaptive firms make locational choices in presence of agglomeration economies. The latter stem from both standard comparative advantage arguments (making some locations inherently more attractive than others) and dynamic increasing returns to scale in agglomeration which can be both sector- and industry-specific. We tested the predictions of the model about the long-run distribution of the size of spatial clusters against data on Italian ‘local system of labor mobility’ (i.e. a proxy for industrial districts).

In each sector under analysis, the accordance of theoretical predictions with data is quite high, with statistically significant, sector- and location-specific, economies of agglomera-
tion. In turn, underlying differences in the modes of innovative exploration and knowledge accumulation, we suggest, are likely to map into sectoral specificities in the strength of geographical clustering.

Of course one may think of several ways forward with respect to the foregoing analysis. First, our basic conjecture on the role of technological specificities as determinants of the intensity of agglomeration, if any, is going to be fully corroborated only by studying many more sectors, possibly in different countries. Second, one may incorporate explicit local interactions among firms and non-linear formulations of the location probabilities. Third, it would be interesting to consider inter-sectoral interactions in location patterns.

Existing evidence does suggest widespread phenomena of clustering of innovation activities (cf. Cowan & Cowan (1998) and Brusco (1982), among others). Moreover, technology-specific interactions between location of innovative activities and location of production in the case of multinational corporations has been identified by Cantwell and collaborators (see e.g. Cantwell (1989) and Feldman (1994)). The foregoing study is, in many respects, complementary to such investigations and, hopefully, moves some steps ahead toward bridging the geography of location with the economics and geography of innovation.
References


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Von Thünen, J. (1826), *Der Isolierte Staat in Beziehung auf Landwirtschaft und Nationalökonomie*, Hamburg, Perthes.

Figure 1: Frequency distributions of MAX (Top-Left), MIN (Top-Right), RANGE (Bottom-Left) and STANDARD DEVIATION (Bottom-Right) statistics computed on the weighted frequency profile of Italian manufacturing business units (BUs) belonging to different industrial sectors (2-digit disaggregation) present in each geographical location in 1996. For each statistics $S$, a circle corresponding to a value $s$ on the $x$-axis represents the % of all locations for which the statistics $S$ (computed on the weighted frequency profile of firms belonging to each industrial sector present in that location) is equal to $s$.

Locations are defined in terms of Local Systems of Labor Mobility (cf. footnote 6).

Source: Our elaborations on ISTAT, Censimento Intermedio dell’Industria e dei Servizi, 1996.
Figure 2: Number of business units belonging to sector \( l \) located in a given Local System of Labor Mobility \( (n_{i,l}) \) vs. the total number of BUs belonging to all sectors but \( l \) \( (n_{i,-} - n_{i,l}) \). Panels: a) Leather products; b) Transport equipment; c) Electronics; d) Financial Intermediation. All variables are in log scale. Estimated Slopes of Linear Regressions (significance of t-test \( \hat{\beta} = 0 \) in brackets): (a) \( \hat{\beta} = 0.443 \) (0.0001); (b) \( \hat{\beta} = 0.798 \) (0.0002); (c) \( \hat{\beta} = 0.727 \) (0.0001); \( \hat{\beta} = 0.746 \) (0.0000). Source: Our elaborations on ISTAT, Censimento Intermedio dell'Industria e dei Servizi, 1996.
Figure 3: **Leather Products.** Observed vs. Theoretical Frequencies of BUs (business units) in LSLM (Local System of Labor Mobility). Y-axis: Frequency of LSLM hosting \( h \) BUs. Source: Our elaborations on ISTAT, 1996 data.

Figure 4: **Transport Equipment.** Observed vs. Theoretical Frequencies of BUs (business units) in LSLM (Local System of Labor Mobility). Y-axis: Frequency of LSLM hosting \( h \) BUs. Source: Our elaborations on ISTAT, 1996 data.
Figure 5: **Electronics**. Observed vs. Theoretical Frequencies of BUs (business units) in LSLM (Local System of Labor Mobility). Y-axis: Frequency of LSLM hosting $h$ BUs. Source: Our elaborations on ISTAT, 1996 data.

Figure 6: **Financial Intermediation**. Observed vs. Theoretical Frequencies of BUs (business units) in LSLM (Local System of Labor Mobility). Y-axis: Frequency of LSLM hosting $h$ BUs. Source: Our elaborations on ISTAT, 1996 data.
<table>
<thead>
<tr>
<th>Sector</th>
<th>ISIC Class</th>
<th>Pavitt’s Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leather</td>
<td>D.19</td>
<td>Supplier Dominated (SD)</td>
</tr>
<tr>
<td>Transport Equipment</td>
<td>D.34, D.35</td>
<td>Scale Intensive (SI)</td>
</tr>
<tr>
<td>Electronics</td>
<td>D.30, D.31, D.32, D.33</td>
<td>Science Based (SB)</td>
</tr>
<tr>
<td>Financial Intermediation</td>
<td>J.65, J.66, J.67</td>
<td>Information Intensive (II)</td>
</tr>
</tbody>
</table>

**Table 1**
The Statistical Classification of the considered Sectors.

<table>
<thead>
<tr>
<th>Sector (l)</th>
<th>$\gamma^*_l$</th>
<th>Confidence Intervals</th>
<th>$\chi^2(f_l, \psi_l(\gamma^*_l))$</th>
<th>Prob{$\chi^2_D &gt; \chi^2(f_l, \psi_l(\gamma^*_l))$}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leather</td>
<td>0.0032</td>
<td>(0.0026, 0.0098)</td>
<td>52.6760</td>
<td>0.3709</td>
</tr>
<tr>
<td>Transport</td>
<td>0.0128</td>
<td>(0.0087, 0.0169)</td>
<td>58.7517</td>
<td>0.1855</td>
</tr>
<tr>
<td>Electronics</td>
<td>0.0376</td>
<td>(0.0301, 0.0462)</td>
<td>54.2862</td>
<td>0.3147</td>
</tr>
<tr>
<td>Financial</td>
<td>0.7871</td>
<td>(0.7101, 0.8005)</td>
<td>44.1767</td>
<td>0.7051</td>
</tr>
</tbody>
</table>

**Table 2**
‘Predicted’ Agglomeration Parameters $\gamma^*_l = \arg \min_{\gamma_l \in G} \chi^2(f_l, \psi_l)$. Confidence Intervals for $\gamma^*_l$ contain all $\gamma_l$ s.t. the 5% Chi-Square test between $\psi_l(h; \gamma_l)$ and $f_l$ is not rejected. Degrees of freedom: $D = 50$. 
<table>
<thead>
<tr>
<th>\chi^2(\psi_l(\gamma_l^<em>), \psi_m(\gamma_m^</em>))</th>
<th>Leather</th>
<th>Transport</th>
<th>Electronics</th>
<th>Financial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leather</td>
<td>\Box</td>
<td>0.0523</td>
<td>0.0002</td>
<td>0.0001</td>
</tr>
<tr>
<td>Transport</td>
<td>0.0523</td>
<td>\Box</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Electronics</td>
<td>0.0002</td>
<td>0.0000</td>
<td>\Box</td>
<td>0.0000</td>
</tr>
<tr>
<td>Financial</td>
<td>0.0001</td>
<td>0.0000</td>
<td>0.0000</td>
<td>\Box</td>
</tr>
</tbody>
</table>

Table 3
Tail probabilities for the Chi-Square test between \( \psi_l(\gamma_l^*) \) (‘predicted’ distribution for sector \( l \)) and \( \psi_m(\gamma_l^*) \) (‘predicted’ distribution for sector \( l \)). Degrees of freedom: \( D = 50 \).

<table>
<thead>
<tr>
<th>\chi^2(\psi_l(\gamma_l^<em>), \psi_m(\gamma_l^</em>))</th>
<th>Leather</th>
<th>Transport</th>
<th>Electronics</th>
<th>Financial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leather</td>
<td>\Box</td>
<td>0.9942</td>
<td>0.0621</td>
<td>0.0000</td>
</tr>
<tr>
<td>Transport</td>
<td>0.9598</td>
<td>\Box</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Electronics</td>
<td>0.0771</td>
<td>0.0000</td>
<td>\Box</td>
<td>0.0000</td>
</tr>
<tr>
<td>Financial</td>
<td>0.0001</td>
<td>0.0000</td>
<td>0.0000</td>
<td>\Box</td>
</tr>
</tbody>
</table>

Table 4
Tail probabilities for the Chi-Square test between \( \psi_l(\gamma_l^*) \) (distribution for sector \( l \) computed at the ‘predicted’ value for sector \( l \)) and \( \psi_m(\gamma_l^*) \) (distribution for sector \( m \) computed at the ‘predicted’ value for sector \( l \)). Degrees of freedom: \( D = 50 \).
Appendices

A Proof of Lemma 1

Point 1. Let $P(n\mid a, b)$ be the generic element of the transition matrix of the Markov chain that describes the dynamics of the system. Moreover, let $p(n^t = n\mid a, b)$ the probability that the chain is in the state $n$ at time $t$ (in the following we will omit, for the sake of simplicity, parameters $a, b$).

We have assumed that in any time period only one firm will exit her current location and only one firm will enter one of the $M$ locations (possibly including the one in which exit has occurred). Therefore, given $n^t = n$, the state at time $t + 1$ must necessarily be such that either (i) there exist two locations, say $k'$ and $k''$, $k' \neq k''$ such that $n_{k't+1} = n_{k't} - 1$ and $n_{k''t+1} = n_{k''t} + 1$; or (ii) $n^{t+1} = n^t$, if the entrant has chosen the same location of the exiting firm. Hence if $n^{t+1} \neq n^t + \Delta_k - \Delta_j$ for all $k, j = 1, ..., M$ then $P(n^t\mid n) = 0$.

Otherwise, if there exist $k, j = 1, ..., M$ such that $n^t = n$ and $n^{t+1} = n + \Delta_k - \Delta_j$, then for any $j, k = 1, ..., M$:

$$
\Pr\{n + \Delta_k - \Delta_j \mid n\} = \Pr\{\text{A Firm Exits from location } j\} \cdot \Pr\{\text{Entrant Chooses location } k \mid \text{A Firm Exits from location } j\}.
$$

As the exiting firm is chosen at random from all incumbent firms then:

$$
\Pr\{\text{A Firm Exits from location } j\} = \frac{n^t_j}{N}.
$$

From (1), we also have that the entrant firm will find $n^t_k$ firms in any location $k \neq j$, while
(if a firm has left location \( j \)) she will find \( n_j - 1 \) firms in location \( j \). Thus:

\[
\Pr\{\text{Entrant Chooses location } k | \text{A Firm Exits from location } j\} \propto \begin{cases} 
  a_k + b_k n_k^j, & k \neq j \\
  a_k + b_k (n_k^j - 1), & k = j
\end{cases}
\]

By imposing the normalizing condition:

\[
H^{-1} \cdot \left( \sum_{j=1}^{M} \sum_{k=1}^{M} \frac{n_j^j}{N} (a_k + b_k n_k^j) + \sum_{j=1}^{M} \frac{n_j^j}{N} (a_j + b_j (n_j^j - 1)) \right) = 1,
\]

we easily get that

\[
H = \sum_{h=1}^{M} a_h + (1 - \frac{1}{N}) \sum_{h=1}^{M} b_h n_h = A + (1 - N^{-1}) \sum_{h=1}^{M} b_h n_h.
\] (13)

This proves Point 1. ■

**Point 2.** For strictly positive \( a_j, j = 1, \ldots, M \), each location has a strictly positive probability of receiving the entering firm, see (2). Therefore any state \( n \in S = \{(n_1, \ldots, n_M) : n_j \geq 0, \sum_{j=1}^{M} n_j = N \} \) is reachable with a positive probability in a suitable number of steps starting from any other state. Hence the Markov chain is irreducible and its evolution reads:

\[
p(n^{t+1} = n) = \sum_{n' \in S} P(n|n')p(n^t = n').
\]

The invariant distribution \( \pi(n;a,b) \) must therefore obey the detailed balance condition:

\[
\pi(n_0;a,b)P(n'|n) = \pi(n'|a,b)P(n|n'),
\] (14)

where \( n, n' \in S \) and \( P \) is the transition matrix. By using eq. (14) and transition probabil-
ities given in (2), one gets:

\[ \pi(n + \Delta_k - \Delta_j; a, b) = \pi(n; a, b) \frac{n_j}{n_k + 1} \frac{a_k + b_k n_k}{a_j + b_j (n_j - 1)}. \]  

(15)

The invariant distribution can thus be obtained by recursively applying (15) for a given initial occupancy vector in \( S \). Let \( \pi^* = (N, 0, \ldots, 0) \) be the state of the system with all firms in the first location and let \( \pi^* = \pi(n^*; a, b) \). Now suppose to move \( n_2 \geq 1 \) firms from the first to the second location. By applying recursively (15) one obtains:

\[
\pi((N - n_2, n_2, 0, \ldots, 0); a, b) = \frac{N!}{(N - n_2)! n_2!} \frac{1}{\prod_{j=1}^{N-n_2} [a_1 + b_1 (j_1 - 1)] \cdot \prod_{j=2}^{n_2} [a_2 + b_2 (j_2 - 1)]} \cdot \prod_{j=1}^{N} [a_1 + b_1 (j_1 - 1)].
\]

(16)

Suppose now to further move \( n_3 \geq 1 \) firms from the first to the third location, while keeping \( n_2 \) firms in location 2. Using again eqs. (15) and (16), one gets:

\[
\pi((N - n_2 - n_3, n_2, n_3, \ldots, 0); a, b) = \frac{N!}{(N - n_2 - n_3)! n_2! n_3!} \frac{1}{\prod_{j_1=1}^{N-n_2-n_3} [a_1 + b_1 (j_1 - 1)] \cdot \prod_{j=2}^{n_2} [a_2 + b_2 (j_2 - 1)] \cdot \prod_{j=3}^{n_3} [a_3 + b_3 (j_3 - 1)]} \cdot \prod_{j=1}^{N} [a_1 + b_1 (j_1 - 1)].
\]

(17)

Notice that the result in (17) does not depend on the order in which firms have been moved from location 1 to locations 2 and 3. Therefore, a clear pattern emerges. Consider a generic occupancy vector \( n=(n_1, n_2, \ldots, n_M) \in S \) obtained by moving exactly \( n_k \geq 1 \) firms from the first to the \( k \)-th location and defining \( n_1 = N - \sum_{k=2}^{M} n_k \). By recursively applying (15), one has:

\[
\pi(n; a, b) = \frac{\pi^*}{\prod_{i=1}^{N} [a_1 + b_1 (i - 1)]} \frac{N!}{n_1! \cdots n_M!} \prod_{j=1}^{M} \vartheta(n_j; a_j, b_j),
\]

(18)

where:
$$\vartheta(n_j; a_j, b_j) = \begin{cases} 
\prod_{h=1}^{n_j} [a_j + b_j(h - 1)] & 1 \leq n_j \leq N \\
1 & n_j = 0 \end{cases}.$$ 

Notice that, if \( n_j = 0 \) for some \( j \), then \( \vartheta(n_j; a_j, b_j) = 1 \) because no factors are generated. Finally, by imposing the obvious condition that probabilities must sum up to one and from the fact that \( \pi \) has to be invariant under symmetric permutations of \( n, a \) and \( b \), one gets:

$$\pi(n; a, b) = Z(a, b, N) \frac{N!}{n_1! \cdots n_M!} \prod_{j=1}^{M} \vartheta(n_j; a_j, b_j).$$

To compute the normalizing constant \( Z(a, b, N) \), let:

$$\theta(k; x, y) = \begin{cases} 
\prod_{h=0}^{k-1} (x + hy) & k > 1 \\
1 & k = 1 \end{cases}.$$ (19)

It is easy to see that:

$$\theta(k; x, y) = \frac{d^k}{ds^k} (1 - sy)^{-x/y} \big|_{s=0}. \quad (20)$$

Indeed consider:

$$\tilde{\theta} = \sum_{k=0}^{\infty} \frac{s^k}{k!} \theta(k; x, y)$$ (21)

and let us rewrite it as a degenerate hypergeometric series:

$$\tilde{\theta}(s; x, y) = 1 + \sum_{k=1}^{\infty} \frac{s^k y^k}{k!} \prod_{h=0}^{k-1} \left( \frac{x}{y} + h \right) = \tF1 (b/a, w; w; b)$$ (22)

for a generic (positive) \( w \). Following Gradshteyn & Ryzhik (2000, p.966), one obtains:

$$\tilde{\theta}(s; x, y) = (1 - sy)^{-x/y}$$ (23)

and eq. (20) immediately follows.
Returning to the expression of the equilibrium distribution, using (20), one gets:

\[ \pi(n, a, b) = \frac{1}{Z} \frac{N!}{\prod j n_j!} (\frac{d}{ds_j})^{n_j} \prod j (1 - s_j b_j)^{-a_j/b_j} |_{s=0} \]  

(24)

where \( s = (s_1, ..., s_M) \). Summing up over all the occupancies, one has:

\[ \sum n \pi(n, a, b) = \frac{1}{Z} (\sum j \frac{d}{ds_j})^N \prod j (1 - s_j b_j)^{-a_j/b_j}. \]  

(25)

Notice also that if \( f_j \) are \( M \) differentiable functions, then the following expression holds:

\[ (\sum j \frac{d}{dw_j})^N \prod j f_j(w_j) |_{w=0} = (\frac{d}{dw})^N \prod j f_j(w) |_{w=0} \]  

(26)

so that one finally obtains:

\[ Z = (\frac{d}{ds})^N \prod j (1 - s_j b_j)^{-a_j/b_j} |_{s=0}. \]  

(27)

B The Invariant Distribution when \( b's \) are Homogeneous across Locations

Suppose that \( b_j = b \) all \( j \). As the probability that an entrant chooses location \( k \) given that a firm exits from location \( j \) is defined up to a proportionality constant - cf. eq. (1) - one can assume \( b = 1 \). Therefore, the proportionality constant in (13) boils down to \( H = A + N - 1 \) and transition probabilities in (2) read:

\[ P(n + \Delta_k - \Delta_j|n) = \begin{cases} \frac{n_j}{N} \frac{a_k + n_k}{A + N - 1} & k \neq j \\ \frac{n_j}{N} \frac{a_k + n_k - 1}{A + N - 1} & k = j \end{cases}. \]  

(28)

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Moreover:

$$
\vartheta(n_j; a_j) = a_j^{[n_j]} = \begin{cases} 
    a_j(a_j + 1) \cdots (a_j + n_j - 1) & 1 \leq n_j \leq N \\
    1 & n_j = 0
\end{cases}. \quad (29)
$$

where $a_j^{[n_j]}$ is the Pochammer’s symbol. From eq. (27), we observe that in this case the normalizing constant $Z(a)$ reduces to:

$$
Z(a) = \left( \frac{d}{ds} \right)^N \prod_{j=0}^{M} (1 - s)^{-a_j} |_{s=0}, \quad (30)
$$

which can be easily computed and reads $Z(a) = A^{[N]}$. Thus the exact formulation for the invariant distribution in the case of homogeneous $b$’s reads:

$$
\pi(n;a) = \frac{N!}{A^{[N]}} \prod_{j=1}^{M} a_j^{[n_j]} \frac{1}{n_j!}. \quad (31)
$$

C Recovering the Polya Approach

Consider the stationary distribution given in (5). Following Johnson et al. (1997, Chs. 40), it is easy to show that (5) can be also interpreted as the time-$t$ probability distribution of a Polya entry process in which firms make irreversible locational choices.

More precisely, suppose an industry (i.e. a urn) with a potentially infinite number of firms (i.e. balls) and $M$ locations (i.e. balls’ colors or types). At time $t = 0$ the state of the system is given by the occupancy vector $\mathbf{n}^0 = (n_1^0, ..., n_M^0)$, $\sum_{i=1}^{M} n_i^0 = N^0$, where $n_i^0$ is the number of balls of color $i$ in the urn. At any time period, a ball at random is extracted from the urn and put once again in the urn together with $c \geq 1$ other balls of the same color. A standard result is that the probability that at time $\tau > 1$ the occupancy vector is $\mathbf{n}^\tau = (n_1^\tau, ..., n_M^\tau) \gg 0$, with $\sum_{i=1}^{M} n_i^\tau = N = N^0 + c\tau$ reads:

$$
p(\mathbf{n}^\tau | \mathbf{n}^0, c) = \frac{\Gamma(N + 1)\Gamma(N^0)}{\Gamma(N^0 + c(N - 1) + 1)} \prod_{i=1}^{M} \frac{\Gamma(n_i^0 + c(n_i^\tau - 1) + 1)}{\Gamma(n_i^0)\Gamma(n_i^\tau + 1)}. \quad (32)
$$
If we are allowed to add only one ball at any time (i.e. \( c = 1 \)), then \( N^t = N^{t-1} + 1 \) and (32) becomes:

\[
p(n^t | n^0) = \frac{N!}{(N^0)!^N} \prod_{i=1}^{M} \frac{(n^0_j)^{n^t_j}}{n^t_j!},
\]

(33)

where \( a[x] = a(a+1) \cdots (a+x-1) \) is the Pochammer’s symbol. Therefore, if \( a_j = n^0_j \), the probability distribution (33) becomes the invariant distribution for the Ehrenfest-Brillouin model (5). This results allows us to directly compute (using standard results for the Polya process theory, cf. Johnson et al. (1997, Chs. 40)), the marginal probability that a site with intrinsic benefit \( a \) will contain in the long-run exactly \( k \) firms. Indeed, the latter marginal distribution is equal to the marginal probability that in an urn with 2 colors there will be \( k \) balls with the first color, when in the urn there are \( N \) balls and the initial number of balls of the first color were \( a \), cf. eq. (6).

Notice finally that, in contrast with the process of firms locational choice presented in Section 3, initial conditions (i.e. the number of firms initially present in each locations) matter in the finite-time probability distribution of the Polya process. As net industry growth rate is positive (entry rate is equal to \( c \) while exit rate is zero) locational decisions are irreversible. Therefore, dynamic increasing returns strongly affect long-run agglomeration patterns as early micro events may cumulatively reinforce the agglomeration benefit of any given location, possibly overcoming ex-ante intrinsic comparative advantages. The impact of additional perturbations caused by entry becomes progressively negligible in the long-run, thus leading to lock-in of the system. On the other hand, the Ehrenfest-Brillouin interpretation is consistent with always-reversible decisions: initial conditions never matter for the invariant distribution. As the size of any individual perturbation does not die out with time, the strength of dynamic increasing returns is much weaker than in the Polya interpretation.