Technology, Entrepreneurship and Inequality:
An interpretative model

Alfonso Gambardella*
and
Davide Ticchi°

* Sant’Anna School of Advanced Studies, Pisa, Italy
° University of Urbino, Italy and
Universitat Pompeu Fabra, Barcelona, Spain
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Alfonso Gambardella
University of Urbino, Urbino, Italy
agambardella@info-net.it

Davide Ticchi
University of Urbino, Urbino, Italy and
Universitat Pompeu Fabra, Barcelona, Spain
davide.ticchi@econ.upf.es

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Abstract

This paper purports to explain some recent trends in several advanced economies. Our model shows that the rise of new scientific and technological opportunities, and particularly the opportunities to develop riskier technological projects, is at the basis of the blossoming of small-medium sized high-tech companies in several regions of the world. This phenomenon, which has been widely documented, and has given rise to several remarks about the growth and employment opportunities of “Silicon Valley” models of industrial activities and employment, is contrasted with an economy based on more stable employment conditions in large firms. Our key result is that both an economy based on large firms and one based on high-tech smaller enterprises lead to higher expected incomes. But while the former implies lower inequality in the sense of lower variance of incomes, the latter implies both higher permanent and transitory inequality. This is consistent with some recent empirical findings about the increase in the variance of incomes in the US and the UK.
1. INTRODUCTION

Entrepreneurship has become a popular concept among business analysts and policy makers. (See for instance OECD, 1998; see also Reynolds, Hay and Camp, 1999.) Several factors account for this recognition. For example, in many high-tech industries, start-ups and small-medium sized companies more generally, have effectively brought new scientific discoveries and technologies into the market (e.g., Arora, Fosfuri and Gambardella, 1999.) Parallel to this, there is a recognition of the economic opportunities that have been created in areas where entrepreneurship, and particularly high-tech entrepreneurship, is especially diffused. For example, Saxenian’s (1994) work on Silicon Valley has emphasized the importance of socio-economic networks of individuals and enterprises, and the effects that this organization of industries and regional economies can have for growth and employment. (See also Porter, 1998.)

In fact, the issue is not limited to some special high-tech regions of the world. Apart from the growth of several of such regions in recent years, even in non G-7 countries (e.g. Ireland, Israel, software in India, electronics in Taiwan), this is not unrelated to some broader changes in the patterns of employment. Particularly, the most dynamic economies are gradually moving away from permanent employment conditions as the natural source of individual incomes. Increasingly, job mobility, risk-sharing employment contracts, new enterprises, or jobs whose rewards are more tightly linked to individual performance, have become the norm rather than the exception, especially for the younger generations and the more educated people.

The goal of this paper is to provide some analytical understanding of these issues. The paper develops a model of the choice of individuals between setting up their own
enterprise -- or choosing employment opportunities whose rewards are more directly linked to performance -- and more stable employment conditions, such as typically fixed wage long-term contracts in large firms. The model studies the effects of changes in the underlying parameters of the economy on the share of the two types of employment; the expected income of the economy, as a measure of its overall performance; and the variance of incomes, as a measure of “inequality”.

Our key result is that both factors that increase the productivity of the “permanent employment” sector of the economy (typically large firms offering fixed wage contracts), and factors that increase the opportunities of independent entrepreneurship, imply higher expected income. But while the former implies lower variance of incomes, the latter implies higher variance of incomes, and hence higher inequality. In our model this result arises from the assumption that there are differences in entrepreneurial or other abilities among individuals. However, these differences translate into ability to earn differential incomes only when the individuals become profit earners in independent enterprises (or when rewards are associated to individual performance), as opposed to earning a given salary determined by the labor market equilibrium.

These issues are related to another important stream of the economic literature. Notably, there is a long literature on wage and income inequality, and Gottschalk and Moffit (1994) and Blundell and Preston (1998 and 1999) have shown that income inequality has increased both in the United States and in the UK. While they both show that part of the increase in inequality is explained by permanent differences across individuals, they also show that a good fraction of it is transitory, and can be attributed to greater short-term uncertainty and instability of jobs, associated with greater mobility of individuals across jobs.

The paper is organized as follows. The next section further motivates our analysis by discussing available evidence and related work in the literature about the rising importance of entrepreneurship especially in high-tech activities. Section 3 develops a basic version of our model. Here we consider an economy composed only of a high-
tech sector, which is populated by large firms and small high-tech enterprises. Section 4 extends the basic model by introducing a “traditional” sector. We show that the main results of our model are not affected by this extension. Section 5 concludes the paper, and speculates on some “social” implications of our model.

2. HIGH-TECH ENTREPRENEURSHIP IN SILICON VALLEY AND ELSEWHERE

Saxenian’s (1994) book is one of the most careful and influential work about what can be labelled as the Silicon Valley “model” of the organization of industries and regional economies. Silicon Valley is a unique phenomenon in the world, and it may seem hard to take it as a representative model of industrial and regional organization of economic activities. But the uniqueness of Silicon Valley has to do mostly with the magnificence of its technological achievements than with its patterns of organizing production and employment. Not only are Silicon Valley’s spreading in other places of the world, even though possibly not with such level of technological performance, but its organizational model is frequently cited as a reference example for many other regions and economies. Moreover, the importance of regional infrastructures, networking, and the like, has been emphasized by other authors, and particularly by Porter (1998).

Saxenian argues that key to the Silicon Valley phenomenon has been the creation of a diffused socio-economic network, whereby a great deal of individual and enterprises interact systematically with one another. This is epitomized by her remark that people commonly think of being employed “by the Valley” rather than by the single enterprises. Apart from infrastructures of various sorts (e.g. venture capital), she stresses “cultural” factors like the propensity to take risk, the low social penalties for economic failures, the interpersonal contacts that make it easier to find and match complementary resources for innovative projects. This has translated into new patterns of industrial organization and employment. Particularly, “permanent” employment conditions are replaced by higher mobility across jobs, a high degree of experimentation in jobs and in innovative projects, an extended propensity towards entrepreneurship. Saxenian compares Silicon Valley with Route 128 in the Boston area, and argues that
the tighter and more limited social networks in the latter, along with greater social
distress for economic failures, lower propensity for risk and economic experimentation,
and the more prominent role played by hierarchically organized and vertically
integrated large firms, can explain a good deal of the differential performance of the two
economies.

At the same time, Silicon Valleys, or allegedly similar models, are flourishing all over
the world. Apart from several US regions, Cambridge and Oxford in the UK are the
most natural European analog. (See *The Economist*, 1999.) Moreover, Ireland and
Israel have shown similar patterns of growth and economic organizations since the past
decade or so, with associated economic and technological performance. Likewise, one
can cite the development of Indian software in the Bangalore region, or the growth of
electronics in the Hsinchu region in Taiwan. McGray (1999) provides a long list of
regions all over the world that are becoming part of he labels the “Silicon Archipelago”.
Even countries that are most typically associated to the permanent employment, large-
firm based model of organizing the economy, such as Germany and Japan, have
recently experienced the growth of small-medium sized high-tech entrepreneurship in
some of their regions. (See *New York Times*, 1999; and *Business Week*, 1999.)

Whether the new Silicon Valleys, or similar models, will all be successful is yet to be
seen. However, what is important is that they all claim to be moving towards models of
organizing production, employment and innovation that rely on systematic
experimentation; higher formation of start-ups and entrepreneurial jobs; a gradual shift
from stable long-term employment conditions towards patterns of employment based on
tighter links of rewards to individual performance; a high degree of job mobility,
failures and new trials. In addition, a recent study by Reynolds, Hay, and Camp (1999)
suggests that the phenomenon of entrepreneurship is not limited to high-tech. (See also
OECD, 1998.)

Apart from the stricter definition of entrepreneurship, i.e. fully independent self-
employment, we already noted that there is a more general trend towards risk contracts
or other institutional set-ups that create tighter associations between rewards and
performance in more traditional forms of employment as well. Whether because of the lower power by the unions, the recognition of the increased importance of individual incentives, or else, even the larger firms are encouraging what is sometimes called “intrapreneurship”. That is, many teams inside large firms or similar organizations increasingly resemble somewhat independent entrepreneurial jobs, both in terms of autonomy of actions and in their relationships to employment opportunities and rewards. To our knowledge, a precise quantitative assessment of this phenomenon has not been made. However, the frequency and extent with which these issues are discussed in trade and business magazines, as well as among policy makers, labor unionists, and other labor or industry analysts, suggests that earnings that reflect more closely the performance of the individuals are becoming a notable phenomenon, also relatively to traditional wage setting mechanisms based on labor market equilibria.

Another important remark is that in the Silicon Valley models of the world, the large firms also play a critical role. In Silicon Valley itself, companies like Fairchild, IBM or the Xerox Technology Park have been major sources of new technologies. (See for instance Kenney and Van Burg, 1999.) Often, they have not used these technologies themselves. They have been exploited by smaller firms, start-ups or even individual employees. Moreover, the large high-tech firms have trained several engineers and researchers, many of whom have later on set-up their companies. At the same time, the opportunity to work in large high-tech firms has encouraged many people to invest in their human capital, which has created a large pool of potential founders of new high-tech enterprises. The larger firms have also nurtured the smaller start-ups, either by linking them to sources of financial capital, or by directly supporting them financially or managerially, or by making them part of their network of suppliers, or by providing a natural “umbrella” for re-employment of individuals whose start-up projects failed. In addition, large high-tech firms often create technology standards around which several smaller firms have produced complementary technologies and components (Kenney and Van Burg, 1999; Langlois and Robertson, 1992).

In short, there is a great deal of complementarity between high-tech large and small firms in many of these areas. This is also apparent in several other examples of the
more recent Silicon Valley’s that have sprung up all over the world. The Irish development for instance owes a great deal to the local subsidiaries of multinational enterprises. They provide a stable and reliable source of high-quality demand for the smaller companies, they train people, and they are a source of technological spillovers. Similarly, the *New York Times* (1999) argues that Deutsche Telecom “is in the vanguard of an unlikely plan to transform Bonn’s culture from bureaucratic stodginess to Silicon Valley sprightliness.” The now privatized German Telecom giant is taking significant steps in the direction of supporting the formation of several high-tech start-ups in the computer, telecommunications and internet businesses in the Bonn’s area.

Finally, there is a natural association between the rise of entrepreneurship, or more generally of more entrepreneurial models of employment, and the extent of income growth and inequality. The new entrepreneurial models of employment tend to be associated with rewards that prize more handsomely individual performance. To the extent that individuals differ in their productive or economic ability, this can translate into a higher spread of individual earnings. As noted in the introduction, this phenomenon has already been documented, particularly for the two leading countries in the world which are more commonly associated with a more entrepreneurial attitude towards jobs and the mobility across them.

Thus, using a wide of sample of family-level income and consumption data, Gottschalk and Moffit (1994) and Blundell and Preston (1998 and 1999) show that a good deal of the increase in the variance of incomes in the US and the UK can be attributed to transitory rather then permanent factors. Particularly, Blundell and Preston argue that the lower increase in the variance of consumption in Britain, as compared to the much faster increase in the variance of incomes during the 1990s, is consistent with the view that the observed phenomenon is likely to stem from more frequent job mobility, differences in earnings, short-term uncertainties and volatility of the individual incomes, rather than permanent gaps among them. Their papers however do not provide any specific explanation of the factors that may account for the sharp changes in the dynamics of the variance of British incomes and consumption in this decade. The
model developed in the remainder of this paper attempts to provide one possible explanation of these trends.

3. THE BASIC MODEL
3.1 Structure of the model
We start with a basic model of an economy composed only of a high-tech sector. There are two employment opportunities in this economy. First, people can be employed in large firms. In this case, their salary is determined by the demand and supply equilibrium in the labor market. Alternatively, people can set up their own business. We assume for simplicity that these independent concerns hire only one individual, the entrepreneur. People choose to be high-tech entrepreneurs vis-à-vis being employed in a large firm when the expected profits from the independent business is higher than the salary that they would obtain by working for the large firms. Since we are dealing with technology- and research-based activities, we assume that production is stochastic. We also assume that the ability of people to set-up a new firm differs among individuals, and these differences are distributed stochastically across the population.

3.2 The large firms
We begin by modelling the demand for labor of the large firms. We assume that there are $M$ large high-tech firms in this economy. The demand for their output is exogeneous, and we label the demand faced by the prototypical firm with $Q$. Note that $Q$ can be thought of as the “size” of the firm. We also assume that $Q$ is distributed across the $M$ firms as $Q \sim S(Q \mid \sigma)$, with $Q \in [Q_A, Q_B]$, $Q_B > Q_A$, and $Q_B < \infty$. As we shall see in the next section, we normalize the size of the small firms to 1. To ensure that the large firms are larger than the small firms, we assume that $Q_A > 1$. The distribution function $S(\cdot)$ depends $\sigma$ which measures the degree of first order stochastic dominance, i.e. $S_\sigma \leq 0$. Higher $\sigma$ implies higher fraction of large firms that are bigger than a given size. This parameter then accounts for how large are the large firms in the economy. The exogeneity of $Q$ is an important assumption. Apart from implying that

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1 We use subscripts to denote first derivatives. Whether subscripts denote derivatives vis-à-vis being the an identifier of a variable will be apparent from the discussion.
our firms operate in a price-taking competitive environment, it denotes that ours is a partial equilibrium model. We can think of our firms as operating in an open economy. Exports and imports are given, and this accounts for the exogeneity of demand, and of firm size.

We model production by assuming that the total revenue of the prototypical large firm is $\mu Q$, with $\mu \sim F(\mu \mid H, x)$. The stochastic element $\mu$ is bounded, i.e. $\mu \in [\mu_A, \mu_B]$, with $\mu_A > 0$, and $\mu_B < \infty$. The distribution of $\mu$ depends on $H$ and $x$. The former is the number of “engineers” employed by the firm (where $H$ stands for “human capital”). $^2$

We assume that a higher number of engineers enables the firm to draw $\mu$ from a “better” distribution, and particularly from a distribution with higher mean. Again, we use the concept of first order stochastic dominance, and assume that $F_H \leq 0$.

The parameter $x$ measures the degree of second order stochastic dominance of the distribution. It is known since the work by Rothschild and Stiglitz (1970), that second order stochastic dominance accounts for the so-called mean-preserving spread of the distribution. This is a measure of the risk associated with the distribution. Higher $x$ means that firms draw $\mu$ from distributions having the same expected value, but higher probability mass at the tails. Formally, second order stochastic dominance is equivalent to the following two conditions (Rothschild and Stiglitz, 1970)

\[
\begin{align*}
(1a) & \quad \int_{\mu_A}^{\mu_B} \mu \ dF(\mu \mid x') = \int_{\mu_A}^{\mu_B} \mu \ dF(\mu \mid x) \quad \forall x' \geq x \\
(1b) & \quad \int_{\mu_A}^{\mu_B} F(\mu \mid x') \ d\mu \geq \int_{\mu_A}^{\mu_B} F(\mu \mid x) \ d\mu \quad \forall x' \geq x \text{ and } \mu_A \leq \tau \leq \mu_B
\end{align*}
\]

After integrating by parts, (1a) also implies that

\[
\int_{\mu_A}^{\mu_B} F(\mu \mid x') \ d\mu = \int_{\mu_A}^{\mu_B} F(\mu \mid x) \ d\mu .
\]

$^2$ In this paper we label the employees of the high-tech sector, whether employed in the large or small firms, as “engineers”. One alternative would be to call them “researchers”. However, the bulk of industrial technological activities in an economy is made by engineers, while the term researchers would give greater emphasis to the role of scientists, which typically play a less prominent role than engineers in industry.
Since $x$ plays a key role in our analysis, it is important to clarify its interpretation. Distributions with higher $x$ can be seen as technological projects that depend on more basic ideas. Typically, basic ideas, such as scientific discoveries, are more risky in the sense that while they can lead to considerable successes, they are also more likely to fail, at least from an economic point of view. We assume that our economy is endowed with a set of “ideas” $(x_1, x_2, \ldots, x_n)$, where the $x$’s are ranked in ascending order. The set of $x$’s is given exogenously, and it stems from the available body of scientific and technological knowledge in the economy (from universities, or elsewhere). The firms choose one of the available $x$’s (up to $x_n$), and by doing so they select the distribution from which they draw $\mu$. Thus, economies wherein firms can select distributions with higher $x$ have the opportunity to undertake more risky technological projects.

To derive the expected profits of the large firms, we need to define their costs. We assume that in the high-tech sectors there are two main sources of costs. First, the large firms need to employ $H$ engineers. If their salary on the labor market is $r$, the total cost for the R&D employees is $rH$. We then assume that these are employed through long-term contracts before the R&D outcomes are observed, i.e. before $\mu$ is drawn. Thus, the engineers employed by the large firms are paid their salaries even if the specific project to which they work fails. This is a natural assumption. In large firms, R&D engineers are not normally hired and fired, and they are typically offered a given salary under relatively long-term employment contracts. (See for instance Chandler, 1990.)

The second source of costs is R&D or manufacturing capital costs, like R&D facilities and equipment, or manufacturing assets. We assume that these costs are divided in two parts. First, the large firms pay a fixed cost $K$ upfront before $\mu$ is realized. Second, they pay a marginal cost $z$ per unit of output produced after $\mu$ is realized, i.e. a total of $zQ$, and they pay it only if they choose to implement the project once $\mu$ is observed. This can be thought of as the use of R&D machinery or capital to further develop the innovation, or of manufacturing asset to produce it.\(^3\) Most importantly, we assume that

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\(^3\) Clearly, the engineers may also provide further work on the innovation, and their costs are already sunk in $rH$. Note also that we assume that no direct labor is required to develop or produce the innovation. We
the marginal cost \( z \) depends negatively on \( K \), i.e. \( z(K) \) with \( z_K < 0 \). This basically says that the firms can choose how much investments to make before and after \( \mu \) is realized. This is another natural assumption. For example, information technology companies can re-use more software codes from previous projects if they have established capabilities in software, typically because they can re-use codes from previous projects. Similarly, large pharmaceutical or chemical companies can enjoy manufacturing scale economies from existing facilities compared to batch processes by smaller biotech companies; or a company like Merck is said to have an established department for dealing with FDA reports which streamlines the procedures that the company has to undertake for any new drug approval (e.g. Gambardella, 1995).

The firm will then develop the project only if the realized \( \mu \) is greater than \( z(K) \). If not, it can limit its losses by not developing the innovation. The gross profits of the firm will then be equal to \((\mu - z(K))Q\) if \( \mu \geq z(K) \), and 0 otherwise. The expected gross profits will be

\[
\int_{z(K)}^{\mu} \mu - z(K) \, dF(\cdot) \cdot Q
\]

After integrating by parts, the expected net profits become

\[
E \Pi_L = \left[ \mu - z(K) \right] - \int_{z(K)}^{\mu} F(\mu \mid H, x) \, d\mu \left[ Q - rH - K \right]
\]

The large firms maximize (2) with respect to \( H \) and \( K \) to obtain the optimal demand for engineers and the optimal level of sunk costs. The first order conditions are

\[
\begin{align*}
(3a) & - \int_{z}^{\mu} F(\mu) \, d\mu \cdot Q - r = 0 \\
(3b) & - z_K [1 - F(z \mid H, x)] Q - 1 = 0
\end{align*}
\]

could introduce direct labor costs, but at the penalty of a technically more complicated model with no real additional insights in its main results. The assumption that in high-tech production the share of direct labor costs is negligible, may not be completely unrealistic if one notes that it typically depends on highly automated manufacturing systems, and the largest share of the value added is generated by the R&D costs.
Assuming that the second order conditions are satisfied, it is not difficult to see that the optimal demands for $H$ and $K$, i.e. $H'(r, Q)$ and $K'(r, Q)$, are non-decreasing in $Q$ and non-increasing in $r$.\(^4\) Clearly, this implies that the optimal $z$ is non-increasing in $Q$ and non-decreasing in $r$.\(^5\)

### 3.3 The small firms

The production of innovations by the small firms is analogous to the large firms. However, we make three assumptions that distinguish them from the latter. First, we assume that the (exogenous) size of the small firms $Q$ is normalized to 1. Second, we assume that they employ only one engineer, notably the entrepreneur, i.e. $H = 1$. Third, we assume that because of the greater reliance of these firms on “individual” ability, i.e. that of the entrepreneur, the expected net profits of these firms also depend on a stochastic element $\varepsilon \geq 0$, which is distributed across the population as $\varepsilon \sim G(\varepsilon)$. Thus, the total revenue of the small firms is $\mu$ distributed as $F(\mu | 1, x)$, with $\mu \in [\mu_A, \mu_B]$, $\mu_A > 0$, and $\mu_B < \infty$. The total costs of the small firms are $z(K)$, which is paid \textit{ex-post}, and $K$, which is incurred \textit{ex-ante}.

The gross profits of the small firms are $\mu - z(K)$ if $\mu \geq z(K)$, and 0 otherwise, which implies that the expected gross profits are $\int_{z(K)}^{\mu_B} \mu - z(K) \, dF(\cdot)$. After integrating by parts, the expected net profits are

$$
(4) \quad E \Pi_S = \left[ \mu_B - z(K) - \int_{z(K)}^{\mu_B} F(\mu | 1, x) \, d\mu \right] - K - \varepsilon
$$

where we assume that the stochastic element $\varepsilon$ enters as an additional component of the

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\(^4\) To see this note that the cross-partial derivatives with respect to any two of $H$, $K$, and $Q$ are non-negative, and that the derivatives of (3a) and (3b) with respect to $r$ are non-positive. Using a well known result about complementarities (e.g. Milgrom and Roberts, 1990), one can sign the optimal demands as above.

\(^5\) For the second order conditions to be satisfied, a necessary condition is that $z_K K > 0$, i.e. increases in $K$ reduce $z$ at a decreasing rate.
ex-ante costs sustained by the entrepreneur.⁶ Since \( H \) is now given, the problem of the small firms boils down to the choice of \( K \). The first order condition of this problem is analogous to (3b) with \( Q = H = 1 \). Since, we showed that the optimal \( K \) increases with \( Q \), it is straightforward that the optimal \( K \) for the small firms is smaller than that of the large firms.⁷ In turn, this implies that the optimal \( z \) of the large firms is smaller than that of the small firms.⁸

3.4 The choice of \( x \) by small and large firms and the share of high-tech employment in small firms

We can now obtain a key proposition of our model about the choice of the degree of risk, \( x \), of the technological projects launched by the large and the small firms. This proposition depends on a critical assumption. Notably, we assume that the large firms are sufficiently large so that, not only is their optimal \( K \) always larger than that of the small firms, but their implied marginal cost \( z \) is always smaller than the lower bound \( \mu_A \) of the value of the innovative projects. By contrast, the optimal \( z \) for the smaller firms is larger than the lower bound. This assumption leads to the following Proposition.

**Proposition 1.** The expected profits of the small firms does not decrease with \( x \), while the expected profits of the large firms are not affected by \( x \). Thus, unlike the large firms, the small firms always choose the riskiest available technological project.

**Proof.** To show that \( \frac{\partial E \Pi_S}{\partial x} \geq 0 \) note from (4) that \( \frac{\partial E \Pi_S}{\partial x} = -\int_{z}^m F_z \, d\mu = \)

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⁶ It can be thought of for instance as differential ability of people in setting up the firm or in performing the ex-ante research.

⁷ Because of the complementarity between \( H \) and \( K \) (see footnote 4), this is further reinforced by the natural assumption that the optimal \( H \) for the larger firms is greater than 1.

⁸ To be sure, the result that the optimal \( K \) (or \( z \)) of the large firms is larger (smaller) than that of the small firms is for any given \( x \). But, as we shall below, large and small firms may choose different \( x \)'s, and this may affect their optimal \( K \)'s in ambiguous directions. It is not difficult to see, however, that whatever the choice of \( x \) by the small and large firms, the optimal \( K \) of the latter is larger if their size \( Q \) is sufficiently large. We then assume that the lower bound on the size of the large firms \( Q_A \) is always high enough to make the optimal \( K \) of any large firm larger than that of the small firms (whatever the \( x \) chosen by the two types of companies). In short, this amounts to defining accordingly the size of the large firms in our economy. Moreover, it appears natural to assume that the large firms have larger sunk assets than a newly founded start-up or small firm.
By (1a) the first term of this expression is equal to zero. Since the optimal z of the small firms is greater than \( \mu_A \), then by (1b) the second term is non-negative. Hence, \( \frac{\partial E \Pi_s}{\partial x} \geq 0 \). The same reasoning applies to show that \( \frac{\partial E \Pi_L}{\partial x} = 0 \).

Here, however, the optimal z is smaller than the lower bound \( \mu_A \) which gives the result. 

**QED**

As noted above, this Proposition depends crucially on an assumption whose intuition can be summarized as follows. The large firms make *ex-ante* investments which implies that they face a fairly small marginal costs of new projects after research has produced some initial outcomes. Because the *ex-ante* investments are sunk, and their costs are incurred in any case, they will only look at the *ex-post* marginal costs when deciding about whether to complete the project. But these marginal costs are low enough (because of their greater productivity from the *ex-ante* investments) that all the projects will be undertaken. Another way to see this is that these are “small” projects for the large firms, and hence they would induce only a minor increment in costs with respect to existing activities. By contrast, the small firms, with no such a high productivity or experience from previous investments, face serious additional costs for implementing the project. The drawback is a higher rate of failures (i.e. zero gross profits, and negative net profits). However, this also enables them to set a lower bound to the losses that they incur. Thus, a probability distribution with the same mean but higher probability mass at the tails increases their expected profits because the increase in probability at the left tail is bounded by the option of not completing the project, while the firm can fully take advantage of the higher probability mass at the right tail.\(^9\)

This means that when faced with a given set of “ideas” \( (x_1, x_2, \ldots x_n) \), the small firms will always choose the distribution with the highest degree of risk. By contrast, the large firms will choose \( x \) exogenously. They will select larger or smaller \( x \)'s from the available pool in the economy according to non-economic factors, like the propensity of the engineers or the managers to select more or less risky technological projects, or the

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\(^9\) See Arora and Gambardella (1994) for a model with a very similar flavor.
"culture" and tradition of the firm, or of its R&D departments.\textsuperscript{10}

We can now determine the share of individuals in the economy who choose to set up their own firms rather than being employed by the large firms. Define expression (4) above for the expected profits of the small firms as \( p(e, x) \). Individuals will choose to set-up their own firms as long as \( p(e, x) \geq r \). Since \( e \) is stochastically distributed across individuals this amounts to defining a threshold level for \( e \), i.e. \( \tilde{e} \), such that all individuals with \( e \) smaller than the threshold will set up their own firms. Using expression (4) for \( p(e, x) \), from \( p \geq r \) the expression for the threshold is

\[
(5) \quad e \leq \tilde{e}(r, x) \equiv \mu_B - z - \int_{z}^{\mu_B} F \, d\mu - K - r
\]

The share of engineers employed by the large firms is then \( 1 - G(\tilde{e}) \), while \( G(\tilde{e}) \) is the share of high-tech entrepreneurs. From (5) it is easy to see that \( \tilde{e}_r = -1 \leq 0 \) and \( \tilde{e}_x = -\int_{z}^{\mu_B} F \, d\mu \equiv \phi \geq 0 \), where \( \phi \geq 0 \) follows from Proposition 1. Thus, the share of high-tech entrepreneurs increases with the availability of basic "ideas" in the economy (riskier distributions), and declines with the market salary of the engineers.

### 3.5 Labor market equilibrium

To obtain the labor market equilibrium level of \( r \), define \( H_S \) to be the supply of engineers in the economy. The equilibrium \( r \) is determined by the equilibrium between demand and supply of engineers. The demand for engineers is the sum of the demands for engineers by all the large firms in the economy. Since there are \( M \) large firms, whose size \( Q \) is distributed according to \( S(Q \mid \sigma) \), the total demand for engineers is

\textsuperscript{10} While noting that this result stems critically from our assumption above, the idea that large and small firms have different propensity to undertake risky technological projects is well grounded in the literature. See for instance Arrow (1983) or Holmström (1989) who provides an explanation based on agency costs and reputation of the large firms on the capital markets. In fact, what is critical for our analysis is that the smaller firms enjoy greater expected profits from riskier technological projects than the large firms. Any theory producing this result would be consistent with our propositions in the following sections.
\[ M \int_{\tilde{Q}} H^*(Q, r) dS(Q \mid \sigma) \mid \sigma. \] The supply of engineers is the number of engineers who do not set-up their own firms, i.e. \( H_s (1 - \tilde{G}) \), where from now on we use the notation \( \tilde{G} \) to denote \( G(\cdot) \) evaluated at the threshold level \( \tilde{e} \). The labor market equilibrium is

\[ (6) \quad M \int_{\tilde{Q}} H^*(Q, r) dS(Q \mid \sigma) - H_s (1 - \tilde{G}) = 0 \]

Comparative statics on (6) will determine the signs of the changes in the equilibrium \( r \) when the exogenous parameters of the model vary. Define \( g(\cdot) \) to be the density function of \( G(\cdot) \), and \( \tilde{g} \) the density evaluated at \( \tilde{e} \). Taking the differentials of (6) with respect to \( r, \sigma, x, \) and \( H_s \), one obtains

\[ (7) \quad \Psi dr - M \int_{\tilde{Q}} H^*_r S_{\sigma} dQ d\sigma + H_s \tilde{g} \phi dx - (1 - \tilde{G}) dH_s = 0 \]

where \( \Psi \equiv M \int_{\tilde{Q}} H^*_r dS - H_s \tilde{g} \) is the derivative of the left hand side of (6) with respect to \( r \), and the negative sign in front of the second term of this expression is obtained after replacing \( \tilde{e}_r = -1 \). Given \( H^*_r < 0 \), then \( \Psi < 0 \). The second term of (7) is obtained after integrating the first term of (6) by parts, and taking the derivative with respect to \( \sigma \).

Since \( H^*_0 > 0 \) and \( S_{\sigma} \leq 0 \), the second term of (7) is negative. Finally, in the third term of (7) we replaced \( \tilde{e}_r \) with its expression \( \phi \geq 0 \). These results imply that the market equilibrium salary \( r(\sigma, x, H_s) \) increases with \( \sigma \) and \( x \), and decreases with \( H_s \), i.e. \( r_{\sigma}, r_x > 0 \), and \( r_{H_s} < 0 \).

3.6 Average income and employment structure of the economy
As an aggregate measure of the performance of our economy, we take the average income of the individuals. We study how it changes with changes in the three main parameters of the model, \( \sigma, x, \) and \( H_s \). The average income of the economy is
\[
\bar{y} \equiv E_y = \int_0^\varepsilon p \, dG + r(1 - \bar{G})
\]

**Proposition 2.** The expected income of the economy increases with \(\sigma\) and \(x\), and it decreases with \(H_S\).

**Proof.** To prove that \(\bar{y}_\sigma > 0\) take the derivative of (8) with respect to \(\sigma\). Using \(\bar{\varepsilon} = -1\), one obtains \(\bar{y}_\sigma = -\tilde{p} \cdot \tilde{g} \cdot r_e + r \cdot \tilde{g} \cdot r_a + r_a (1 - \bar{G})\), where \(\tilde{p}\) is (4) evaluated at \(\bar{\varepsilon}\). Since \(\tilde{p} = r\), \(\bar{y}_\sigma = r_a (1 - \bar{G}) > 0\). Similarly, using \(\tilde{p} = r\), and the fact that \(\phi\) is independent of \(\varepsilon\), \(\bar{y}_x = \phi \bar{G} + r_a (1 - \bar{G}) > 0\). Finally, using the fact that neither \(\varepsilon\) nor \(p\) depend directly on \(H_S\), \(\bar{y}_{H_S} = r_{H_S} (1 - \bar{G}) < 0\). **QED**

The employment structure of the economy as \(\sigma\), \(x\), and \(H_S\) change is summarized by the following Proposition.

**Proposition 3.** As \(\sigma\) increases the size of the employment in large firms increases, and the size of the entrepreneurial sector decreases. The opposite is true as \(x\) or \(H_S\) increase.

**Proof.** See the Appendix.

Propositions 2 and 3 combined underlie some of the key results of our analysis. First, Proposition 3 says that a high-\(\sigma\) economy is associated with a more prominent role of the large firm sector, and therefore with more extensive permanent employment conditions. By contrast, a high-\(x\) economy, which features widespread technological and scientific ideas, is based on diffused entrepreneurship and greater short-term uncertainty in employment conditions. It is not difficult to see that these features characterize some alternative types of existing economies. California, and Silicon Valley in particular, exhibit some key features of high-\(x\) economies, while some of the
leading European countries (e.g. Germany) and Japan are more typically high-$\sigma$ economies.

Proposition 2 then says that both types of economies can be successful in generating higher expected income. This suggests that there is no inherent superiority of one of the two types of industrial or employment structures. Economies based on larger firms and more stable employment conditions can be highly productive, and so can be economies based on more extended entrepreneurial jobs and greater focus on riskier technological ideas.

The results about a larger supply of labor are also interesting. First, Proposition 3 suggests that as $H_S$ increases, entrepreneurship also increases. The intuition of this result is that the effect of an increase in labor supply is to reduce the market salary. This encourages more people to seek alternative opportunities via more entrepreneurial jobs, whereby their incomes depend to a greater extent on their individual abilities. This is suggestive for example of one seemingly surprising result of the study by Reynolds, Hay and Camp (1999) mentioned earlier. They find that Italy is one of the most entrepreneurial countries in Europe, with a higher percentage of the population that has undertaken independent jobs than the UK, and quite higher than countries like Germany or France. They also find that the degree of entrepreneurship in Italy is especially high for younger people. The crowded and highly regulated Italian labor market prevents the younger generations to find stable jobs at reasonably high salaries. Their response is to look for more entrepreneurial and independent opportunities.

Proposition 2 however suggests that more crowded labor markets imply lower expected income. This is natural, as in our model the size and productivity of the large firm sector is given. Thus, increases in labor supply, with no effects on the output side of the industries, lead to lower productivity, and hence to lower expected incomes.

3.7 Variance of incomes and “inequality”
We finally look at how the variance of incomes in our economy changes with changes in $\sigma$, $x$, and $H_S$. The variance of incomes is
The following Proposition summarizes our results.

**Proposition 4.** The variance of incomes is not increasing in $\sigma$. The opposite is true for $x$ and $H_S$. A high-$\sigma$ economy is then more “equal”, while a high-$x$ economy is more “unequal”. An economy with a higher supply of labor is also more unequal.

**Proof.** See the Appendix.

Proposition 4 is another key result of our analysis. It shows that high-$\sigma$ economies, like some continental European economies or Japan, which rely on large high-tech firms offering long-term employment conditions, induce lower inequality in the sense of lower variance of incomes. This is because high-$\sigma$ induces a higher demand for engineers by the large firms, which translates into higher salary in equilibrium. This encourages more people to accept jobs in the large firms rather than undertaking their own entrepreneurial initiatives. We then showed that the combined effect of higher market salary for a larger share of people in the economy and the lower share of the population with variable incomes, implies that the overall variance of incomes is smaller. An economy based on large firms is then more egalitarian.

By contrast, an economy with significant technological opportunities, i.e. high-$x$ economy, induces a greater degree of entrepreneurship. A larger fraction of the population earns variable incomes, at least in the short-run. We noted earlier that when $x$ increases the market salary of the individuals working in the large firms also increase. However, the overall effect is to increase the variance of incomes. Thus, Silicon Valley type economies are likely to be more unequal. This suggests that the more economies are based on the utilization of potentially new and riskier ideas coming from advances in basic science and technologies, the more one is likely to observe greater inequality in incomes. Our model says that one mechanism by which this effect may arise is that the

\[
V(y) = \int_0^\xi p^2 dG + r^2 (1 - \tilde{G}) - \bar{y}^2
\]
new opportunities are more effectively exploited by individuals who run their own independent business, or that are employed under conditions whereby their incomes are more tightly linked to their performance.

Moreover, if we combine these results with those obtained in the previous Propositions, we find that both high-σ and high-χ economies produce a higher expected income, and in both economies the market salary of the individuals employed under permanent employment conditions is higher. However, the former is associated with less entrepreneurship; reduced short-term uncertainty in incomes; more extensive permanent employment conditions; and a lower variance of income. The opposite is true for high-χ economies.

This also provides one possible explanation of the empirical results by Gottschalk and Moffitt (1994) and by Blundell and Preston (1998 and 1999). The rise of several new scientific and technological opportunities, and more generally of new entrepreneurial opportunities, along with new employment arrangements based on flexible salaries linked to performance, can explain the greater share of individuals that rely on earnings that are more unstable in the short-run. While this may well imply greater expected income, it also comes with greater variance of incomes.

Finally, Proposition 4 suggests that, coeteris paribus, economies with a larger available supply of labor are also more unequal. This is because crowded labor markets induce a decline in the market salary of the permanent employment sector, and at the same time it encourages more people to seek more entrepreneurial jobs. If we combine this result with the one in Proposition 2, economies with a larger supply of labor can be both poorer and more unequal.

4. THE EXTENDED MODEL: HIGH-TECH VS. “TRADITIONAL” SECTORS

4.1 Set-up of the model
We extend the model developed in the previous section by assuming that, apart from a high-tech sector, our economy features a “traditional” sector. Apart from making the
model more articulated and realistic, the goal of this extension is twofold. First, we endogenize the number of individuals that work in the high-tech sector, and in so doing we endogenize the size of the high-tech sector itself. Second, we want to check whether the results of the simpler model are robust to this extension.

In the extended model, the structure (and notation) of the high-tech sector is totally analogous to the previous model, with people choosing whether to be employed in large firms or found their own firms. But now people have an additional alternative. They can choose to be employed in the traditional sector. We assume that, like for the large high-tech firms, people employed in the traditional sector face long-term employment conditions, and they are paid a wage determined by the labor market equilibrium in this sector. To keep a distinctive terminology, we label the individuals employed in the high-tech sector (whether large or small firms) as engineers, and those employed in the traditional sector as workers. The engineers employed by the large high-tech firms are paid a salary, while the workers are paid a wage.

To distinguish between the choice of working in the high-tech vs. low-tech sector, we assume that the individuals who choose to become engineers face an investment cost which nets their income. This can be thought of as investment costs that they have to sustain in order to maintain their human capital. This cost is faced by both the engineers who work for the large firms and by the high-tech entrepreneurs. People who work in the traditional sector do not bear this cost. We assume that the investment cost to maintain one’s human capital is distributed stochastically across the individuals in the population. Moreover, we assume that when the decision to become engineer is faced, people do not know their individual $\epsilon$ but only the distribution $G(\epsilon)$, while they know their human capital investment cost. This appears to be a natural assumption as people choose their education quite a few years before they enter the workforce. To keep the model technically simple, we make the additional assumption that the individual $\epsilon$’s and the stochastic human capital investment costs are independent.

These assumptions imply that people will choose to become engineers as long as their expected income from being employed in the high-tech sector (i.e. the weighted average
between the large firm salary and the individual profits obtained by setting up an independent firm), net of the individual human capital, is higher than the market wage in the traditional sector. The marginal engineer is the one whose expected income net of the human capital cost is equal to the latter wage. Once in the high-tech sector, $e$ is revealed to each individual, and people will choose whether to work for the larger firm or set up their own company according to $p(e, x) \geq r$.

Another assumption that is embedded in our set-up is that once they become workers or engineers people cannot move back and forth between these two categories of employment. This may imply for instance that people will work in large high-tech firms even if the wage in the traditional market is higher than the salary paid by the large high-tech firms. There are some justifications to this assumption. Typically, it is rare that people who invested in education to become engineers seek employment as production workers, even if the wages in the latter sector were higher than the engineers’ income.

But another way to justify this assumption is that, for those who chose to become engineers, there is mobility between working in high-tech large firms or setting up one’s own company. Although our model is totally static, one can think of a situation in which small high-tech companies last for only one period, and each engineer faces a new $e$ in every period. Over time, they can then switch back and forth between large firms and high-tech companies, and their expected income is equivalent to their permanent income over time. In this case, they will not have an incentive to return to the low tech sector because the engineers’ expected income net of the human capital costs is higher than the wage in the traditional sector. As we shall see, we will also use this assumption to distinguish between the variance of permanent and transitory incomes in the economy.

4.2 Notation and structure of the extended model
While the problem of the large and small high-tech firms is analogous to the simpler model, we define the profits of the traditional sector to be $\frac{1}{\alpha} \cdot T \cdot L^a - wL$ where $T$ is
the (exogenous) size of the traditional sector, $L$ is the quantity of labor, $w$ is the wage of the traditional sector, and $\alpha \in (0, 1)$. It is easy to see that the first order conditions of this problem imply $w = T \cdot L^{\alpha-1}$. The human capital investment cost of each individual is $c$, and we assume that $c$ is distributed across the population of individuals as $c \sim E(c)$, with $c \geq 0$. As noted earlier, we also assume that $c$ and $\varepsilon$ are independent. The size of the population is $N = H_S + L_S$, where $H_S$ is the number of engineers in the economy and $L_S$ is the number of workers.

The number of engineers $H_S$ is determined by the condition $\bar{y} - c \geq w$, where $\bar{y}$ is the expected income from working in the high-tech sector defined by (8) above. Since $c$ is distributed stochastically across individuals, the “marginal” engineer is the one with $c = \bar{c}$ where $\bar{c} : \bar{y} - \bar{c} = w$. The threshold $\bar{c}$ determines the number of engineers in the economy, notably $H_S = N \cdot \bar{E}$, where analogously to the earlier notation for $G$, we use $\bar{E}$ to denote $E$ evaluated at $\bar{c}$, i.e. $\bar{E} \equiv E(\bar{c})$. This also implies that the workers are the individuals with $c > \bar{c}$, and therefore $L_S = N(1 - \bar{E})$.

The equilibrium of the extended model is obtained by solving three equations for the three equilibrium variables of the model. The three equations are the labor market equilibrium equation in the traditional sector, the labor market equilibrium equation in the large firm high-tech sector, and $\bar{y} - \bar{c} = w$. The three equilibrium variables are the wage of the traditional sector $w$, the salary of the large high-tech firms $r$, and the threshold $\bar{c}$ which determines the number of engineers and the number of workers in the economy, i.e. $N \cdot \bar{E}$ and $N(1 - \bar{E})$. Given the independence between $c$ and $\varepsilon$, $\bar{c}$ also enables us to determine the size of the employment in the large high-tech firms and the number of high-tech entrepreneurs, notably $N \cdot \bar{E} (1 - \bar{G})$ and $N \cdot \bar{E} \cdot \bar{G}$.

We study how the three endogenous variables change with changes in the parameters of the model, and particularly the size and productivity of the traditional sector $T$, the size of the population $N$, the size of the large firms $\sigma$, and the technological opportunities of the economy $x$. Moreover, using the computed effects of the exogenous variables on the
equilibrium variables of the model, we determine the sign of the effects of the exogenous variables on three measures of the performance of the extended economy -- the expected income of the economy and two types of variances.

The first type of variance is what we label the “variance of consumption”, or the “variance of permanent income”. To understand the nature of this variance note that, as suggested earlier, the individuals working in the high-tech sector can in the long-run switch back and forth from working in large firms or setting up their own company. As a result, their long-run net income is $\bar{y} - c$. This can be thought of as a measure of their “permanent income”. Alternatively, if people smooth out their consumptions over time, this is a measure of their consumption. Clearly, workers in the low-tech sector earn $w$ both in the short- and in the long-run, and this is measures their consumption.

The second type of variance takes into account that in each period $N \tilde{E} (1-\tilde{G})$, people working in the high-tech sector earn $r - c$, while $N \tilde{E} \tilde{G}$ of them earn $p(\varepsilon) - c$. This can be thought of as the variance of “short-term” or “transitory” income of the economy. By looking at the effects of the exogenous variables of the model on these two types of variances, we can make statements about changes in permanent and transitory inequality.

### 4.3 Equilibrium

To solve for the equilibrium of the model, we have to specify our three equilibrium equations. The labor market equilibrium equation for the traditional sector is given by the first order condition of the low-tech sector optimization problem, i.e. $w = T \cdot L^{\alpha-1}$, where we replace $L$ with the expression for the labor supply, notably $N(1 - \tilde{E})$. The labor market equilibrium for the large high-tech firms is expression (6) in the simpler model, after replacing $H_S$ with $N \tilde{E}$. Finally, the third equation is $\bar{y} - \tilde{c} = w$. The equilibrium system of equations is then
To examine the effects of the changes in the exogenous variables $T, N, \sigma,$ and $x$ on the endogenous variables $w, r,$ and $\tilde{c},$ we differentiate this system with respect to the endogenous and exogenous variables. The system of differentials is

\[
\begin{align*}
\Psi dT &- N(1 - \tilde{G}) \tilde{d}c = M^* d\sigma + \tilde{E}(1 - \tilde{G}) dN - N \tilde{g} \tilde{E} \phi dx \\
\Psi dw & - \tilde{G} d\tilde{c} = \phi \tilde{G} dx
\end{align*}
\]

where \( \Psi \equiv M \int_{Q} H^* dS - N \tilde{g} \tilde{E} < 0; \) \( M^* \equiv M \int_{Q} H^* S_{\sigma} dQ < 0; \)

\[
W \equiv T \cdot N^{\alpha-1} \cdot (1 - \tilde{E})^{\alpha-1} > 0; \quad W^* \equiv (1 - \alpha) \cdot \frac{W}{1 - \tilde{E}} > 0; \quad \tilde{c} \text{ is the density of } E(c) \text{ evaluated at } \tilde{c}; \text{ and } \tilde{g} \text{ and } \phi \text{ have been defined in the previous section. After some algebra, the determinant of (11) is } D \equiv \Psi \cdot (1 + W^*) - N \cdot \tilde{c} \cdot (1 - \tilde{G})^2 < 0. \text{ The system can be solved for the derivatives of } w, r, \text{ and } \tilde{c} \text{ with respect to } T, N, \sigma, \text{ and } x \text{ by the Cramer’s rule.}
\]

4.4 Expected income, variances of permanent and transitory incomes

The expected income of this economy is the weighted average of the incomes of the individuals employed in the high-tech sector and of the individuals employed in the traditional sector. The weights are the shares of people employed in the two sectors. Furthermore, the average income of the engineers in the high-tech sector is the weighted average between those employed in the large firms and those that set-up their company. The weights in this case are the shares of the two types of employment. Clearly, the expected income of the high-tech engineers, net of the human capital cost $c$ is $\bar{y} - c$.

The expected income of the economy $\bar{Y}$ is then
where we used the fact that \( \bar{y} \) does not depend on \( c \) to take it out of the integral sign; we integrated \( -\int_{0}^{\tilde{c}} c \, dc \) by parts; and we used the fact that \( \bar{y} - \tilde{c} = w \).

To obtain what we called the variance of permanent income or the variance of consumption, we take the income of the high-tech engineers to be their long-run (net) expected income \( \bar{y} - c \). The variance of consumption \( V^C \) is then

\[
V^C = \int_{0}^{\tilde{c}} (\bar{y} - c)^2 \, dc + w^2 (1 - \tilde{E}) - \bar{Y}^2
\]

The variance of transitory income takes into account the differences in the short-term incomes of the individuals employed in the high-tech sector. Thus, \( V^Y \) is

\[
V^Y = \int_{0}^{\tilde{c}} \int_{0}^{\tilde{c}} (p - c)^2 \, dG + (r - c)^2 (1 - \tilde{G}) \, dE + w^2 (1 - \tilde{E}) - \bar{Y}^2
\]

From (13), the last two terms of this expression are equal to \( V^C - \int_{0}^{\tilde{c}} (\bar{y} - c)^2 \, dc \). Hence, \( V^Y \) becomes

\[
(14) \quad V^Y = V^C + \int_{0}^{\tilde{c}} \int_{0}^{\tilde{c}} (p - c)^2 \, dG + (r - c)^2 (1 - \tilde{G}) - (\bar{y} - c)^2 \, dE
\]

Note that the second term of this expression is the average variance of income of the high-tech engineers. Since any variance is a positive number, this confirms the well known theoretical and empirical result that, with consumption smoothing over time, the
variance of income is higher than the variance of consumption. (See for instance Cutler and Katz, 1992.) Our model is then consistent with the permanent income hypothesis.

4.5 Changes in $T$, $N$, $\sigma$, and $x$

- **Changes in the size of the traditional sector $T$**

To examine the structure of an economy with a large traditional sector, we solve system (11) for changes in $T$, i.e. $dT$, other things being equal. We obtain the following Propositions.

**Proposition 5a.** Increases in the size of the traditional sector imply that the wage of the traditional sector increases, the salary of the engineers increases, and the size of the employment in the high-tech sector decreases.

**Proof.** By applying the Cramer’s rule to (11) one obtains the following derivatives

\[ w_T = D^{-1} \frac{W}{T} (\Psi - N \tilde{e} (1 - \tilde{G})^2) > 0 \]

\[ r_T = -D^{-1} \frac{W}{T} N \tilde{e} (1 - \tilde{G}) > 0 \]

\[ \tilde{c}_T = -D^{-1} \frac{W}{T} \Psi < 0 \]

Note also that this implies $w_T = (1 - \tilde{G}) \cdot r_T - \tilde{c}_T$. *QED*

The intuition of these results can be summarized as follows. The increase in the wage of the traditional sector is an obvious implication of the increase in its size and productivity. This induces more people to work for the traditional sector rather than investing in human capital. This reduces the supply of engineers, and increases their salary. The increase in salary however is not sufficient to increase the expected income of the engineers enough to encourage more people to become engineers. Clearly, the number of high-tech entrepreneurs is also reduced.
**Proposition 5b.** Increases in the size of the traditional sector imply higher expected income in the economy, lower variance of both permanent and transitory income, and a smaller differences between the variance of transitory and permanent income.

**Proof.** See the Appendix.

Proposition 5b says that economies characterized by a large traditional sector exhibit high expected incomes, and lower permanent and transitory inequality. Moreover, in these economies there is lower short-term uncertainty in incomes and more stable occupations.

- **Changes in the size of the population N**

**Proposition 6a.** Increases in the size of the population imply lower wages in the traditional sector, and lower salaries paid by the large high-tech firms. The effect on the size of the high-tech sector is ambiguous.

**Proof.** Apply Cramer’s rule to (11). One obtains

\[ i) \quad w_N = D^{-1} \left[ (1 - \alpha) \frac{W}{N} (-\Psi + N \bar{c}(1 - \bar{G})^2) + W \bar{E} \cdot (1 - \bar{G})^2 \right] < 0 \]

\[ ii) \quad r_N = D^{-1} (1 - \bar{G}) \cdot \left[ \bar{E}(1 + W^+) + (1 - \alpha) W \bar{c} \right] < 0 \]

\[ iii) \quad \bar{c}_N = D^{-1} \left[ \Psi (1 - \alpha) \frac{W}{N} + \bar{E} \cdot (1 - \bar{G}) \right] > or < 0 \]

Moreover, this implies \( w_N = (1 - \bar{G}) \cdot r_N - \bar{c}_N \). Since \( w_N < 0 \), then \( \bar{c}_N > r_N (1 - \bar{G}) \).

**QED**

Proposition 6a says that, other things being equal, a larger labor supply reduces the wage and salaries that are formed in the labor markets. Since this reduces both the worker’s income and the engineers expected income, the effects on the number of engineers in the economy is ambiguous.
**Proposition 6b.** Increases in the size of the population reduces the expected income of the economy. The effect on permanent and transitory inequality is ambiguous. However, if a larger $N$ is associated with an increase in high-tech entrepreneurship, both permanent and transitory inequality increases.

**Proof.** See the Appendix.

Proposition 6b confirms the negative effect of increases in population size on the expected income of the economy shown by the simpler model. However, we can only develop a sufficient condition for the effects on inequality. The result that an increase in the number of educated people leads to greater permanent and transitory inequality suggests that increases in population size can give rise to “dual” economies. Part of the population is employed under flexible labor market conditions, and these people earn lower wages and salary. Part of the population undertakes entrepreneurial opportunities, with higher incomes.

**Larger high-tech large firms, $σ$**

**Proposition 7a.** Increases in the size of the large firm high-tech sector imply higher wages in the traditional sector, higher salaries in large high-tech firms, and a larger size of the high-tech sector.

**Proof.** Apply Cramer’s rule to (11). One obtains

\[
i) \quad w_α = D^{-1} [W^+ M^* (1 - \tilde{G})] > 0 \\
ii) \quad r_α = D^{-1} M^* (1 + W^+) > 0 \\
iii) \quad \tilde{c}_α = D^{-1} M^* (1 - \tilde{G}) > 0
\]

Moreover, this implies $w_α = (1 - \tilde{G}) \cdot r_α - \tilde{c}_α$. Since $w_α < 0$, then $\tilde{c}_α < r_α (1 - \tilde{G})$.

**QED**

Larger and more productive large high-tech firms induce a higher salary for their
employees. The increase in the expected income of the high-tech sector also induces more people to become engineers. Note that this also means that there is an increase in the number of high-tech enterprises in the economy, which is suggestive of the role of the large high-tech firms in encouraging the rise of such firms. The higher number of engineers implies a reduction in the supply of traditional workers, which increases their wage.

**Proposition 7b.** Increases in the size of the large firm high-tech sector increases the expected income of the economy and the permanent inequality. The effect on the transitory inequality is ambiguous.

**Proof.** See the Appendix.

The increase in permanent inequality produced by large high-tech firms stems from the mobility between large firms and high-tech enterprises by the qualified personnel. This is because the increase in $\sigma$ produces an increase in the expected income of the engineers that is higher than the increase in the wages of the traditional sector. Moreover, there are more engineers, and thus more people enjoying the higher expected income of the high-tech sector.

The unambiguous effect on long-term inequality critically depends on the fact that the economy exhibits high job mobility between large high-tech firms and smaller enterprises. If this was not so, people working in the large firms may stay in these firms for very long periods, and similarly entrepreneurs (including those whose companies fail) may find it difficult to change job. In this case, the long-term variance of incomes would be closer to our variance of transitory income $V^\beta$, which was showed to have an ambiguous sign. To put these remarks in perspective, we are saying that increases in the size and productivity of the large high-tech firms in a society with high mobility of jobs like Silicon Valley leads to greater permanent inequality. By contrast, a similar increase in a country with lower mobility across jobs, like Germany or Japan, may not produce an increase in inequality.
Changes in technological opportunities, $x$

**Proposition 8a.** Increases in technological opportunities $x$ imply higher wages in the traditional sector and a higher size of the high-tech sector. The effect on the salary of the engineers employed by the large high-tech firms is ambiguous.

**Proof.** From Cramer’s rule applied to (11),

\[
\begin{align*}
\text{i)} & \quad w_x = D^{-1} \phi \cdot W^* \left[ \tilde{G} \cdot \Psi - (1 - \tilde{G}) \cdot N \tilde{E} \tilde{g} \right] > 0 \\
\text{ii)} & \quad r_x = D^{-1} N \cdot \phi \cdot \left[ \tilde{G} \tilde{c} (1 - \tilde{G}) - \tilde{E} \tilde{g} (1 + W^*) \right] > 0 \quad \text{or} < 0 \\
\text{iii)} & \quad \tilde{c}_x = D^{-1} \phi \cdot \left[ \tilde{G} \cdot \Psi - (1 - \tilde{G}) \cdot N \tilde{E} \tilde{g} \right] > 0
\end{align*}
\]

It is also easy to see that $w_x = W^* \tilde{c}_N$. Moreover, we know from the previous section that $\bar{y}_x = \phi \tilde{G} + r_x (1 - \tilde{G})$. If we replace $r_x$ with expression (ii) above, and use the full expression for the determinant $D$, after some tedious algebra one obtains

\[
\bar{y}_x = w_x + \tilde{c}_x = \tilde{c}_x (1 + W^*) \quad QED
\]

Technological opportunities increase the expected income of the engineers, which encourages more people to invest in human capital. The reduced supply of workers increases their wage in the traditional sector. In the high-tech sector, we already saw that increases in $x$ favor the formation of the high-tech small firms. The combined effect of a higher fraction of engineers in the population, and the attractiveness of smaller firms for these engineers, has an ambiguous effects on the supply of engineers to the large high-tech companies. This in turn means that the effect of higher $x$’s on the salary of the engineers employed by large firms is ambiguous.

**Proposition 8b.** Increases in technological opportunities raise the expected income of the economy, as well as both permanent and transitory inequality.

**Proof.** See the Appendix.

Thus, an economy based on increasing scientific and technological opportunities grows
faster, but it is also bound to be more unequal.

5. CONCLUSIONS

This paper attempted to interpret some underlying phenomena that have been observed in advanced, and to some extent non-advanced economies. Particularly, we tried to link a few trends: First, the observed increase in the variance of incomes, especially in the US and the UK; second, the increasing reliance of several economies on scientific and technological opportunities, and the related formation of high-tech small-medium enterprises. In addition, these patterns are not unrelated to a more general trend towards more “entrepreneurial” jobs, which show tighter relationships between rewards and individual performance, along with greater mobility of individuals across jobs and short-term uncertainty of occupations.

The key results of our model can be summarized as follows. An economy characterized by high scientific and technological opportunities stimulates the formation of small-medium sized high-tech enterprises, which we showed to enjoy greater advantages from undertaking riskier technological projects. This implies that when high scientific and technological opportunities are available there is greater formation of independent high-tech enterprises, and a greater fraction of the population earn incomes that differ according to the different abilities of the individuals. Apart from increasing the average income of the economy, this leads to greater variance of incomes, and hence higher inequality. We contrasted this situation with an economy characterized by a large “traditional” sector with long-term stable occupations. We found that such an economy also fares higher expected incomes, but lower variance and inequality. We also examined some intermediate cases, and particularly the effects of larger high-tech firms, and a larger supply of labor.

Our analysis suggests some additional remarks. First, our model assumes that both the firms and the individuals are risk-neutral. Risk aversion would diminish the incentives of the individuals to undertake more unstable job opportunities, and this would affect our analysis in favor of the more stable occupations. However, risk aversion would only
affect the “levels” rather than the “derivatives” of the model. Put differently, other things being equal, risk aversion would imply a higher threshold for the formation of high-tech enterprises, and this would produce a lower variance of income. But even with risk aversion, an increase in technological opportunities $x$ would imply the formation of new enterprises, and hence a greater variance of incomes. Similarly, a larger traditional sector would induce fewer high-tech firms, and a smaller variance of incomes. Risk aversion only means that the compensation for undertaking risky project has to be higher.

Second, our analysis suggests some broader speculations about the meaning of inequality. In some sense, one can ask – given that it produces higher expected income, is an increase in scientific and technological opportunity, and the implied increase in inequality, harmful for society? Stated as such, the answer is “no”. Particularly, note that increases in $x$ raise the wages in the traditional sector. Thus, society is more unequal, but this comes with an increase of both the higher and the lower incomes. Inequality increases only because the high incomes rise faster. This is totally different from a situation in which the incomes of the bottom part of the distribution decline. As shown by our model, this is the type of inequality produced by increases in $N$.

One reason why society may be worst-off with increases in $x$ would be if people do not care only about their income, but also about their income relative to that of the others. Particularly, suppose that people in the lower part of the income distribution would be worst-off if the income of the upper part increases faster than theirs. They would then be willing to accept lower incomes, provided that the income of the upper earners did not increase that fast. Societies with such sociological underpinnings would see lower earners resist against increases in incomes by the upper earners, even if this costed part of their income. The issue is neither trivial nor totally speculative. For example, would people prefer an increase in their income by 10% and an increase in their neighbor’s income by 100%, or increases by 5% and 6% respectively?

In many modern societies this question does not have an obvious answer. Similarly, in many cases unions representing one category of workers ask for larger increases in
wages simply because other categories of workers have had larger increases in wages. Thus, societies in which people care only about their income are more likely to accept the inequality implied by high-\(x\) economies. Vice versa, when there are social pressures against differential incomes, high-\(x\) economies are less likely to arise, even if they would imply higher incomes of both the lower and upper classes.

While these remarks suggest that increases in inequality may be perfectly acceptable if people cared only about their incomes, there are reasons that warn against increases in inequality even if they produced higher incomes by lower earners. In our model, increases in inequality are brought about by the fact that incomes depend to a greater extent on different individual-specific abilities. But this also means that in these societies, income earners, and particularly the top and most able ones, may be less willing to give up some of their earnings to sustain the incomes of less capable people. Put simply, such societies may be less solidaristic because people perceive that their higher incomes stem from their higher abilities, and thus feel that their differential position in the income distribution is a “just” reward to their skills.

These are issues that we did not model in our analysis. However, we can speculate that such an attitude may reduce the willingness, especially by the higher income earners, to pay taxes to sustain lower income earners, as this would be seen as an unjustified reduction of the rewards to their abilities, as Paganini’s reply to the horse-cart conductor in our epitaph seems to suggest. To the extent that redistributive issues are important to diffuse services or other opportunities to larger fractions of the population, an increase in inequality, even when it produces higher incomes by the bottom part of the distribution, may increase the poverty of part of the population because of the reduced access to such services or to related indirect sources of wealth. In short, what is not so obvious is that in society rewards have to accrue only on the basis of economic abilities.

Appendix

Proofs of Propositions 3, 4, 5b, 6b, 7b, 8b

Proof of Proposition 3. To prove the first part of this Proposition, note that \(H_5\) is
given, and it does not depend on \( \sigma \). Hence, to study the changes in the total employment by the large firms, i.e. \( H_s (1 - \tilde{G}) \), and in the number of entrepreneurs, \( H_s \tilde{G} \), one simply needs to look at the changes in the share \( \tilde{G} \). Using \( \tilde{e}_s = -1 \), \( \tilde{G}_a = -\tilde{g} \cdot r_a < 0 \). Hence, as \( \sigma \) increases the size of the employment in large firms increases, and the extent of entrepreneurship decreases.

Similarly, to examine the effects of changes in \( x \), we only need to look at the sign of \( \tilde{G}_x = -\tilde{g} \cdot r_x + \tilde{g} \cdot \phi = \tilde{g} (\phi - r_x) \). Using the expression for \( r_x \), which can be derived from (7),

\[
\phi - r_x = \phi + \frac{H_s \tilde{g} \phi}{\Psi} = \frac{\phi}{\Psi} \left( M \int_{\tilde{g}}^{0} H_s^{-} dS \right) > 0.
\]

Hence, increases in \( x \) lead to fewer people employed in large firms, and more extended entrepreneurship.

Finally, \( \frac{\partial H_s \tilde{G}}{\partial H_S} = \tilde{G} - H_s \tilde{g} \cdot r_{h_s} \). Using the expression for \( r_{h_s} \), which can be obtained from (7), and using the expression for \( \Psi \), after some algebra one obtains

\[
\frac{\partial H_s \tilde{G}}{\partial H_S} = \tilde{G} \int_{0}^{\tilde{g}} H_s^{-} dS - H_s \tilde{g} > 0.
\]

Hence, a larger supply of engineers implies more entrepreneurship, and fewer employees in the larger firm sector. \( QED \)

**Proof of Proposition 4.** \( V_a = -\tilde{p} \tilde{g} \cdot r_a + \tilde{g} \cdot 2 r r_a (1 - \tilde{G}) - 2 \tilde{y} \tilde{y}_a \). Since \( \tilde{p} = r \), and using \( \tilde{y}_a = r_a (1 - \tilde{G}) \), one obtains \( V_a = 2 r a (1 - \tilde{G}) (r - \tilde{y}) \). But the entrepreneurs in this economy are the individuals for which \( p \geq r \), which means that \( \tilde{y} \geq r \). Hence, \( V_a \leq 0 \).

To show how the variance of income varies with \( x \), note that

\[
V_x = \tilde{p} \tilde{y} \cdot \tilde{e}_x - r \tilde{y} \cdot \tilde{e}_x + 2 \int_{0}^{\tilde{e}} \tilde{p} \cdot \phi \ dG + 2 r r_x (1 - \tilde{G}) - 2 \tilde{y} \tilde{y}_x.
\]

Using \( \tilde{y}_x = \phi \tilde{G} + r_x (1 - \tilde{G}) \); \( \int_{0}^{\tilde{e}} \tilde{p} \ dG = \tilde{y} - r (1 - \tilde{G}) \); and the fact that \( \phi \) is independent of \( \varepsilon \); after some algebra one obtains \( V_x = 2 (1 - \tilde{G}) (\tilde{y} - r) (\phi - r_x) \). We showed in Proposition 3 above that \( \phi - r_x > 0 \). Hence, \( V_x \geq 0 \).
For the change in the variance of income with respect to $H_S$, $V_{H_S} = 2r_{H_S}(1 - \tilde{G}) - 2\tilde{y} \tilde{y}_{H_S}$. Since $\tilde{y}_{H_S} = r_{H_S}(1 - \tilde{G})$, and $r_{H_S} < 0$, one obtains $V_{H_S} = 2r_{H_S}(1 - \tilde{G})(r - \tilde{y}) > 0$. \textit{QED}

\textbf{Proof of Proposition 5b.} Using (12), $\tilde{y}_r = w_r + \tilde{E} \cdot \tilde{c}_r = (1 - \tilde{G}) \cdot r_r - (1 - \tilde{E}) \cdot \tilde{c}_r > 0$, where we used the fact that $w_r = (1 - \tilde{G}) \cdot r_r - \tilde{c}_r$. To compute the effects on the variance of consumption, the derivative of (13) with respect to $T$ is

$$V_T^C = 2\int_0^c (\tilde{y} - c) \cdot \tilde{y}_r \, dE + 2 \cdot w \cdot w_r (1 - \tilde{E}) - 2 \cdot \tilde{E} \cdot \tilde{y}_r$$

Note that $\tilde{y}_r = r_r (1 - \tilde{G}) = w_r + \tilde{c}_r$, and that this expression is independent of $c$. Hence, it can be taken out of the integral sign. Moreover

$$\int_0^c (\tilde{y} - c) \, dE = \tilde{E} \cdot \tilde{y} - \tilde{c} \tilde{E} + \left[ E \, dc = \tilde{E} w + \left[ E \, dc = \tilde{y} - w(1 - \tilde{E}) \right.$$ \n
Using $\tilde{y}_r = w_r + \tilde{E} \cdot \tilde{c}_r$, one obtains $V_T^C = 2\tilde{c}_r (1 - \tilde{E}) (\tilde{y} - w)$. But $\tilde{y} - w = \int_0^c (\tilde{y} - c - w) \, dE > 0$ because $\tilde{y} - c \geq w \forall c \leq \tilde{c}$. Hence, $V_T^C > 0$.

Finally, from (14), $V_T^Y = V_T^C + \tilde{V}_T^\gamma \cdot \tilde{c}_T + \int_0^c 2 (r - c) \cdot r_r (1 - \tilde{G}) - 2 (\tilde{y} - c) \cdot \tilde{y}_r \, dE$, where $\tilde{V}_T^\gamma \equiv V_T^\gamma (\tilde{c})$ is the variance of the incomes in the high-tech sectors for the marginal engineer with $c = \tilde{c}$. Using $\tilde{y}_r = r_r (1 - \tilde{G})$, the term under the integral sign is equal to $2 \cdot r_r (1 - \tilde{G}) \cdot (r - \tilde{y}) \cdot \tilde{E} < 0$. Since $V_T^C < 0, \tilde{c}_T < 0, and \tilde{V}_T^\gamma > 0$, then $V_T^Y < 0$.

Moreover, $V_T^Y - V_T^C < 0$. \textit{QED}

\textbf{Proof of Proposition 6b.} From (12), $\tilde{y}_N = w_N + \tilde{E} \cdot \tilde{c}_N = (1 - \tilde{G}) \cdot r_N - (1 - \tilde{E}) \cdot \tilde{c}_N$.

But $\tilde{c}_N > r_N (1 - \tilde{G})$ implies $\tilde{y}_N < (1 - \tilde{G}) \cdot r_N - (1 - \tilde{E}) \cdot r_N (1 - \tilde{G}) = r_N (1 - \tilde{G}) \cdot \tilde{E} < 0$. To compute the effects on the variances of consumption and income, we use the same strategy used in the proof of Proposition 5b. Take the derivative of (13) with respect to $N$, and use

$$\int_0^c (\tilde{y} - c) \, dE = \tilde{E} \cdot \tilde{y} - \tilde{c} \tilde{E} + \left[ E \, dc = \tilde{E} w + \left[ E \, dc = \tilde{y} - w(1 - \tilde{E}) \right.$$ \n
$\tilde{y}_N = w_N + \tilde{E} \cdot \tilde{c}_N$, and after re-arranging terms one obtains $V_N^C = 2\tilde{c}_N (1 - \tilde{E}) (\tilde{y} - w)$. Since $\tilde{y} - w > 0$, then $V_N^C > 0$ if $\tilde{c}_N > 0$. 

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From (14) \( V_N^Y = V_N^C + \tilde{V} \cdot \tilde{c}_N + 2 \cdot r_N (1 - \tilde{G})(r - \tilde{y}) \cdot \tilde{E} \). Hence, \( \tilde{c}_N > 0 \rightarrow V_N^C > 0, V_N^Y > 0 \) & \( V_N^Y - V_N^C > 0 \). \textit{QED}

**Proof of Proposition 7b.** From (12), \( \tilde{V}_a = w_a + \tilde{E} \cdot \tilde{c}_a = (1 - \tilde{G}) \cdot r_a - (1 - \tilde{E}) \cdot \tilde{c}_a \). But \( \tilde{c}_a < r_a (1 - \tilde{G}) \) implies \( \tilde{V}_a > (1 - \tilde{G}) \cdot r_a - (1 - \tilde{E}) \cdot r_a (1 - \tilde{G}) = r_N (1 - \tilde{G}) \cdot \tilde{E} > 0 \). To compute the effects on the variances of consumption and income, we use the usual strategy. We first obtain \( V_a^C = 2 \tilde{c}_a (1 - \tilde{E}) \tilde{Y} - w > 0 \). We then obtain
\[
V_a^Y = V_a^C + \tilde{V}_a \cdot \tilde{c}_a + 2 \cdot r_a (1 - \tilde{G}) (r - \tilde{y}) \cdot \tilde{E}
\]
\( V_a^Y \) has an ambiguous sign, and so does \( V_a^Y - V_a^C \). \textit{QED}

**Proof of Proposition 8b.** \( \tilde{Y}_x = w_x + \tilde{E} \cdot \tilde{c}_x = (W^* + \tilde{E}) \cdot \tilde{c}_x > 0 \). Using this expression for \( \tilde{Y}_x \), along with \( \tilde{y}_x = \tilde{c}_x (1 + W^*) \), and \( \int (\tilde{y} - c) dE = \tilde{Y} - w(1 - \tilde{E}) \), after some algebra one obtains that \( V_x^C = 2 \tilde{c}_x (1 - \tilde{E}) \tilde{Y} - w > 0 \). As far as the variance of transitory income is concerned, note that
\[
V_x^Y = V_x^C + \tilde{V}_x \cdot \tilde{c}_x + \int_0^\tilde{y} \left( (p - c) \Phi \right) dG + 2 \cdot (r - c) \cdot (1 - \tilde{G}) \cdot r_x - 2(\tilde{y} - c) \cdot \tilde{y}_x dE
\]
Replace \( r_x (1 - \tilde{G}) \) with \( \tilde{y}_x - \Phi \tilde{G} \). Since \( \phi \) is independent of \( \epsilon \), it can moved out of the first integral sign. Rearranging terms, the last three terms of this expression become
\[
2 \int_0^\tilde{y} \left( (p - r) \right) dG - \tilde{y}_x (\tilde{y} - r) dE.
\]
Since \( \tilde{y} - r = \int_0^\tilde{y} (p - r) dG \), one obtains
\[
2 \cdot \int_0^\tilde{y} (\tilde{y} - r) \cdot (\phi - \tilde{y}_x) dE.
\]
Using \( \tilde{y}_x = \Phi \cdot \tilde{G} + r_x (1 - \tilde{G}) \), one can write
\[
(\phi - \tilde{y}_x) = (1 - \tilde{G}) \cdot (\phi - r_x).
\]
Use the expression for \( r_x \) in \( ii \) above. By using the full expression for the determinant \( D \), after some algebra one obtains that
\[
\phi - r_x = \Phi \int_{\tilde{G}}^{-N \Phi (1 - G)} + (1 + W^*) M \int_{\tilde{G}}^0 H^* dS > 0.
\]
This implies that \( V_x^Y > 0 \) and that \( V_x^Y > V_x^C \). \textit{QED}
References


