The Dynamics of Norms and Conventions under Random Matching

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Abstract

Focusing on 2x2 coordination games, the concept of stochastic stability as developed by Young (1993, 1998) is extended to take account of state dependent error and sample sizes. Both, error and sample size are supposed to be correlated with the loss that occurred, if a player chooses a non-best response strategy. The original predictions are robust to this change if the game's pay-off matrix exhibits a form of symmetry, or if only the relative potential loss from idiosyncratic play defines the state dependent variable. If neither of these conditions is met, the state dependent version will not necessarily determine the same Stochastically Stable State (SSS) as the original approach. Even if these conditions are met, it is shown that in the context of state dependence, the minimum stochastic potential is a necessary but insufficient condition to determine a convention: The SSS must further sufficiently risk dominate the other equilibrium.

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1. Introduction

In the spirit of Max Weber's "Wirtschaft und Gesellschaft" (Weber, 2007), the role of culture, as an important economic determinant, has experienced a strong revival in the scientific literature since the late 80s (Harrison and Huntington, 2000; Huntington and Harrison, 2004; Welzel and Inglehart, 1999; Huntington, 1997; Ades and Di Tella, 1996; Bollinger and Hofstede, 1987), illustrating the fundamental impact of social norms and conventions on economic development.[2] Culture does not only determine the institutional framework, but also the behaviour of economic agents. Consequently, social processes and culture, though often neglected, are essential variables of economic theory. "Culture is the mother, institutions are the children" (Etounga-Manguelle, 2004, p. 135). The dynamics determining social conventions and norms must be therefore of special economic interest.

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[2]I will use the terms *norm* and *convention* interchangeably, since the transition between both concepts is completely smooth and seamless - in fact Weber speaks of conventional norm (see Weber, 2007, Ch. 4).

Along this line, "The Evolution of Convention" by Young (1993) is a well-known approach that is part of a larger literature on stochastic stability (Turnovsky and Weintraub, 1971; Foster and Young, 1990; Kandori et al., 1993; Blume, 1993; Ellison, 1993, 2000; Samuelson, 1994, 1997; Orléan, 1995; Robson and Vega-Redondo, 1996; Morris, 2000; Bowles, 2006). The approach allows to discriminate between potential conventions, being synonymically defined as pure Nash equilibria. In his approach, Young does not try to explain the subtle workings of how a transition between conventions occurs, but why we observe certain conventions to be more stable and to persist longer than others. This article takes Young's approach as a basis and guideline. Consequently, the sociological intricacies of norms and conventions are not the issue of this article, but conventions in the abstract form of the stochastic stability approach. Since the assumption of state independent error size has been criticised, the focus will be on the state dependence both of the sample process and the error probability by relaxing and altering some of the assumptions of the original approach. We will observe under which conditions the original predictions are maintained.

This article focuses solely on the issue of state independence, but maintains the remaining original assumptions, such as random, but global interactions and adaptive play. Each player samples from the set of previous interactions and is paired randomly with any other player in his population with positive probability.[3] The first section will illustrate that reasonable assumptions on the error and sample size will strengthen the original results for symmetric games, though not necessarily for asymmetric games.

The second section points out a more fundamental issue in this context. The original framework neglects the effect of random choice that occurs in the presence of a state dependent error size, since error rates cannot be assumed to converge to zero.[4] If error rates are high, the occurring *randomness* from idiosyncratic play will necessitate a larger basin of attraction as a counter-force, in order for a Stochastically Stable State (*SSS*) to evolve in the long-term. Hence, for interactions with a low potential loss owing to erroneous play, the stochastic stability approach cannot be applied directly. A second condition must be additionally fulfilled: the one-third rule.

1.1. A short Introduction to Stochastic Stability

In the context of conventions several questions arise: Which strategies constitute a possible convention? Why do certain conventions persist, whilst others are rather short-lived? Why does a specific convention (and thus norm) emerge and not another, i.e. why do we see both similar (so called *evolutionary universals*, see Parsons (1964)) but also entirely different behavioural patterns in locally separated parts of the world? Classical game theory provides an answer to the first question. The strategy profile defined by the convention consists of the best response strategies of each player (type) to the strategy of the other players, implying that if a sufficient number of individuals is believed to follow the convention, it is individually pay-off maximising to do the same. Thus a convention describes a stable pure Nash equilibrium of an n-person game. The second question, however, can only be insufficiently answered. Obviously, since the conventional strategy is best response to the strategies played by all other individuals, it is best not to deviate from the strategy

[3] Player pairing is only restricted by affiliation to types. If types exist, a player can only be paired with players of a different type. In "the Battle of Sexes" a man can be paired with any woman in his population, but not with another man. Issues resulting from the restrictive assumption of *global* interactions and sampling will not be dealt with in this article, and are left to a different research project.

[4] Notice that although Theorem 4.2. in Young (1998) provides a condition for the case of non-zero error rates, the results obtained here are different as some of the original assumptions are changed.

prescribed by the convention. Yet, when it comes to interactions, in which more than one Nash equilibrium in pure strategies exist, the determination of a long-term convention is ambiguous. Why is money accepted in exchange for goods and services? Why are economic interactions determined by certain informal rules and not others? Why do people first let others exit the coach and only after that enter the train? The inverse behaviour could also define a convention. The answer often given to the third question is that the choice between conventions follows a non-ergodic (or path dependent) process. This answer leaves to much space to chance events and unknown exogenous variables to be able to explain the similarity of conventions in separate regions of the world.

A more adequate explanation to this question is, however, strongly connected to the second question. The history leading to a new convention is fundamentally shaped by the underlying conventions and norms that currently prevail. Hence, conventions and norms at one point in time will define the historical circumstances that determine future norms or conventions (see Bicchieri, 2006 for an overview of the current literature on how existing norms affect players' choices). The third question thus collapses to the second. This circumstance requires an approach to discriminate between various conventions, answering the question of why certain types of conventions prevail over others. Kandori et al. (1993) and Young (1993) have developed similar approaches to this question. Since it constitutes the basis for subsequent derivations, this article will focus on Young's approach on stochastic stability and will elaborate its basic reasoning in the following. Readers familiar with the concept can skip to the following section 2.

Assume that for a finite player population, n different sub-populations exist, each indicating a player type participating in the game. Strategies and preferences are identical for all individuals in the same sub-population. The game is defined by $\Gamma = (X_1, X_2, ..., X_n; u_1, u_2, ..., u_n)$, where X_i indicates the strategy set and u_i the utility function of individuals of type i. Hence for simplicity, define each individual in such a sub-population C_i as player i. Assume that one individual from each sub-population C_i is drawn at random in each period to play the game. Each individual draws a sample of size $s < \frac{m}{2}$ from the pure-strategy profiles of the last m rounds the game has been played.[5] The idea is that the player simply asks around what has be played in past periods. Hence, the last m rounds of play can be considered as the collective memory of the player population. In addition to Young's assumptions, I assume explicitly that m and s are large. This assumption is made to guarantee that the minimum rate in the sample required to switch best-response strategy can take any value between 0 and 1. (The example in AppendixA on page 15 illustrates an instance, in which this is not the case.)

Each state is thus defined by a history $h = (x^{t-m}, x^{t-m+1}, \ldots, x^t)$ of the last m plays and a successor state by $h' = (x^{t-m+1}, x^{t-m+2}, \ldots, x^t, x^{t+1})$ for some $x^{t+1} \in X$, with $X = \prod X_i$, which adds the current play to the collective memory of fixed size m, deleting the oldest. Each individual is unaware of what the other players will choose as a best response. He thus chooses his best reply strategy with respect to the strategy frequency distribution in his sample (fictitious play with bounded memory, which Young called *adaptive play*). He chooses, nonetheless, any strategy in his strategy profile with a positive probability. Consequently, suppose that there is a small probability that an agent inadequately maximizes his choice, and commits an error or simply experiments. The probability of this error equals the rate of mutation $\varepsilon > 0$, i.e. with probability

[5] More precisely; the general condition is defined as $s \leq \frac{m}{L_\Gamma + 2}$, with L_Γ being the maximum length of all shortest directed paths in the best reply graph from a strategy-tuple x to a strict Nash equilibrium (see Young, 1993). Since here the analysis is restricted to 2×2 coordination games, the simplified assumption suffices.

ε an individual j in C_j does not choose his best response $x_j^* \in X_j$ to his sample of size s from a past history of interactions.[6] Instead he chooses a strategy at random from X_j. Since each state is reachable with positive probability from any initial state if $\varepsilon > 0$, the process is described by an irreducible Markov chain on the finite state space $\Omega \subset (X_1 \times X_2 \times ... \times X_n)^m$. Not all states are, however, equally probable. In order to shift a population from some stable equilibrium (i.e. convention), at which players only remember to have always played the same strategy, defined by strategy profile $x^{*t} = (x_1^*(t), x_2^*(t), \ldots, x_n^*(t))$ and history $h_k = (x^{*t-m}, x^{*t-m+1}, \ldots, x^{*t})$ to some other stable equilibrium defined by $x^{'t}$ and $h_l = (x^{'z-m}, x^{'z-m+1}, \ldots, x^{'z})$ in time z, requires that a sufficiently large number of individuals idiosyncratically chooses a non-best response strategy to move the population out of the basin of attraction of the equilibrium defined by h_k into the basin of attraction of another equilibrium, so that $x_i^{'}$ is eventually a best response to any sample drawn from m.

For each pair of recurrent classes E_i, E_j from the set of recurrent classes $E_1, E_2, \ldots, E_k$ in the non-perturbed Markov process, a directed ij-path is defined by a sequence of states $(h_1, h_2, ..., h_z) \in \Omega$ that goes from E_i to E_j. Define the resistance $r(h, h^{'})$ as the number of mistakes (perturbations) necessary to cause a transition in each period from any current state h to a successor state h' connected by a directed edge, implying that the transition from h to the successor state $h^{'}$ in an n-person game is of order $\varepsilon^{r(h,h^{'})}(1-\varepsilon)^{n-r(h,h^{'})}$. (If $h^{'}$ is a successor of h in the non-perturbed process resistance is 0. If $h^{'}$ is not a successor state both in the perturbed and unperturbed process, the resistance is equal to ∞.) The resistance of this path is given by the sum of the resistances of its edges, $r_\sigma = (r(h_1, h_2) + r(h_2, h_3) + \ldots + r(h_{z-1}, h_z))$. Let r_{ij} be the least resistance over all those ij-paths. Hence, there exists a tree rooted at vertex j for each recurrent class E_j that connects to *every* vertex different from j. Notice that connections can be defined by a direct or indirect path leading from any other vertex k for E_k to j for E_j, with $k \neq j$, in the perturbed process. A path's resistance is thus given by the sum of the least resistances r_{ij} over all the edges in the tree. The stochastic potential for any E_j is defined as the least resistance among all these trees leading to vertex j. The recurrent class with least stochastic potential determines the Stochastically Stable State. Remember the least resistance path can be direct or indirect, and takes further account of all strategies in the strategy set. In other words, an *SSS* is the equilibrium that is the easiest accessible from *all other states combined*.

The assumption of the stochastic stability approach that errors are state and pay-off independent has been criticized (Bergin and Lipman, 1996; Bowles, 2006). Yet, the same type of criticism applies to the state independent sample size. It will be illustrated in this article that major changes are not required in Young's approach to take this circumstance into account. The following section will thus include pay-off dependent sample and error size into the calculation of the resistances. The method is based on an approach of Young (see Young, 1998, Theorem 4.1) and the work of (van Damme and Weibull, 1998). This article will show that stochastic stability still holds under the assumption of state dependence under most conditions. It also illustrates that there is a significant difference between assuming state dependent sample size and error size, if different player types interact. Sample size affects the rate at which a player type directly *observes* mutations and is dependent on the pay-off this player type has at the current equilibrium state. In contrast, error

[6] Strictly speaking the error rate is given by $\lambda_j \varepsilon$ for player j and has full support, i.e. all strategies in X_j are played with positive probability whenever an error occurs or the player experiments. Note, however, in the standard case the *SSS* is independent of λ_j and the probability, with which a strategy is randomly chosen.

size affects the rate at which an error is *committed* by the other player types: It therefore depends on the pay-off of those other player types. Section 3 will show, however, that these assumptions raise a crucial issue, since the error rate can be expected to be high in certain states, thus causing a potential disruption in the transition from one convention to another.

2. State Dependent Sample and Error Size

This section will follow the approach of van Damme and Weibull (1998) to some extent. It only considers sample and error size as loss-dependent, but generally assumes type independence (if not mentioned otherwise), i.e. error and sample size are defined by a function that has only the pay-offs as its arguments and is not type specific. Findings are rather similar, but will differ in various details. Van Damme and Weibull assume that an individual can choose his error level, but has to pay a control cost. The *control cost function* $v(\epsilon_i(\omega))$ is defined as a function of individual's i error size at the current state ω. Furthermore, the *control cost function* is supposed to be decreasing, strictly convex, and twice differentiable. As in van Damme and Weibull, this article will only consider 2×2 coordination games with two strict Nash equilibria in pure strategies, generally of the form presented in matrix 1, with $a_i > c_i$ and $d_i > b_i$. (In the following index 1 is always assigned to row players and index 2 to column players.)

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} a_1, a_2 & b_1, c_2 \\ c_1, b_2 & d_1, d_2 \end{pmatrix} \end{array} \tag{1}$$

Define $g_i(\omega) = max[\pi_i(A, \omega), \pi_i(B, \omega)]$, and $w_i(\omega) = min[\pi_i(A, \omega), \pi_i(B, \omega)]$ and thus $l_i(\omega) = g_i(\omega) - w_i(\omega)$, given the current conventional state ω. The expected pay-off is then determined by $\pi_i = g_i(\omega) - \epsilon_i(\omega) l_i(\omega) - \delta v(\epsilon_i(\omega))$, where l defines the loss in the case, where an error is committed and the non-conventional (non-best response) strategy is played. In addition to the approach of van Damme and Weibull, ϵ_i is assumed either to be a function of the sample size $s_i(\omega)$, implying that the larger $s_i(\omega)$ the lower the probability of drawing a skewed sample from the collective memory m. Additionally, the error probability is assumed to be directly controllable by each individual and is determined by an exponent $\gamma_i(\omega)$ and the exogenous "baseline error ε", such that $\epsilon_i(\omega) = \varepsilon^{\gamma_i(\omega)}$. The idea is that individuals try to *stabilise their trembling hand* if stakes are high, whereas they are more inclined to explore alternative strategies if potential loss is small. Further let us assume δ equals 1, ϵ to be restricted to the unit interval, and the baseline error ε to be exogenous. Therefore expected profit is given by

$$\pi_i = g_i(\omega) - \epsilon_i(s_i(\omega), \gamma_i(\omega); \varepsilon) l_i(\omega) - v(\epsilon_i(s_i(\omega), \gamma_i(\omega); \varepsilon)) \tag{2}$$

Given the previous assumptions, $v(\epsilon_i(s_i(\omega), \gamma_i(\omega); \varepsilon))$ is strictly convex and twice differentiable in ϵ_i, and ϵ_i is strictly decreasing both in $s_i(\omega)$ and $\gamma_i(\omega)$. Hence the marginal cost function $-v^{'}(.)$ will be decreasing in ϵ_i and increasing in s_i and γ_i. Maximizing the expected pay-off yields

$$l = -\frac{\partial v[\epsilon_i(s, \gamma; \varepsilon)]}{\partial \epsilon_i}. \tag{3}$$

For the general 2×2 coordination game assume that ω and ω' denote the two possible conventional states of the world. As before $\epsilon_i(\omega)$ is the mutation probability of type $i = 1, 2$ in states ω, $s_i(\omega)$

is the corresponding sample size, and $\gamma_i(\omega)$ is defined as such that $\epsilon_i(\omega) = \varepsilon^{\gamma_i(\omega)}$. Given the definition above $l_i(\omega)$ defines the loss function of player i in state ω as the loss that occurs if player i erroneously plays his non-best response with respect to state ω. Thus the loss function is defined as $l_1(\omega) = \pi^1_{\omega\omega} - \pi^1_{\omega'\omega}$ for the row players, and $l_2(\omega) = \pi^2_{\omega\omega} - \pi^2_{\omega'\omega}$ for column players, if π^1 and π^2 indicate the corresponding pay-offs in the pay-off matrix for each player type and the first part of the index the state to which the player chooses the best response strategy and the second part the actual state. Notice that error and sample rate depend only on the expected loss, but are type independent, since the cost control function and the baseline error is identical for all types. It follows from equation 3 that

$$\begin{aligned} l_i(\omega) < l_j(\omega') \Leftrightarrow \epsilon_i(\omega) > \epsilon_j(\omega') \Leftrightarrow s_i(\omega) < s_j(\omega') \\ \Leftrightarrow \gamma_i(\omega) < \gamma_j(\omega'), \quad \text{for } i, j = 1, 2. \end{aligned} \tag{4}$$

In 2×2 coordination games, only two equilibria in pure strategies exist and both equilibria are connected by only direct paths. The *reduced resistance* is defined by the minimum share of non-best response plays in the sample, which is necessary to induce best response players to switch their strategy. If the sample and error size are state and type independent, both can be normalised to 1 and the reduced resistances will equal the stochastic potential. It suffices thus to compare only the two reduced resistances along the direct paths (one for each player type).

For 2×2 coordination games and state independent sample and error size, the reduced resistances are defined as follows:

$$r_{AB} = min\left(\frac{a_1 - c_1}{a_1 - b_1 - c_1 + d_1}, \frac{a_2 - c_2}{a_2 - b_2 - c_2 + d_2}\right) \text{ and} \tag{5a}$$

$$r_{BA} = min\left(\frac{d_1 - b_1}{a_1 - b_1 - c_1 + d_1}, \frac{d_2 - b_2}{a_2 - b_2 - c_2 + d_2}\right) \tag{5b}$$

$$\text{or more succinctly: } r_{AB} = \alpha \wedge \beta \text{ and } r_{BA} = (1 - \alpha) \wedge (1 - \beta) \tag{5c}$$

where A and B describe the pure Nash equilibria defined by their corresponding strategies, and α and β define the minimum population frequencies in the sample, necessary to induce best-response players to switch to strategy B. Obviously in this case the *SSS* is equivalent to the risk dominant Nash equilibrium. (for detailed proofs, refer to Young, 1993, 1998).

The symmetric case describes a game, in which a player's position is irrelevant, i.e. pay-offs are independent of the indices in matrix 1. The following two propositions hold in the presence of state dependent sample size $s(\omega)$ given convention ω (see AppendixA for proofs) of this section:

Proposition 1. *For the symmetric case with state dependent sample size the resistances are determined by*

$$r^s_{AB} = \alpha s(A) \tag{6a}$$
$$r^s_{BA} = (1 - \alpha)s(B) \tag{6b}$$

Proposition 2. *In the case of two different player types $i = 1, 2$ and state dependent sample size $s_i(\omega)$, the resistances are defined by*

$$r^s_{AB} = \alpha s_1(A) \wedge \beta s_2(A) \tag{7a}$$

$$r^s_{BA} = (1 - \alpha)s_1(B) \wedge (1 - \beta)s_2(B). \tag{7b}$$

Since the matrix's pay-offs can be changed by a positive affine transformation and only relative sample values are of interest, suppose that in the case of symmetric pay-offs it holds by normalisation that $s(A) \neq s(B) = 1$. Then the equilibrium sample size s^*, at which both equilibria are stochastically stable, is given by $s^* = \frac{1-\alpha}{\alpha}$. For all $s(A) > s^*$, h_A is the sole Stochastically Stable State. In the case of $s(A) < s^*$ the *SSS* is defined by h_B.

For state dependent error size, defined by $\varepsilon^{\gamma_i(\omega)} = \epsilon_i(\omega)$, and normalised state independent sample size ($s_{1,2} = 1$), the following two propositions hold:

Proposition 3. *In the symmetric case with state dependent error size, resistances are given by*

$$r^{\gamma}_{AB} = \alpha\gamma(A) \tag{8a}$$

$$r^{\gamma}_{BA} = (1-\alpha)\gamma(B). \tag{8b}$$

Hence, a decrease (increase) in error size from $\varepsilon(\omega)$ to $\varepsilon(\omega^{'})$, with $\varepsilon(\omega') = \varepsilon(\omega)^{\zeta}$ and $\zeta > 1$ ($\zeta < 1$), is equivalent to an increase (decrease) of the sample size $s_i(\omega)$ by ζ.

Proposition 4. *In the general case with state dependent error size, the resistances are given by*

$$r^{\gamma}_{AB} = \alpha\gamma_2(A) \wedge \beta\gamma_1(A) \tag{9a}$$

$$r^{\gamma}_{BA} = (1-\alpha)\gamma_2(B) \wedge (1-\beta)\gamma_1(B). \tag{9b}$$

In the symmetric case, the "*speed*", at which the boundary of the basin of attraction $\mathfrak{B}_A$ is approached, directly depends on $\gamma(A)$. A $\gamma(A) > 1$ reduces the error rate and decreases the "*step size*" and thus steepens the basin of attraction. The relation between sample size and error size in the symmetric case is reasonable. A higher sample rate should decrease the probability of an error occurring in a symmetric game.

It follows that, for the symmetric case, the unique invariant distribution of the unperturbed Markov process, described by $h^* = h^*P$, with transition matrix P and history (distribution) h^*, can be generalised to (see Bergin and Lipman, 1996 for details).

$$\frac{h^{'}_A}{h^{'}_B} = \varepsilon^{m-i^*+1-\gamma i^*}\frac{k_A[1+f_A(\varepsilon)]}{k_B[1+f_B(\varepsilon)]}$$

where i^* indicates the number of players who chose strategy B at the interior mixed equilibrium state, at which state the error rate changes from some error rate ε^{γ} to another defined by ε. In other words, if p_{ij} represents the probability of moving from state i to state j in the unperturbed Markov process, then $\exists i^*$ such that $p_{i0} = 1$ if $i < i^*$, and $p_{im} = 1$ if $i > i^*$). For m very large and $\varepsilon \to 0$ this can be normalized and rewritten as:

$$\frac{h^{'}_A}{h^{'}_B} = \varepsilon^{1-\alpha-\gamma\alpha}\frac{k_A}{k_B}$$

for α defined as before. In the case of $\gamma > \frac{1-\alpha}{\alpha} = \gamma^*$ the exponent is negative and the ratio goes to ∞. Hence $h^{'}_A \to 1$. In the case of $\gamma < \gamma^*$ the ratio goes to zero and $h^{'}_B \to 1$. For $\gamma = \gamma^*$, $\frac{h^{'}_A}{h^{'}_B} \to \frac{k_A}{k_B}$.

The general pay-off matrix in 1 can have 4 different pay-off structures. The symmetric case is generally defined in the literature as above, i.e. it does not matter whether an individual is a column or row player. This situation occurs in a population with only one player type. If two player types exist, their interests can be diametrically opposed, i.e. pay-offs are defined by a matrix, in which $a_i = d_j$ and $c_i = b_j$ for $i \neq j$. In such games pay-offs for both players are identical, but mirrored on both diagonals of the pay-off matrix. Hence, I define such pay-off matrices as "double mirror-symmetric". Finally, a third type of pay-off symmetry may occur. If $a_i = d_j$, $b_i = b_j$ and $c_i = c_j$, pay-offs are only mirrored on the main diagonal. I define such a pay-off matrix as "mirror-symmetric". In the case, where the original pay-off matrix cannot be transformed into one of the previous structures by a positive affine transformation of all pay-off values that is identical for both player types (thus maintaining the relative values of all four losses), the pay-off matrix is defined as "asymmetric". Using the previous definitions, the following result is obtained:

Proposition 5. *Assume the case, where condition 4 on page 6 holds and resistances are defined by equations 7 in the case of state dependent sample size and equations 9 in the case of state dependent error size.*

In both cases, the original results of stochastic stability are confirmed if the pay-off structure exhibits some form of symmetry, i.e. if it is either symmetric, mirror-symmetric or double mirror-symmetric. Results do not necessarily coincide if pay-offs are asymmetric. Yet, the asymmetric case confirms the results if the sample and error size are a function of the relative instead the absolute potential loss, i.e. if both variables independent of any positive affine transformation of the pay-off matrix.

Two games are equivalent, if the pay-off matrix of a game can be transformed by a positive affine transformation for each player type into the pay-off matrix of the other game. Consequently if players consider only the relative expected loss[7], also the long-term evolutionary properties of both games remain identical. In the case, where both error and sample size are state dependent, the reduced resistances for pay-off matrix 1 propositions 2 and 4 lead to

$$r^{s\gamma}_{AB} = \alpha s_1(A)\gamma_2(A) \wedge \beta s_2(A)\gamma_1(A) \tag{10a}$$

$$r^{s}_{BA} = (1-\alpha)s_1(B)\gamma_2(B) \wedge (1-\beta)s_2(B)\gamma_1(B). \tag{10b}$$

Furthermore, note that this approach also yields a positive relation between risk-aversion and surplus share, if the assumption of type independence of error and sample size is relaxed:

Proposition 6. *In a double mirror-symmetric coordination game with two pure Nash equilibria, the player type that is less risk-averse can appropriate the greater share of the surplus in the case, where sample size is state dependent. In the case, where error size is state dependent, this result holds if $\gamma_i(\omega)$ is strictly convex in $l_i(\omega)$. If the function is strictly concave, the more risk-averse player appropriates the greater surplus share.*

Being more open to taking risks can, ceteris paribus, benefit the player type. This result is coherent with findings in the economic literature (King, 1974; Rosenzweig and Binswanger, 1993; Binmore, 1998 and for a critical discussion of empirical studies, see Bellemare and Brown, 2009)

[7] implying that the effect of the perceived loss on error and sample size is scale independent

on the positive correlation between wealth and risk. If we take risk as a measure of need[8], this analytical result is the obvious relation that the needier one group is and the less it has to lose (from punishment, social shunning, non-conformity etc.) the more likely the convention will be defined in its favour.[9]

Intuitive assumption about the sample size, thus lead to the confirmation of the approach of Young for most interactions. If equation 4 holds, the approach of Young is unaffected in the case of state dependence for symmetric pay-off configurations or if loss is regarded in relative and not absolute terms. Yet, the results of the state dependent approach, only constrained by equation 4, will not coincide with the standard approach for all pay-off configurations.

Also notice the impact of state dependence on the time a player population remains on average in an equilibrium, i.e. the number of interactions a convention endures. In a symmetric game with N players, waiting time w is given by

$$w = \left[\sum_{i=\alpha s}^{N} \left(\binom{N}{i} \varepsilon^{\gamma i} (1-\varepsilon)^{\gamma(N-i)} \right) \right]^{-1}$$

w is strictly increasing in s and γ. This illustrates that not only the interpretation of what defines an interaction (period) determines the transition time, but also the way in which potential loss affects unconventional strategy choice. This mitigates the argument against Young's approach that transition should require to much time. If players interact very frequently, and error and sample rate are low, waiting time is marginal. Yet, the critique is valid for another reason, since the state dependence of the error and sample size entails a fundamental issue that will be discussed in the following section. Given the assumption of state dependence, it is not guaranteed that a convention truly evolves. In this context, it is doubtful that a repeated transition between pure equilibria is generally possible. It might be that a completely mixed state is more likely to evolve in the long-term than any of the two pure Nash equilibria.

3. The one-third Rule and State Dependency

In the previous analysis, I have only considered pay-off losses in the case, where a population is located close to one of the equilibria, i.e. if a distinct convention prevails. Hence, the loss $l_i(\omega) = \pi^i_{\omega\omega} - \pi^i_{\omega'\omega}$ has been defined as the pay-off difference that occurs if a player chooses his best strategy with respect to the absorbing state ω', though the actual current state is defined by the pure Nash equilibrium state ω. It has been thus assumed that a player only considers maximal potential loss and assigns unit probability to the strategy profile defining the current convention. On the one hand, this is a reasonable assumption if a player considers only pure conventional strategies (or in the case of very high discount rates). On the other hand, it might be more realistic to assume that a player evaluates his potential loss according to his sample.

Suppose that a player has to decide whether to experiment or not. Whenever the player population is in a state of transition and moves against the force of the basin of attraction of the current

[8] "Recall that need is to be measured in terms of the risks that people are willing to take to satisfy their lack of something important to them." (Binmore, 1998, p. 463)

[9] This also conforms with Young, 1998, Theorem 9.1., which shows that conventions are close to a social contract that maximises the relative pay-off of the group with the least relative pay-off.

convention towards the other equilibrium, the player observes that other players have been experimenting before. Assume that he perceives previous “experimenters” at a rate of p in his sample. Based on this, he might expect that in the current play his counterpart will also experiment with probability p. He thus evaluates his potential loss based on the mixed state, defined by his sample, and on the expected loss value, by considering the loss function $l_1(\omega^p) = \pi_{\omega\omega^p} - \pi_{\omega'\omega^p}$, given that ω^p indicates a state in which players experiment at rate p. Hence, for pay-offs as in matrix 1 a player i will compare $l_i(A^p) = (1-p)a_i + pb_i - (1-p)c_i - pd_i$ and $l_i(B^p) = pc_i + (1-p)d_i - pa_i - (1-p)b_i$. Notice that the case in section 2 is obtained by setting $p = 0$, i.e. the player does not expect his counterpart to experiment. Yet given a rate p of experimenters, not playing the conventional strategy in h_A will always incur an expected loss greater than playing the non-conventional strategy in h_B as long as $a - c > d - b$.

The propositions could be expanded from the case of $l_1(\omega)$ to the extended case $l_1(\omega^p)$, and the general results with respect to stochastic stability should persist. This is not done here, since I believe a more fundamental issue is raised by the transition: Relaxing Young’s condition of a state independent error entails that its frequency cannot be assumed as being generally small. If we consider $l_i(\omega^p)$, the absolute size of the potential expected loss varies with the strategy distribution in the sample, i.e. with the number of experimenters. That implies that $l_i(\omega^p) \to 0$, as the distribution approaches the interior equilibrium. As the loss grows smaller, the error size increases. At the interior equilibrium expected pay-off from both strategies is identical and hence, no expected loss results from choosing any of the two strategies at random. As a consequence, error rates will be close to 1 in the vicinity of the mixed equilibrium distribution; the zero limit of the error size is inapplicable. Also in other situations error rate can generally be expected to be high and expected loss is generally low. This is the case if potential loss is generally relatively low in comparison to the pay-offs received in both equilibria, or cases in which sampling of information is very costly and individuals are only weakly affected by expected pay-offs.

Consider the example of driving on the left or right. Since most people are right handed, keeping left was indeed risk dominant. Nowadays, both conventions can be observed and are stable. They are imposed by law and risks are high to be punished in the case of infraction. For pedestrians this is not the case. Although to keep on the same side as driving a car is the marginal risk dominant strategy, the costs of walking on the same side as the vis-à-vis are low and people pass both on the left and right. Thus, we can observe a mixed equilibrium. Similar reasoning holds for the convention to stop at a red light. A stable convention thus requires additional properties.[10]

In the case of state dependence, random choice plays a substantial role in the determination of a convention. Furthermore, it is more realistic to assume that it is not necessarily the last element in the collective memory, which is forgotten, since this also requires the individual capability to exactly define the sequence of interactions. In the original approach, it is supposed that an individual can sample at most half of the history of past interactions, but is still capable to assign a time frame to each interaction. In this section, equal “death” probability is assigned to all elements, taking into account the tendency of individuals to remember and over-evaluate rare events more strongly than common events. A rare and extraordinary event is thus less likely to be chosen for death, since it’s rate of occurrence is lower in m. The process by which the collective memory of size m is updated can then be described by a stochastic death and birth process - a Moran process (for details, refer

[10]Observing southern European road users illustrates that stopping at red lights or keeping on the right lane is only a stable convention if it is sufficiently enforced by law.

to AppendixB).

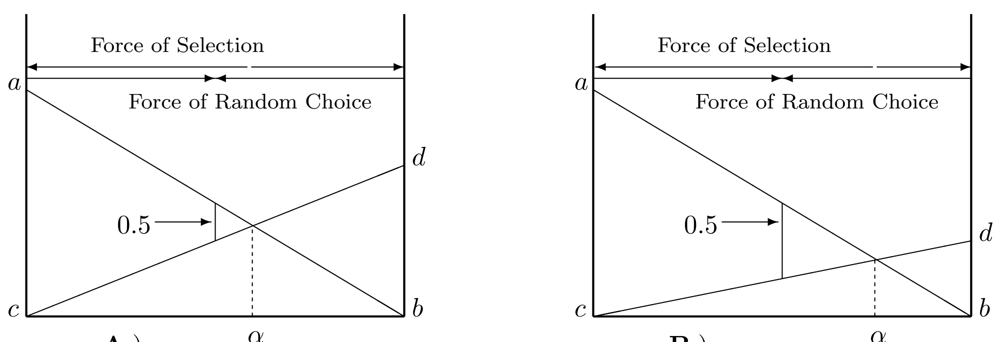
Adapting the approach of Nowak (2006) shows that relaxing the assumption of an overall small error rates would additionally require that an equilibrium generates a larger basin of attraction in order to eligible as a long-term conventional strategy. A predominance of random errors creates an additional invasion barrier and further, once sufficiently mixed, random choice will continuously push the population towards a completely mixed strategy profile in which both strategies are played with roughly equal probability. This counter-acts the selection process, which gravitates the population towards the Nash equilibrium inside the basin of attraction. Hence, a minimum basin of attraction is necessary for an equilibrium to exercise sufficient gravitational pull on a population at the completely mixed state to overcome the adverse effect of random choice. This translates to a sufficiently small resistance on the path towards this equilibrium.

The fixation probability defines the probability of a mutant strategy to cause a switch from some convention to that defined by the mutant strategy. It thus defines the likelihood of a switch in conventions, i.e. the force of *selection* that pushes a population to the other equilibrium. If strategy choice were completely random, fixation probability should be $1/m$. This represents the force of random choice that gravitates a population to a completely mixed strategy, since any new strategy is added to the collective memory with equal likelihood in this case. In the presence of high error rates it holds:[11]

Proposition 7. *Given a large player population, playing a coordination game with two pure-strategy Nash equilibria, a symmetric pay-off matrix and normalised sample rate $s_i(\omega) = 1$. The unstable interior equilibrium is given by a frequency of $(\alpha = (a-c)/(a-b-c+d))$ players choosing strategy B. If the error rate reaches levels close to 1, the reduced resistance r_{AB} (or r_{BA}) must be smaller than $\frac{1}{3}$ for selection to favour the convention h_B (or h_A). If $\alpha \in (\frac{1}{3}; \frac{2}{3})$, the fixation probability is less than $1/m$ for both strategies, and random choice superimposes selection.*

The graph 1 helps to illustrate proposition 7. A.) and B.) represent the expected, normalised

Figure 1: The one-third rule



and symmetric pay-offs of two games, defined as in matrix 1 on page 5, with $a_1 = a_2$, $d_1 = d_2$, and $c_1 = c_2 = b_1 = b_2 = 0$. The frequency of strategy B players defines the abscissa, the intersecting functions show the expected pay-off for each of the strategies on the ordinate. Consequently, if a

[11]The proof, based on Nowak (2006), as well as some extensions can be found in AppendixB.

player encounters strategy B players with a frequency of f his expected pay-off is either $(1-f)a$ if he plays A, or fd if he plays B. In A.) pay-off d is only marginally smaller than a. The equilibrium frequency of strategy B players given by α thus lies close to the 0.5 frequency. The *Force of Random Choice* from stochastic replacement of memories pushes the distribution towards this completely mixed $half : half$ distribution. The *Force of Selection* pushes the distribution to the pure equilibria. It increases with the distance from α and is determined by the vertical gap between the $(a-b)$ and the $(c-d)$ line. We observe that at the completely mixed distribution the Force of Selection is very small in the direction of h_A. Random choice is frequent with respect to the best response play, and so random replacement of old memories will superimpose selection by invasion at the completely mixed distribution. In contrast, the Force of Selection is very strong at the pure equilibria, i.e. both at h_A and h_B, and reduce non-best response play. This is not the case in B.). At the completely mixed distribution the Force of Selection is much stronger in the direction of h_A, hence, the Force of Selection is sufficiently strong to push the population to h_A. In addtion, at h_B the Force of Selection is very weak in the direction of h_A, and random choice will favour a transition out of the basin of attraction of h_B, after which point a transition to h_A will occur.

In other words, stochastic stability is a necessary condition for a stable convention, but will not suffice in the case of a high stochastic error rate. If the basin of attraction of all equilibria is insufficiently large, i.e. smaller than 2/3 of the distance between both equilibria, and hence the transition has a reduced resistance larger than 1/3, the stochastic strategy choice does not favour any strategy. The question to be answered is indeed, whether or not it is a reasonable assumption to expect a low mutation rate, when explaining the evolution of conventions. The higher the risk and hence the potential loss, the longer a convention will persist, since its sample size will be proportionally greater and its error size proportionally smaller. The idea that a society shifts between equilibria, however, requires a population to move through the interior equilibrium during the turnover process. Only if selection based on best response play is strong enough, the population will be able to attain the Stochastically Stable State.[12]

Notice, however, that the one-third rule applies to the extreme case, in which error rate is generally considered to be large. This will not be the case close to the pure equilibrium states, at which only very few players experiment, and whenever the potential loss is high. In these cases, the minimum basin of attraction lies between two thirds and one half. A solution to this issue is postponed to later research. Yet, it may be prudent to constrain consideration to direct or indirect paths with edges of reduced resistances smaller than one third as valid potential paths towards a convention defining the stochastic potential.

4. Discussion and Conclusion

The discussion in this article does not fundamentally challenge the application of the stochastic stability approach as an equilibrium refinement mechanism, since it has found identical results under specific conditions; yet it questions the *general* viability of the original results. In the context of state dependence, findings in this article contrast with the original stochastic stability approach of Young in two important points: First, we observe that Stochastically Stable States, defined by the

[12]Remember that in the underlying approach, transition only occurs spontaneous and involuntarily, and is not subject to conscious and deliberate (revolutionary) choice; or following Carl Menger, the institutions in this approach are purely organic.

state dependent and independent approach, do not necessarily coincide. Both approaches predict the same *SSS* in the case of some symmetry between player types in the pay-off matrix or if error and sample rates are related to the *relative* potential loss that occurs if players do not coordinate. Yet, in the general case the *state dependent SSS* might not be the one predicted by the original approach.

Second, the correlation between error and sample rate, and individual pay-off casts doubt on whether all possible paths towards an equilibrium can be taken into account, when calculating the *SSS*. If a path is defined by two equilibria that generate approximately the same average pay-offs and thus risk rates, one Nash /conventional strategy cannot successfully invade another. It turns out that in this context, minimum stochastic potential is a necessary, but not a sufficient condition for an *SSS*.

There are two possible interpretations of the one-third rule and thus extensions of the stochastic stability criterion that instantly come to mind. Either the rule implies that a Nash equilibrium with resistance larger than one third cannot be invaded by another Nash strategy. It follows that any path (i.e. edge) with too high resistance is considered invalid and an i-tree can only be constituted by valid edges. This might eventually lead to a situation, in which no equilibrium is *SSS* as no valid i-tree exists. Alternatively, the rule also entails that if the unstable interior equilibrium is sufficiently close to the centre (i.e. at $1/2$; the $50:50$-state), random choice superimposes selection. The player population will be continuously drawn back to the interior. Hence, a completely mixed interior equilibrium at $1/2$ can be considered as a potential *SSS*, if it lies on such an invalid path. Which interpretation applies depends on whether a population is a priori located in a mixed state and then moves to an equilibrium, or if it starts out from a convention and then shifts to another equilibrium.[13]

The article further illustrates the difference between sample and error size. In the case of more than one player type, both variables affect the stochastic potential of an equilibrium in a different way. This difference is of special interest for the determination of an *SSS* for general games without any symmetry in the pay-off matrix. The sole focus on a state dependent error rate is insufficient in these cases. The state dependence of both variables can affects the player type that provokes the transition. It not generally the case, as has been in the original approach, that the player type that has more to lose from a shift in convention causes it (for a detailed illustration of this issue, see Bowles, 2006, Ch. check).

Yet, the one-third rule only defines the extreme assumption, in which conventions are completely subject to random choice (i.e. random replacement of memories). Usually, this is the case for a small region around the interior equilibrium. One half and one third are consequently the upper and lower bound of the condition that defines the maximum viable reduced resistance. The actual threshold value should be adapted to the type of game and its context. It is therefore necessary to expand the rule in a way that defines a contextual threshold value for the resistances that lies between both values.

Further notice that the results obtained here also refer to coordination games with a larger set of strategies. In the case of more pure Nash equilibria, the reduced resistances are given by the minimum sum of the resistances of the edges of each directed graph towards an equilibrium in the set of all i-trees. Yet, as is done in Young's approach, a mere summation of the least resistances

[13] The former is the case if players have no knowledge of a convention and choose their first strategy at random. The latter is the case if a convention exists and a historical change discloses an alternative convention.

without weighting them seems problematic. If a population moves along an indirect path, it will spend time in the basin of attraction of an equilibrium that lies on that path, making it a temporal convention. During this time, the population will play a strategy profile close to that dictated by the equilibrium. This might be one that strongly inhibits idiosyncratic play and will be robust against individual errors. The state dependent error and sample size thus also applies to the resistances along the indirect paths. The approach described herein is applicable to games with larger strategy sets and more equilibria by weighting each individual resistances with the sample or error size according to its convention.

AppendixA. Proofs for Stochastic Stability

An example will help to understand the intuition before coming to the proofs . In order to simplify as much as possible, for the length of this example, I will abstract from the loss - error rate relation 4 on page 6, and from the assumption of a relatively large sample size as well as the condition that $s \leq \frac{m}{2}$. (The example will also make it evident why this has been initially assumed.)

Example: Consider two players, who meet each other on a narrow road once a day, and have to decide whether to cross on the left or right. Hence, they play a 2×2 coordination game. Assume that players have a very short memory and remember only the last 2 moves ($m_{i,t} = (x_{j,t-1}, x_{j,t})$). Memory size is identical to sample size. Each state of the game can thus be represented by a vector of four components ($h_t = (m_i, m_j)$). Further assume that players are symmetric, therefore $h_t = (m_i, m_j) = (m_j, m_i)$. The 10 possible states are then defined as *(ll,ll), (ll,lr), (ll,rl), (ll,rr), (lr,lr), (lr,rl), (lr,rr), (rl,rl), (rl,rr), and (rr,rr)*. Each player chooses his best response to his memory of the opponent's last two actions. Obviously *(ll,ll)* and *(rr,rr)* are absorbing states, as the best response to rr is always r and to ll always l. Assume that both equilibria provide the same strictly positive pay-off, and that mis-coordination gives zero pay-off. In the case, in which a player has a "mixed memory" of the opponent's play, i.e. rl or lr, he chooses l or r both with probability $\frac{1}{2}$. In the unperturbed Markov process, states *(ll,ll)* or *(rr,rr)* will persist forever, once they are reached. State *(ll,lr)* will move to state *(ll,rl)* or *(lr,rl)*, each with probability $\frac{1}{2}$.

Now assume that a player commits an error with a low probability and does not choose his best response strategy. Let the case, in which he has memory ll and chooses r, occur with probability λ and the second case, in which he has memory rr and chooses l, occur with probability ε. Let the states' position be as in the previous enumeration, starting with *(ll,ll)* and ending with *(rr,rr)*. The transition matrix of the perturbed Markov process is then defined as in matrix A.1

$$P^{\varepsilon} = \begin{pmatrix} (1-\lambda)^2 & 2(1-\lambda)\lambda & 0 & 0 & \lambda^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & (1-\lambda)/2 & \lambda/2 & 0 & (1-\lambda)/2 & \lambda/2 & 0 & 0 & 0 \\ (1-\lambda)/2 & \frac{1}{2} & 0 & 0 & \lambda/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \varepsilon(1-\lambda) & \varepsilon\lambda & 0 & (1-\varepsilon)(1-\lambda) & (1-\varepsilon)\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon/2 & \frac{1}{2} & (1-\varepsilon)/2 \\ \frac{1}{4} & \frac{1}{2} & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \varepsilon/2 & \varepsilon/2 & 0 & (1-\varepsilon)/2 & (1-\varepsilon)/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon^2 & 2(1-\varepsilon)\varepsilon & (1-\varepsilon)^2 \end{pmatrix} \tag{A.1}$$

$P^{\sigma} = \lim_{n \to +\infty; \varepsilon, \lambda \to 0} P^{\varepsilon}$ defines the limit distribution with ε and λ approaching zero at the same rate. If $\lambda = \varepsilon$ each row vector of P^{σ} has components $(0.5 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0.5)$. Thus, both equilibrium states occur with equal probability. If $\lambda < \varepsilon$ state *(*ll,ll) is *SSS*, if $\lambda > \varepsilon$ state *(*rr,rr) is *SSS*.[14]

If we assume that equilibrium *(*l,l) generates a larger pay-off than equilibrium *(*r,r), all states, except *(*rr,rr), will converge to state *(*ll,ll) in the unperturbed Markov process. Ceteris paribus, the transition matrix looks as in matrix A.2

[14] e.g. if $\varepsilon = 0.0001$ and $\lambda = \varepsilon^{1.5}$, the population remains in state *(*ll,ll) almost all time (99%) and basically never in state *(*rr,rr) (< 1%).

$$
P^{\varepsilon} = \begin{pmatrix}
(1-\lambda)^2 & 2(1-\lambda)\lambda & 0 & 0 & \lambda^2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & (1-\lambda)^2 & (1-\lambda)\lambda & 0 & (1-\lambda)\lambda & \lambda^2 & 0 & 0 & 0 \\
(1-\lambda)^2 & 2(1-\lambda)\lambda & 0 & 0 & \lambda^2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \varepsilon(1-\lambda) & \varepsilon\lambda & 0 & (1-\varepsilon)(1-\lambda) & (1-\varepsilon)\lambda & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & (1-\lambda)^2 & 2(1-\lambda)\lambda & \lambda^2 \\
0 & 0 & (1-\lambda)^2 & (1-\lambda)\lambda & 0 & (1-\lambda)\lambda & \lambda^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon(1-\lambda) & (1-\varepsilon)(1-\lambda)+\varepsilon\lambda & (1-\varepsilon)\lambda \\
(1-\lambda)^2 & 2(1-\lambda)\lambda & 0 & 0 & \lambda^2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \varepsilon(1-\lambda) & \varepsilon\lambda & 0 & (1-\varepsilon)(1-\lambda) & (1-\varepsilon)\lambda & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon^2 & 2(1-\varepsilon)\varepsilon & (1-\varepsilon)^2
\end{pmatrix} \tag{A.2}
$$

Since state *(rr,rr)* has no basin of attraction for $m = 2$, we cannot calculate the resistances for various pay-offs. Yet, a change in the relative error size can still shift the *SSS*. For $\lim_{n\to+\infty} P'^{\varepsilon,\lambda}$, $\varepsilon = 0.00001$ and $\varepsilon^{\frac{1}{5}} = \lambda$ each row vector is defined by approximately (0.03 0.01 0.01 0.00 0.00 0.00 0.00 0.00 0.00 0.96). We observe that though state *(ll,ll)* is risk dominant, the players will spend approximately 96% of the time in state *(rr,rr)*.[15]

Proof of Proposition (1). (This proof is with the exception of minor changes identical to the one of Young, 1998, Theorem 4.1) Let G be a 2×2 coordination game with the corresponding conventions (pure Nash equilibria) $h_A = (A, A)$ and $h_B = (B, B)$. Let $\mathfrak{B}_i$, with $i = A, B$ represent the equilibria's basins of attraction. In addition, let the pay-offs of the game be symmetric. Assume that sample size is dependent on the pay-offs at the current convention. Hence, as long as the population is inside the basin of attraction of convention h_A, players sample at a size $s(A)$, in the case they are in $\mathfrak{B}_B$, sample size is $s(B)$. Further, let the memory m be sufficiently large ($s(\omega) \leq m/2$). Let r_{AB} denote the reduced resistance for every path on the z-tree from h_A to h_B as a function of the sample size $s(A)$. Since after entering $\mathfrak{B}_B$ the system converges to h_B without further errors, r_{AB} is the same as the reduced resistance for all paths from h_A to $\mathfrak{B}_B$. Let α be defined as above and suppose that the population is in h_A for a sufficiently large time, so that all players have chosen strategy A for m periods in succession. For a player to choose strategy B and for the system to enter $\mathfrak{B}_B$ there must be at least $\alpha s(A)$ times strategy B in the player's sample. This can only happen with positive probability if $\alpha s(A)$ players successively commit the error of choosing action B. The probability of this to occur is at least $\varepsilon^{\alpha s(A)}$. The same logic holds for convention h_B, only that $(1-\alpha)s(B)$ players successively have to make the mistake. This event then happens with order $\varepsilon^{(1-\alpha)s(B)}$. It follows that the resistance from h_A to h_B is thus $r^s_{AB} = \alpha s(A)$ and from h_B to h_A is $r^s_{BA} = (1-\alpha)s(B)$. h_A is stochastically stable iff $r^s_{AB} \geq r^s_{BA}$.

Proof of Proposition (2). Assume the same conditions as before except that row players have sample size $s_1(A)$ near h_A and $s_1(B)$ near h_B, and the column players have sample size $s_2(A)$ and $s_2(B)$ respectively and pay-offs are not necessarily symmetric (i.e. interaction pairs are given by one row and one column player). Keep in mind that α refers to the share of column players and β the share of row players. Hence, a row player 1 currently playing strategy $x_1 = A$ will only change strategy if there is a sufficient number of column players playing $x_2 = B$ in his sample. For a positive probability of this to happen there must be at least $\alpha s_1(A)$ players committing an error

[15] Notice that, however, in this example the *SSS* will ultimately switch to *(ll,ll)* as $\varepsilon \to 0$, since *(rr,rr)* has no basin of attraction. A larger memory of 3 would require a transition matrix of size 36×36

in subsequent periods, occurring with probability $\varepsilon^{\alpha s_1(A)}$. For a column player 2 with $x_2 = A$ to switch there must be a sufficient number of row players playing $x_1 = B$ in his sample. Hence, there must be again at least $\beta s_2(A)$ of these players in m, happening with probability of at least $\varepsilon^{\beta s_2(A)}$. The same reasoning holds for the transition from h_B to h_A. Hence, $r^s_{AB} = \alpha s_1(A) \wedge \beta s_2(A)$ and $r^s_{BA} = (1-\alpha)s_1(B) \wedge (1-\beta)s_2(B)$.

Proof of Proposition (3). Now suppose that the rate of mutation is $\epsilon(A) = \varepsilon^{\gamma(A)}$ in $\mathfrak{B}_A$ and $\epsilon(B) = \varepsilon^{\gamma(B)}$ in $\mathfrak{B}_B$ and that pay-offs are symmetric. Assume that sample size is constant and normalised at $s(A), s(B) = 1$, thus is state and pay-off independent. Other conditions are equal to the first proof. Starting in h_A for a system to enter $\mathfrak{B}_B$ with positive probability, again a share of α players successively has to commit the error of choosing action B. For a player to change strategy from A to B there must be thus at least αs players playing strategy B in m, in order to sample a share of α B players with positive probability. By the same logic as above this event occurs with probability $\varepsilon^{\gamma(A)\alpha}$. Congruently, a switch from h_B to h_A happens with probability $\varepsilon^{\gamma(B)(1-\alpha)}$. The resistance from h_A to h_B is thus $r^{\gamma}_{AB} = \gamma(A)\alpha$ and from h_B to h_A is $r^{\gamma}_{BA} = \gamma(B)(1-\alpha)$.

Proof of Proposition (4). As in the second proof suppose that pay-offs are not necessarily symmetric and that there exist two inter-acting types of players with state dependent error size $\epsilon_i(\omega)$. Row players have error size $\varepsilon^{\gamma_1(A)} = \epsilon_1(A)$ near h_A and $\varepsilon^{\gamma_1(B)} = \epsilon_1(B)$ near h_2, and column players have error size $\varepsilon^{\gamma_2(A)} = \epsilon_2(A)$ and $\varepsilon^{\gamma_2(B)} = \epsilon_2(B)$ respectively. For convenience assume that sample rate is normalised to $s(A), s(B) = 1$. A row player 1 currently playing strategy A will only change his strategy if there is a sufficient number of column players playing B, i.e. if he encounters a proportion of at least α column players choosing strategy B in his sampled set. For this event to happen with positive probability, there must be $s_1(A)\alpha$ of this column players in m. For a normalised sample size $s_{1,2} = 1$ this happens with a probability of $\epsilon_2(A)^{\alpha} = \varepsilon^{\alpha\gamma_2(A)}$. A column player has to meet a portion of β row players erroneously playing strategy B. Hence, there must be at least βs_2 such players in m, which occurs with probability $\epsilon_1(A)^{\beta} = \varepsilon^{\beta\gamma_1(A)}$. For h_B the argument is analogous. Thus $r^{\gamma}_{AB} = \gamma_2(A)\alpha \wedge \gamma_1(A)\beta$ and $r^{\gamma}_{BA} = \gamma_2(B)(1-\alpha) \wedge \gamma_1(B)(1-\beta)$.

If we define the state dependent sample size as in the proof of proposition 3 and error size as in proposition 4, it follows that in the case of both state dependent error and sample size the least resistances are given by equation 10 on page 8.

Proof of Proposition (5). Assume condition 4 on page 6 holds, again defined as:

$$\begin{aligned} l_i(\omega) < l_j(\omega') &\Leftrightarrow \epsilon_i(\omega) > \epsilon_j(\omega') \Leftrightarrow s_i(\omega) < s_j(\omega') \\ &\Leftrightarrow \gamma_i(\omega) < \gamma_j(\omega'), \quad \text{for } i,j = 1,2. \end{aligned} \tag{A.3}$$

Assume the general case of 2×2 conflict-coordination games, with the asymmetric pay-off structure as in matrix A.4.

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} a_{11}, b_{11} & a_{12}, b_{12} \\ a_{21}, b_{21} & a_{22}, b_{22} \end{pmatrix} \end{array} \Rightarrow \begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} a, b & 0, 0 \\ 0, 0 & c, d \end{pmatrix} \end{array} \tag{A.4}$$

The first pay-off matrix is equivalent to the second by transformation, given that $a = a_{11} - a_{21}$, $b = b_{11} - b_{12}$, $c = a_{22} - a_{12}$ and $d = b_{22} - b_{21}$. The definition in the right matrix will be used in

the following, as the transformation will not affect the loss size and thus results, but will simplify notation. For this pay-off matrix the frequencies are given by $\alpha = \frac{a}{a+c}$, and $\beta = \frac{b}{b+d}$. Define a positive, continuous and strictly increasing function μ and η, such that $s_i(\omega) = \mu(l_i(\omega))$, and $\gamma_i(\omega) = \eta(l_i(\omega))$. If for both player types the same equilibrium risk dominates, the solution is trivial. For $a > c$ and $b > d$, it always holds that
$min\{\alpha s_1(A); \beta s_2(A)\} > min\{(1-\alpha)s_1(B); (1-\beta)s_2(B)\}$ and also
$min\{\alpha\gamma_2(A); \beta\gamma_1(A)\} > min\{(1-\alpha)\gamma_2(B); (1-\beta)\gamma_1(B)\}$. Hence, h_A is *SSS*. The inverse holds for $a < c$ and $b < d$.

For $a > c$ and $d > b$ we obtain $\alpha > 1-\alpha$ and $1-\beta > \beta$. Hence, $\alpha > \beta$ and $1-\beta > 1-\alpha$. Consequently, there are two possibilities. Either $\beta > 1-\alpha$ (h_A is *SSS*) or $\beta < 1-\alpha$ (h_B is *SSS*).

State dependent sample rate: Define as before that $s_i(\omega) = \mu(l_i(\omega)$. Then by assumption $\mu(a)\alpha > \mu(c)(1-\alpha)$ and $\mu(b)\beta < \mu(d)(1-\beta)$. Under these conditions four cases can occur:
1. case: If $\mu(a)\alpha < \mu(b)\beta$, then $c < a < b < d$ and thus, $\mu(c)(1-\alpha) < \mu(d)(1-\beta)$. In this case h_A is *SSS*.
2. case: If $\mu(c)(1-\alpha) > \mu(d)(1-\beta)$,then $b < d < c < a$ and thus, $\mu(a)\alpha > \mu(b)\beta$. In this case h_B is *SSS*.
Hence, the results for the state dependent sample size do not necessarily coincide with the state independent case.
3. case: The indeterminate case occurs, if $\mu(a)\alpha > \mu(b)\beta$ and $\mu(c)(1-\alpha) < \mu(d)(1-\beta)$. Depending on the relative size of b and c and the order of $\mu(l_i(\omega)$ the state dependent solution will differ from the original approach.
4. case: A contradiction occurs, if $\mu(a)\alpha < \mu(b)\beta$ and $\mu(c)(1-\alpha) > \mu(d)(1-\beta)$. The case contradicts with the assumption that $a > c$ and $d > b$.

As a result only if $\mu(a)\alpha < \mu(b)\beta$ and $\beta > 1-\alpha$, and if $\mu(c)(1-\alpha) > \mu(d)(1-\beta)$ and $\beta < 1-\alpha$, the state dependent and independent results coincide.

In the case of state dependent error rate: As before define $\eta(l_i(\omega)) = \gamma_i(\omega)$. The reduced resistances are then given by $r^{\gamma}_{AB} = \eta(b)\frac{a}{a+c} \wedge \eta(a)\frac{b}{b+d}$ and $r^{\gamma}_{BA} = \eta(d)\frac{c}{a+c} \wedge \eta(c)\frac{d}{b+d}$. Without further assumptions on $\eta(l_i(\omega))$ no definite results can be obtained.

Assume that both $\hat{\mu}(l_i(\omega))$ and $\hat{\eta}(l_i(\omega))$ are defined as such that they are not subject to any positive affine transformation, thus $s_1(A) = \hat{\mu}(\alpha)$ and $\gamma_1(A) = \hat{\eta}(\alpha)$, and define the remaining sample and error rates equivalently. This implies that a player only regards his potential loss in relative terms and not in absolute pay-offs. For the state dependent sample size the resistances are $r^s_{AB} = \hat{\mu}(\alpha)\ \alpha \wedge \hat{\mu}(\beta)\ \beta$ and $r^s_{BA} = \hat{\mu}(1-\alpha)\ (1-\alpha) \wedge \hat{\mu}(1-\beta)\ (1-\beta)$. For $1-\alpha < \beta$, it follows that h_A is *SSS*; for $1-\alpha > \beta$, it is obtained that h_B is *SSS*.

For the state dependent error size the resistances are thus $r^{\gamma}_{AB} = \hat{\eta}(\beta)\ \alpha \wedge \hat{\eta}(\alpha)\ \beta$ and $r^{\gamma}_{BA} = \hat{\eta}(1-\beta)\ (1-\alpha) \wedge \hat{\eta}(1-\alpha)\ (1-\beta)$. Hence, for $1-\alpha < \beta$ and given the former assumptions, it must be that $\alpha > 1-\beta > \beta > 1-\alpha$ and thus $min\{\hat{\eta}(\beta)\ \alpha; \hat{\eta}(\alpha)\ \beta\} > min\{\hat{\eta}(1-\beta)\ (1-\alpha); \hat{\eta}(1-\alpha)\ (1-\beta)\}$. As a consequence, it follows that h_A is *SSS*. In the same way, if $1-\alpha > \beta$ it must hold that $r^{\gamma}_{AB} < r^{\gamma}_{BA}$ and h_B is *SSS*.

Given that pay-offs are *symmetric* entails that $a = b$ and $c = d$. Without loss of generality, assume that $a > c$ and thus $b > d$. Since, $\alpha = \beta > 1-\alpha = 1-\beta$ is follows that $h_A = SSS$. In the case of state dependent sample size we obtain $\mu(a)\ \alpha = \mu(b)\ \beta\ >\ \mu(c)\ (1-\alpha)\ = \mu(d)\ (1-\beta)$, and $h_A = SSS$. In the case of state dependent error size, it holds that $\eta(b)\ \alpha = \eta(a)\ \beta\ >\ \eta(d)\ (1-\alpha) =$

$\eta(c)\ (1-\beta)$, and $h_A = SSS$.

If pay-offs are *double symmetric*, then $a = d$ and $= c$. Without loss of generality, assume that $a > c$, thus $d > b$, leading to $\alpha = 1-\beta > 1-\alpha = \beta$, and hence both equilibria are SSS. In the case of state dependent sample size it holds that $\mu(a)\alpha = \mu(d)\ (1-\beta)\ >\ \mu(c)\ (1-\alpha) = \mu(b)\ \beta$, and both are SSS. In the case of state dependent error size $\eta(b)\ \alpha = \eta(c)\ (1-\beta)\ >\ \eta(d)\ (1-\alpha) = \eta(a)\ \beta$, and both are SSS.

Given a 2×2 conflict games, with a *mirror-symmetric* pay-off structure as in matrixA.5:

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} a,d & b,c \\ c,b & d,a \end{pmatrix} \end{array} \tag{A.5}$$

Assume without loss of generality that $a > d > b, c$, then $r_{AB} = \beta$ and $r_{BA} = (1-\alpha)$. Equilibrium (A,A) will be the *SSS*, iff $d-c > d-b$, hence iff $c < b$. Assume further that this is the case, then $l_1(A) = a-c, l_2(A) = d-c, l_1(B) = d-b, l_2(B) = a-b$. For the state dependent error size $s_1(A) > s_2(A) > s_1(B)$ and $s_1(A) > s_2(B)$, since A.3 holds. Hence $r_{AB} = \mu(d-c)(\frac{d-c}{(.)})$ and $r_{BA} = \mu(d-b)(\frac{d-b}{(.)})$, where $(.) = (a-b-c+d)$. Since $d-c > d-b$, equilibrium (A, A) will be *SSS*. The same argument holds for $c > b$, in which case h_B is *SSS*.

In the case of state dependent error size and for assumption $c < b$, we obtain $\gamma_1(A) > \gamma_2(A) > \gamma_1(B)$ and $\gamma_1(A) > \gamma_2(B)$, and thus $r^{\gamma}_{AB} = \eta(d-c)(\frac{a-c}{(.)}) \wedge \eta(a-c)(\frac{d-c}{(.)})$ and $r^{\gamma}_{BA} = \eta(a-b)(\frac{d-b}{(.)}) \wedge \eta(d-b)(\frac{a-b}{(.)})$. Hence, if $c < b$ it must hold that $\min\left\{\eta(d-c)(\frac{a-c}{(.)}); \eta(a-c)(\frac{d-c}{(.)})\right\} > \min\left\{\eta(a-b)(\frac{d-b}{(.)}) \wedge \eta(d-b)(\frac{a-b}{(.)})\right\}$ and h_A is the *SSS*. By the same reasoning, for $c > b$ it holds that $r^{\gamma}_{AB} < r^{\gamma}_{BA}$ and thus h_B is the *SSS*.

Consequently, in the case losses are considered relative and are independent of a positive pay-off transformation that does not change the game structure, state dependence confirms the results obtain in the standard approach. This is not necessarily the case for any function of the sample and error size, if pay-offs show no form of symmetry.

Example: A short example will illustrate these results. Suppose the following pay-off matrix:

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} 16,6 & 0,0 \\ 0,0 & 10,8 \end{pmatrix} \end{array} \tag{A.6}$$

Hence, $\alpha = \frac{8}{13}$, $(1-\alpha) = \frac{5}{13}$, $\beta = \frac{3}{7}$, and $1-\beta = \frac{4}{7}$. As a result it holds, that $h_A = SSS$. For the general case we obtain $r^s_{AB} = \mu(6)\ \frac{3}{7}$ and for h_A to be *SSS* under the assumption of state dependent sample size it must hold that $\mu(6)\ \frac{3}{7} > \mu(10)\ \frac{5}{13}$ (and thus also that $\mu(10)\ \frac{5}{13} < \mu(8)\frac{4}{7}$), which is not the case for all functional forms of $\mu(.)$.[16]

For the state dependent error size it holds $r^{\gamma}_{BA} = \eta(8)\ \frac{5}{13}$. Hence, this must be strictly smaller than $\min\left\{\eta(6)\ \frac{8}{13}\ ,\ \eta(16)\ \frac{3}{7}\right\}$, which again is not fulfilled for all functional forms of $\eta(.)$.

If we restrict the form of $\mu(.)$ and $\eta(.)$ to the assumptions above, we obtain: $r^{s'}_{AB} = \min\left\{\hat{\mu}(\frac{8}{13})\ \frac{8}{13}\ ,\ \hat{\mu}(\frac{3}{7})\ \frac{3}{7}\right\}$ and $r^{s'}_{BA} = \min\left\{\hat{\mu}(\frac{5}{13})\ \frac{5}{13}\ ,\ \hat{\mu}(\frac{4}{7})\ \frac{4}{7}\right\}$. Thus, $r^{s'}_{AB} > r^{s'}_{BA}$. Further

[16]Notice that $b < d < c < a$ is not a sufficient condition for $SSS = h_B$, see case 2. above.

$r^{\gamma'}_{AB} = \min\left\{\hat{\eta}(\frac{3}{7})\ \frac{8}{13}\ ,\ \hat{\eta}(\frac{8}{13})\ \frac{3}{7}\right\} > r^{\gamma'}_{BA} = \min\left\{\hat{\eta}(\frac{4}{7})\ \frac{5}{13}\ ,\ \hat{\eta}(\frac{5}{13})\ \frac{4}{7}\right\}$, $r^{\gamma'}_{AB} > r^{\gamma'}_{BA}$. Consequently, in the constrained case, $h_A = SSS$ both for state dependent sample and error size.

Proof of Proposition (6). Given a normalised double-mirror symmetric game with two Nash equilibria

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} \text{a,b} & \text{0,0} \\ \text{0,0} & \text{b,a} \end{pmatrix} \end{array} \tag{A.7}$$

In this case the frequencies are as such that $\alpha = 1-\beta$ and $1-\alpha = \beta$. Hence in $r_{AB} = r_{BA}$ and each equilibrium is *SSS* in the state independent case. Assume without loss of generality that player type 1 (row player) is less risk averse than player type 2 (column player) and that he has a higher surplus in h_A than in h_B and the inverse for type 2, i.e. $a > b$. Since player 1 is less risk averse, it can either be expected that $s_1(\omega) < s_2(\omega')$, or $\gamma_1(\omega) < \gamma_2(\omega')$, where ω and ω' indicate state h_A or h_B. Further, we know that $\alpha > (1-\alpha)$.

In the case of state dependent sample size the resistances are rewritten as: $r^s_{AB} = \alpha\ s_1(A) \wedge (1-\alpha)\ s_2(A)$ and $r^s_{BA} = (1-\alpha)\ s_1(B) \wedge \alpha\ s_2(B)$. It must hold that $s_1(A) > s_1(B)$ and $s_2(B) > s_2(A)$, but also that $s_1(A) < s_2(B)$ and $s_1(B) < s_2(A)$. Hence, $r^s_{BA} = (1-\alpha)\ s_1(B) < r^s_{AB}$. Consequently, h_A is *SSS*. Hence, the less risk averse player type 1 can gain a higher surplus.

For the case of state dependent error size, define two positive and strictly increasing functions u and v as such that $u(.) > v(.)$, $u(0), v(0) = 0$ (from pay-off function 2 on page 2) and $u'(.), v'(.) > 0$. Let $\gamma_1(\omega) = v(l_1(\omega))$ and $\gamma_2(\omega) = u(l_2(\omega))$, and hence, $\gamma_1(A) = v(a)$, $\gamma_1(B) = v(b)$, and $\gamma_2(A) = u(b)$, $\gamma_2(B) = u(a)$. The resistances are $r^\gamma_{AB} = \alpha u(b) \wedge (1-\alpha)v(a)$ and $r^\gamma_{BA} = (1-\alpha)u(a) \wedge \alpha v(b)$.

Four possible outcomes can occur -

1. case: $r^\gamma_{AB} = \alpha u(b)$ and $r^\gamma_{BA} = \alpha v(b)$. From the minimum conditions of the resistances it must be that $\frac{a}{b} < \frac{v(a)}{u(b)} < \frac{u(a)}{v(b)}$. It must further hold that $u(.)$ and $v(.)$ are convex in pay-offs a and b. As $u(.) > v(.)$, $h_A = SSS$.

2. case: $r^\gamma_{AB} = \alpha u(b)$ and $r^\gamma_{BA} = (1-\alpha)u(a)$. In this case it must hold $\frac{u(a)}{v(b)} < \frac{a}{b} < \frac{v(a)}{u(b)}$. This contradicts the assumptions that $u(.) > v(.)$.

3. case: $r^\gamma_{AB} = (1-\alpha)v(a)$ and $r^\gamma_{BA} = \alpha v(b)$. Thus, $\frac{u(a)}{v(b)} > \frac{a}{b} > \frac{v(a)}{u(b)}$. Since $u(.) > v(.)$, no further restrictions on the functions' slope can be derived. For $h_A = SSS$ only if $\frac{a}{b} < \frac{v(a)}{v(b)}$, which holds in the case $v(.)$ is a strictly convex function. If the function is strictly concave $h_B = SSS$.

4. case: $r^\gamma_{AB} = (1-\alpha)v(a)$ and $r^\gamma_{BA} = (1-\alpha)u(a)$. For this inequality to occur, it must be that $\frac{a}{b} > \frac{u(a)}{v(b)} > \frac{v(a)}{u(b)}$. It must further hold that $u(.)$ and $v(.)$ are concave in pay-offs a and b. As $u(.) > v(.)$, $h_B = SSS$.

Hence, if $u(.)$ and $v(.)$ are strictly convex, then h_A is *SSS*. If both functions are strictly concave, then h_B is *SSS*.

AppendixB. The one-third rule

Assume as before a general 2×2 coordination game with two strict Nash equilibria in pure strategies. For simplicity assume that pay-offs are symmetric as in matrix:[17]

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} \text{a,a} & \text{b,c} \\ \text{c,b} & \text{d,d} \end{pmatrix} \end{array} \tag{B.1}$$

Further assume that two players are randomly paired. Since none can observe a priori the other player's strategic choice, each asks s other players among the m players in his population what strategy they have chosen in previous periods. (An alternative interpretation of adaptive play is that the same player faces an identical choice m times during his life, but recalls only for s incidences which strategies has been chosen by his counterparts.) Based on his sample, each chooses the strategy for the current game.

The collective memory of the other players' strategy choice (and his own) during $\frac{m}{2}$ past plays is sufficiently described by a matrix of size $m \times m$, since it resembles the following random Moran process: After each interaction both players retain the memory of the other's current play (alternatively they retain their own strategy), and two "old" memories from previous play are forgotten. Yet, for simplicity this process can be approximated by a sequence in which one new memory is born and added to the memory, an old is lost and deleted from the memory. Thereafter the new memory of the second player is added and an old is forgotten. Hence, each play defines two periods in the birth-death process.[18]

Furthermore the old memory, which "dies", is not necessarily the oldest memory. It is in fact supposed that any memory of previous play can be forgotten with equal probability $\frac{1}{m}$. This relaxation both simplifies the following analysis but also augments the degree of realism. The original approach by Young is overly restrictive. A general assumption that the last element in the collective memory dies, requires players to keep track of the precise order of events. Hence, in the following it is assumed that the death of an element (or rather its omission) is not deterministically, but stochastically defined.

In such a process two different events can occur in each period. Either a strategy in m substitutes an element indicating a different strategy or the same strategy. Hence, the rate, at which a certain strategy has been played in the memory of m players[19] (or $\frac{m}{2}$ previous plays) can either decrease, increase (each by one unit) or remain unchanged. Define i as the number of memories that strategy

[17] Here I will follow mainly the assumptions and proof of Nowak (2006), except for minor adaptations and extensions to the given context.

[18] The reason for these simplifications is the following: If the process is determined by the sequence two births and subsequently two deaths the transition of a state is not bounded to its immediate successor or predecessor (one element more or one element less) but can directly transit to the second-order successor or predecessor (two elements more or two elements less), resulting in 4 state dependent transition probabilities instead of two, which tremendously complicates the following derivation. The reason for the strict pay-off symmetry is that it allows to neglect types. In the general case it is required to define two Moran processes. We then obtain two interdependent systems of equations not easily solvable in closed form. Yet, I believe the illustrative purpose of this section is not upset by these additional assumptions as the intention is to illuminate the general dynamics of the process in the case of a high level of idiosyncratic play.

[19] Assuming that players are only drawn once. Hence, more correctly it is m memories of some number of players. If a player had been drawn twice he retains two memories.

$$T = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ p_{1,0} & 1-p_{1,0}-p_{1,2} & p_{1,2} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & p_{m-1,m-2} & 1-p_{m-1,m-2}-p_{m-1,m} & p_{m-1,m} \\ 0 & 0 & 0 & \dots & 0 & 0 & 1 \end{pmatrix} \tag{B.2}$$

A has been played, i.e. i is equal to the frequency with which strategy A occurs in the history of past plays. Consequently, the probability that an element defining strategy A is forgotten is equal to $\frac{i}{m}$, and similarly for the strategy B, this probability equals $\frac{m-i}{m}$. (One might also interpret this assumption as more unique events being less likely to be forgotten.)

Given the underlying transition probabilities $p_{i,i+1}$ and $p_{i,i-1}$ to move from state i to i+1 and i-1 respectively, the Markov process is defined by a three-diagonal transition matrix of the form of matrix B.2.

Hence, i=0 (corresponding to state h_B) and i=m (corresponding to state h_A) are absorbing states. The fixation probability ρ_x defines the probability that an individual strategy x, mutant to the current convention of $m-1$ states that are only defined by strategy x', can sufficiently proliferate to finally reach the absorbing state, in which all individuals played strategy x in memory m, i.e. the probability of switching conventions. Following Nowak (2006) we obtain:

$$\rho_A = \frac{1}{1+\sum_{j=1}^{m-1}\prod_{k=1}^{j}(\frac{p_{k,k-1}}{p_{k,k+1}})} \tag{B.3a}$$

$$\rho_B = \frac{\prod_{k=1}^{m-1}(\frac{p_{k,k-1}}{p_{k,k+1}})}{1+\sum_{j=1}^{m-1}\prod_{k=1}^{j}(\frac{p_{k,k-1}}{p_{k,k+1}})} \tag{B.3b}$$

Now assume that an individual calculates his pay-off according to matrix B.1 on page 21 and memory size m. If he were capable to draw an unbiased sample from the entire memory, i.e. $distribution(s) = distribution(m)$, and if he expects this to be a representative account of the strategy distribution in the entire population for the coming period, the expected pay-off is simply defined by the relative frequencies of both strategies. Consequently, he expects that the population consists of i individuals playing A and $m-i$ individuals playing B. Define $\Pi_x(i)$ as the expected pay-off that an individual receives, when playing strategy x in such a population, where the frequency of strategy A is defined by i.[20]. The expected pay-offs for each strategy are then given by

$$\Pi_A(i) = \frac{a(i-1)+b(m-i)}{m-1} \tag{B.4a}$$

$$\Pi_B(i) = \frac{ci+d(m-1-i)}{m-1} \tag{B.4b}$$

Adaptive play assumes that sample size is smaller than memory size ($m \geq 2s$) and, thus, distributions do not necessarily coincide. This can also be represented by a new element in m,

[20]Keep in mind that there are $m-1$ other individuals and $i-1$ strategy A players and $m-i$ strategy B players for an individual playing A and similarly for an individual playing B.

which is not deterministically defined solely by equations B.4, but by a stochastic process. An error occurs, when a player draws a skewed sample or idiosyncratically chooses an action at random. The higher the relative expected pay-off of the best response strategy, the more likely this strategy will define the new element in m. This is simply equivalent to inducing an individual to play a non-best response strategy requires a larger share of adverse states (i.e. of non-best strategies) in sample s than the share defined by the unstable interior equilibrium. In other words, the sample needs to be sufficiently skewed, which happens with decreasing probability as the collective memory includes a larger share of states to which the strategy is a non-best response. Since all elements in m are sampled with equal probability, the likelihood of a sufficiently skewed sample will decrease as the relative expected pay-off of the best-response strategy increases in the current state.

Adaptive play thus supposes that with a certain probability $\varepsilon > 0$ individuals choose a response strategy at random, i.e. they experiment. With probability $\varepsilon(l_i(\omega^p))$ the process does not behave according to the relative expected pay-offs defined in B.4, but completely random. In order to weight the intensity of strategic selection, based on expected pay-offs, and the random choice, write the pay-off functions as

$$\pi_A = \varepsilon + (1-\varepsilon)\Pi_A(i) \tag{B.5a}$$

$$\pi_B = \varepsilon + (1-\varepsilon)\Pi_B(i) \tag{B.5b}$$

with $\varepsilon \in (0,1)$. Strong selection is thus defined by $\varepsilon \to 0$, which is the case underlying the intuition of Kandori, Mailath and Rob, and Young. Weak selection occurs in the case of a high error rate $\varepsilon \to 1$, if $l_i(\omega^p) \to 0$. These assumptions allow to define the transition probabilities of the Moran process described in B.2. The probability that a new element in the collective memory defines either strategy A or strategy B is

$$\frac{i\pi_A}{i\pi_A + (m-i)\pi_B} \text{ or } \frac{(m-i)\pi_B}{i\pi_A + (m-i)\pi_B}.$$

Consequently,

$$p_{i,i+1} = \frac{i\pi_A}{i\pi_A + (m-i)\pi_B}\frac{m-i}{m} \tag{B.6a}$$

$$p_{i,i-1} = \frac{(m-i)\pi_B}{i\pi_A + (m-i)\pi_B}\frac{i}{m} \tag{B.6b}$$

For the neutral case of $\varepsilon = 1$, in which selection favours neither A nor B, we obtain a completely random process, such that fixation probability is defined as $\rho_x = \frac{1}{m}$.[21] This is intuitive, since each value in the matrix describing the Moran process, has the same probability to spread over the entire memory. Setting equations B.5a and B.5b into equation B.3a, we obtain

$$\rho_A = \frac{1}{1 + \sum_{j=1}^{m-1}\prod_{k=1}^{j}(\frac{\pi_B}{\pi_A})} \tag{B.7}$$

Based on Nowak (2006), the Taylor expansion for $\varepsilon \to 1$ (or $(1-\varepsilon \to 0)$ gives

$$\rho_A = \frac{1}{m}\frac{1}{1-(\gamma m - \delta)(1-\varepsilon)/6}, \quad \text{with} \tag{B.8}$$
$$\gamma = a + 2b - c - 2d \text{ and } \delta = 2a + b + c - 4d$$

[21]Remember that here $p_{i,i-1} = p_{i,i+1}$.

In order for selection to favour strategy A, its fixation probability must be greater than in the neutral case, i.e. $\rho_A > \frac{1}{m}$ and it must be that $\gamma m > \delta$. Hence,

$$a(m-2) + b(2m-1) > c(m+1) + d(2m-4) \tag{B.9}$$

If the memory size m is sufficiently large, we obtain

$$a - c > 2(d-b) \tag{B.10}$$

Setting the pay-offs from B.4 or B.5 equal, we obtain for a large memory size that the unstable interior equilibrium is defined by a frequency of $\alpha = \frac{a-c}{a-c-b+d}$ players choosing strategy B, defining the basins of attraction for both strict Nash equilibria. By putting in the results of the previous equation B.10, we obtain

$$\alpha > 2/3 \tag{B.11}$$

Thus, in order for equilibrium (A, A) to be a *SSS*, the minimum frequency of idiosyncratic players choosing strategy A to cause a switch of best-response players to this strategy, and thus the reduced resistance to (A,A), is defined by $r_{BA} = 1 - \alpha$, and has to be less than one third in the context of weak selection and high error rate. Redefining yields the same result with respect to the resistance of (B, B), i.e. $r_{AB} < 1/3$, whence proposition 7.

AppendixC. Bibliography

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